ELE617E
Lectures

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Algorithmic Strength Reduction

- It leads to a reduction in HW complexity by exploiting substructure sharing,
- reduce area or power,
- or iteration periode.

L-parallel circuit requires an L $\times$ Area!
Question: to realize parallel FIR filtering structure that consume less area.

Parallel FIR Filters

$$y(n) = h(n) \star x(n) \quad \rightarrow Y(z) = H(z)X(z)$$

$$\begin{aligned} X(z) =& x(0) + x(1)z^{-1} + x(2)z^{-2} + x(3)z^{-3} + x(4)z^{-4} + \dots \\ =& x(0) + x(2)z^{-2} + x(4)z^{-4} + \dots + z^{-1}\{x(1) + x(3)z^{-2} + x(5)z^{-4} + \dots\} \\ =& X_0(z^2) + z^{-1}X_1(z^2) \end{aligned}$$

where $X_0(z^2) = \mathcal{Z}(x(2k))$ and $X_1(z^2) = \mathcal{Z}(x(2k+1))$. $X(z)$ is decomposed into **two** poly phases.

$$H(z) = H_0(z^2) + z^{-1}H_1(z^2)$$

$$H_0(z) = h(0) + h(2)z^{-2} + h(4)z^{-4} + \dots$$

$$H_1(z) = h(1) + h(3)z^{-2} + h(5)z^{-4} + \dots$$

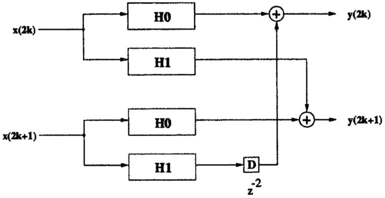
$$\begin{aligned} Y(z) &= (X_0(z^2) + z^{-1}X_1(z^2))(H_0(z^2) + z^{-1}H_1(z^2)) \\ &= \{X_0(z^2)H_0(z^2) + z^{-2}X_1(z^2)H_1(z^2)\} + \{X_0(z^2)H_1(z^2) + X_1(z^2)H_0(z^2)\} \end{aligned}$$

Parallel FIR Filters

$$Y(z) = \{X_0(z^2)H_0(z^2) + z^{-2}X_1(z^2)H_1(z^2)\} + \{X_0(z^2)H_1(z^2) + X_1(z^2)H_0(z^2)\}$$

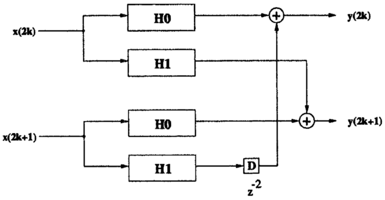
$$\begin{aligned} Y(z) &= Y_0(z^2) + z^{-1}Y_1(z^2) \\ Y_0(z^2) &= X_0(z^2)H_0(z^2) + z^{-2}X_1(z^2)H_1(z^2) \\ Y_1(z^2) &= X_0(z^2)H_1(z^2) + X_1(z^2)H_0(z^2) \end{aligned}$$

$$\begin{bmatrix} Y_0 \\ Y_1 \end{bmatrix} = \begin{bmatrix} H_0 & z^{-2}H_1 \\ H_1 & H_0 \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \end{bmatrix}$$



Parallel FIR Filters

$$\begin{bmatrix} Y_0 \\ Y_1 \end{bmatrix} = \begin{bmatrix} H_0 & z^{-2}H_1 \\ H_1 & H_0 \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \end{bmatrix}$$



- 2-parallel FIR filtering structure.
- 2N MUL and 2(N-1) ADD

L-parallel FIR

$$\begin{bmatrix} Y_0 \\ Y_1 \\ \vdots \\ Y_{L-1} \end{bmatrix} = \begin{bmatrix} H_0 & z^{-L}H_{L-1} & \dots & z^{-L}H_1 \\ H_1 & H_0 & \dots & z^{-L}H_2 \\ \vdots & \vdots & \ddots & \vdots \\ H_{L-1} & H_{L-2} & \dots & H_0 \end{bmatrix} \begin{bmatrix} X_0 \\ X_1 \\ \vdots \\ X_{L-1} \end{bmatrix}$$

Polyphase Filters:

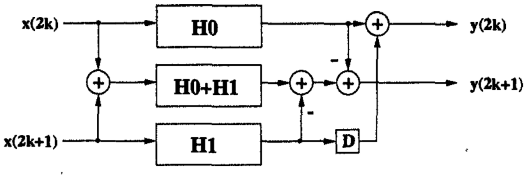
Filter	MUL	ADD	sub-filter
FIR	N	N-1	1
2-Parallel	2N	2(N-1)	4
3-Parallel	3N	3(N-1)	9
L-Parallel	LN	L(N-1)	L^2

However, they give L samples each cycle
See 3-parallel FIR filter implementation Fig. 9.2 (page 259.)

Fast FIR Filters

$$Y(z) = Y_0(z^2) + z^{-1}Y_1(z^2)$$
$$Y_0(z^2) = X_0(z^2)H_0(z^2) + z^{-2}X_1(z^2)H_1(z^2)$$
$$Y_1(z^2) = X_0(z^2)H_1(z^2) + X_1(z^2)H_0(z^2)$$
$$= \{X_0(z^2) + X_1(z^2)\}\{H_0(z^2) + H_1(z^2)\} - X_0(z^2)H_0(z^2) - X_1(z^2)H_1(z^2)$$

PS: $Y_1 = f(X_0, X_1, H_0, H_1, H_0 + H_1, X_0 + X_1)$
Filters: $H_0 = \{h_0, h_2, h_4, h_6\}$, $H_1 = \{h_1, h_3, h_5, h_7\}$,
$H_0 + H_1 = \{h_0 + h_1, h_2 + h_3, h_4 + h_5, h_6 + h_7\}$



$D = z^{-2}$, $3N/2$ MUL and $3(N/2-1)+4$ ADD operation.

Fast FIR Filters

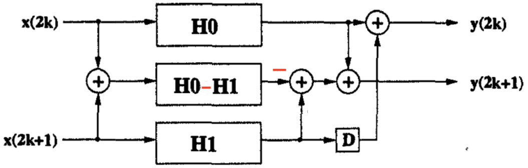
$$Y(z) = Y_0(z^2) + z^{-1}Y_1(z^2)$$
$$Y_0(z^2) = X_0(z^2)H_0(z^2) + z^{-2}X_1(z^2)H_1(z^2)$$
$$Y_1(z^2) = X_0(z^2)H_1(z^2) + X_1(z^2)H_0(z^2)$$
$$= X_0(z^2)H_0(z^2) + X_1(z^2)H_1(z^2) - \{X_0(z^2) - X_1(z^2)\}\{H_0(z^2) - H_1(z^2)\}$$

PS: $Y_1 = f(X_0, X_1, H_0, H_1, H_0 - H_1, X_0 - X_1)$

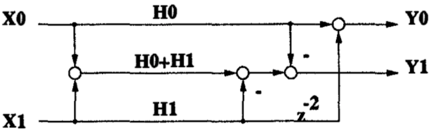
Filters:

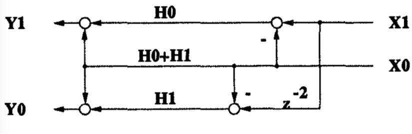
$H_0 = \{h_0, h_2, h_4, h_6\}$, $H_1 = \{h_1, h_3, h_5, h_7\}$,

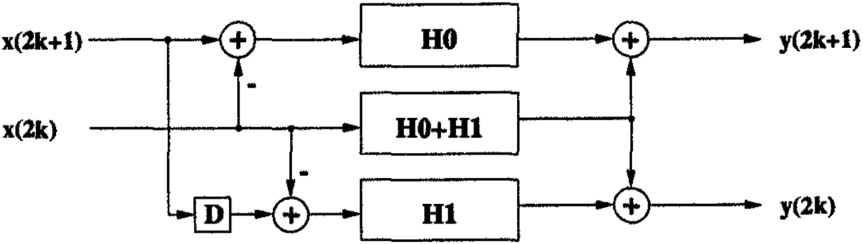
$H_0 + H_1 = \{h_0 - h_1, h_2 - h_3, h_4 - h_5, h_6 - h_7\}$



Transpose of Fast FIR

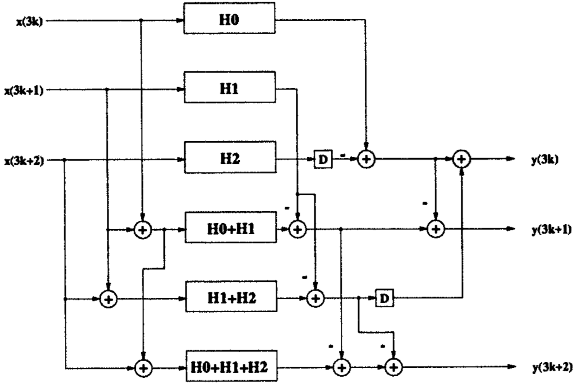






Transposition does not change the complexity of the filter algorithm.

$$Y_0 = X_0H_0 - z^{-3}X_2H_2 + z^{-3}[(H_1 + H_2)(X_1 + X_2) - H_1X_1]$$
$$Y_1 = [(H_0 + H_1)(X_0 + X_1) - H_1X_1] - [H_0X_0 - z^{-3}H_2X_2]$$
$$Y_3 = ...$$



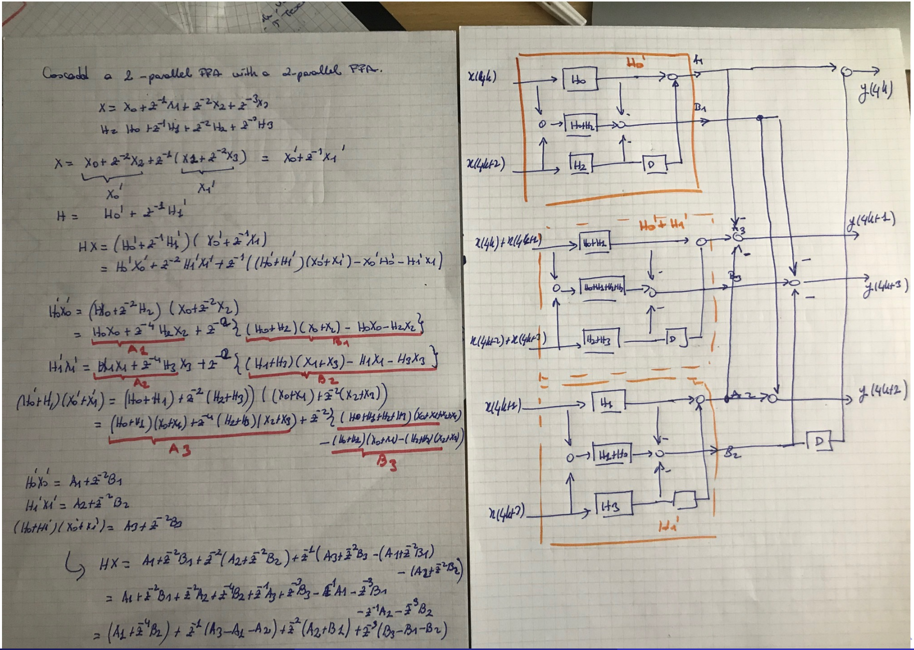
$6(N/3)$ MUL and $6(N/3 - 1) + 10$ ADD operation and %33, 3 saving !

Comparison Polyphase Filters

Filter	MUL	ADD	Reduced MUL	Reduced ADD
FIR	N	N-1		
2-Parallel	2N	2(N-1)	3N/2	3N/2+1
3-Parallel	3N	3(N-1)	2N	2N+4
4-Parallel	4N	4(N-1)	9 N/4	20+9(N-1)

for 4-parallel case : MUL 4N to 9 N/4 hence %xxx

4-parallel FFA



6-parallel FFA