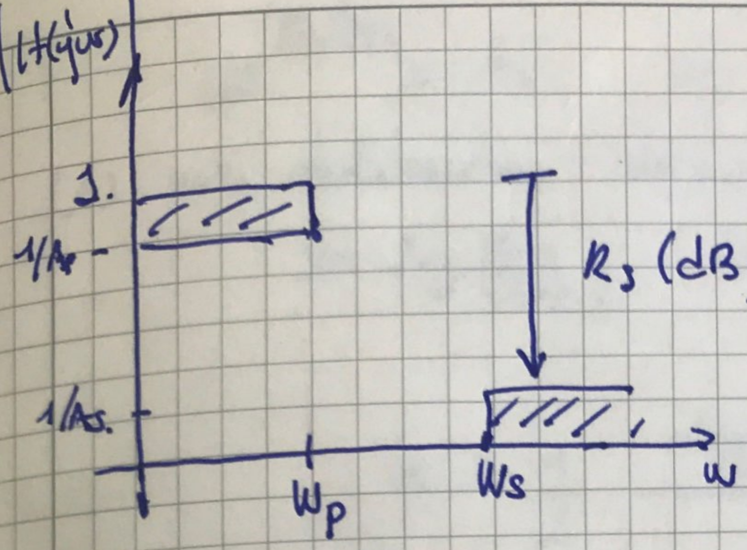


Butterworth $\frac{1}{A_s^2} = \frac{1}{1+\omega_s^{2n}}$ ve $\frac{1}{A_p^2} = \frac{1}{1+\omega_p^{2n}}$

$$A_s^2 = 1+\omega_s^{2n}$$
$$A_p^2 = 1+\omega_p^{2n}$$

$$\frac{A_s^2-1}{A_p^2-1} = \left(\frac{\omega_s}{\omega_p}\right)^{2n}$$

$$2n \log\left(\frac{\omega_s}{\omega_p}\right) = \log\left(\frac{A_s^2-1}{A_p^2-1}\right) \quad (1)$$

$$= \log(A_s^2-1) - \log(A_p^2-1)$$

suppose $A_s >>$ ve $A_p << 1$

$$\Rightarrow 2n \log\left(\frac{\omega_s}{\omega_p}\right) \cong \log A_s^2 - 0$$

$$\boxed{n \log\left(\frac{\omega_s}{\omega_p}\right) \cong \log A_s}$$

ws'de gelen $\frac{1}{A_s}$'e düşmekte

Zayıflama $20 \log A_s = R_s$.

$$\Rightarrow n \cong \frac{R_s}{20 \log(\omega_s/\omega_p)}$$

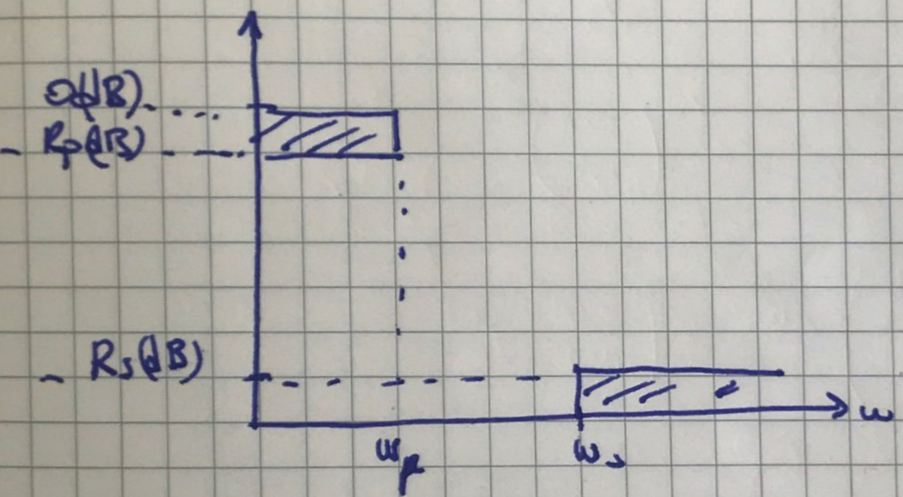
$10^{R_s/20} = A_s$ ve $10^{R_p/20} = A_p$ olmak üzere

(1) nolu denklemde devam edelim.

$$2n \log\left(\frac{w_s}{w_p}\right) = \log\left(\frac{10^{R_s/10}-1}{10^{R_p/10}-1}\right)$$

$$n = \frac{1}{2} \cdot \frac{\log\left(\left(10^{R_s/10}-1\right)/\left(10^{R_p/10}-1\right)\right)}{\log\left(w_s/w_p\right)}$$

BU FORMUL YARDIMIYLA.



grafiği yardımıyla n'nin derecesi bulunabilir.
Bu hesaplamada yukarı yuvarlama unutulmamalıdır.

$$n = \frac{1}{2} \cdot \frac{\log\left[\frac{\left(10^{R_s/10}-1\right)}{\left(10^{R_p/10}-1\right)}\right]}{\log\left(\frac{w_s}{w_p}\right)}$$

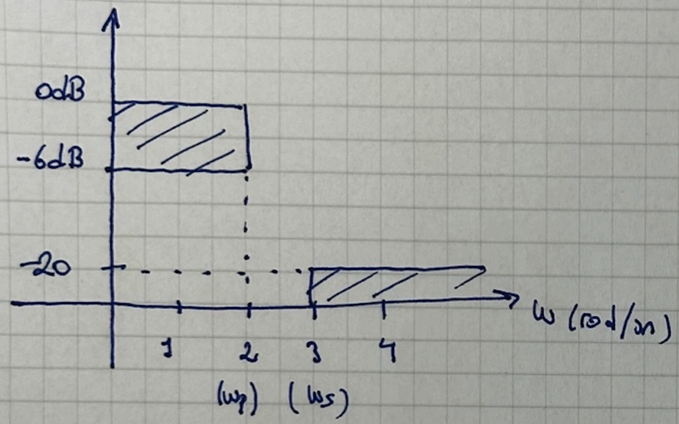
Bir kesim frekansı Butterworth için
$R_p = 3dB$ zayıflamanın olduğu
frekans olarak seçiyoruz.

ÖRNEK

$0 \le w \le 2$ rad/sn için $-6dB \le |H(jw)| \le 0$

3 rad/sn $\le w$ için $|H(jw)| \le -20dB$

istenmektedir. Bu tanıma uygun Butterworth tipi filtrenin derecesini ve kesim frekansına karşılık düşen frekansı hesaplayın.



$$20\log\left(\frac{1}{A_p}\right) = -6dB$$

$$A_p \cong 2$$

$$20\log\left(\frac{1}{A_s}\right) = -10dB$$

$$A_s \cong 10$$

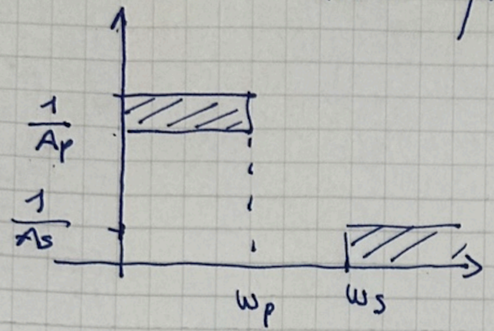
$$n = \frac{1}{2} \cdot \frac{\log\left[\dfrac{(10^{R_s/10}-1)}{(10^{R_p/10}-1)}\right]}{\log\left(\dfrac{w_s}{w_p}\right)}$$

$$= \frac{1}{2} \cdot \frac{\log\left[\dfrac{(10^{20/10}-1)}{(10^{6/10}-1)}\right]}{\log\left(3/2\right)}$$

$$= 4,319$$

$\Rightarrow n=5$ olmalıdır.

Kesim frekansının hesabı:



$$\frac{1}{A_s} = \frac{1}{\sqrt{1+w_s^{2n}}}$$ seçilen $n=5$ için $$\frac{1}{A_s} \geqslant \frac{1}{\sqrt{1+w_s^{10}}}$$

$$1+w_s^{10} \geqslant A_s^2$$

$$w_s^{10} \geqslant A_s^2 - 1$$

$$\frac{1}{A_p} = \frac{1}{\sqrt{1+w_p^{2n}}}$$ seçilen $n=5$ için $$\frac{1}{A_p} \leqslant \frac{1}{\sqrt{1+w_p^{10}}}$$

$$1+w_p^{10} \leqslant A_p^2$$

$$w_p^{10} \leqslant A_p^2 - 1$$

w_c için $$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+w_c^{2n}}}$$

$n=5$ için $$\frac{1}{2} = \frac{1}{1+w_c^{10}}$$

$$w_c^{10} = 1$$

$$\left.\begin{aligned} w_s^{10} &\geqslant A_s^2 - 1 \\ w_c^{10} &= 1 \end{aligned}\right\} \rightarrow \text{oranla} \quad \left(\frac{w_s}{w_c}\right)^{10} \geqslant A_s^2 - 1$$

$$\frac{w_s}{w_c} \geqslant \left(A_s^2 - 1\right)^{1/10}$$

$$w_s \cdot \left(A_s^2 - 1\right)^{-1/10} \geqslant w_c$$

$$\left.\begin{aligned} w_p^{10} &\leqslant A_p^2 - 1 \\ w_c^{10} &= 1 \end{aligned}\right\}$$

$$\left(\frac{w_p}{w_c}\right)^{10} \leqslant A_p^2 - 1$$

$$w_p\left(A_p^2 - 1\right)^{-1/10} \leqslant w_c$$

$$1{,}7931 = w_p\left(A_p^2 - 1\right)^{-1/10} \leqslant w_c \leqslant w_s \cdot \left(A_s^2 - 1\right)^{-1/10} = 1{,}8948$$