Circuit and System Analysis
EHB 232E

Prof. Dr. Müştak E. Yalçın

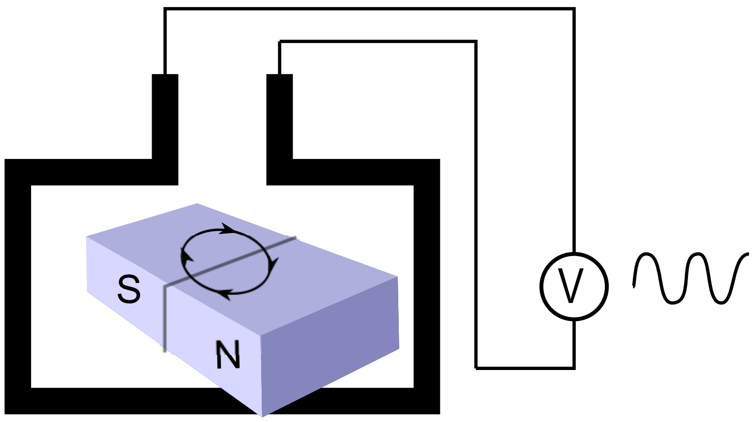
Istanbul Technical University
Faculty of Electrical and Electronic Engineering

mustak.yalcin@itu.edu.tr

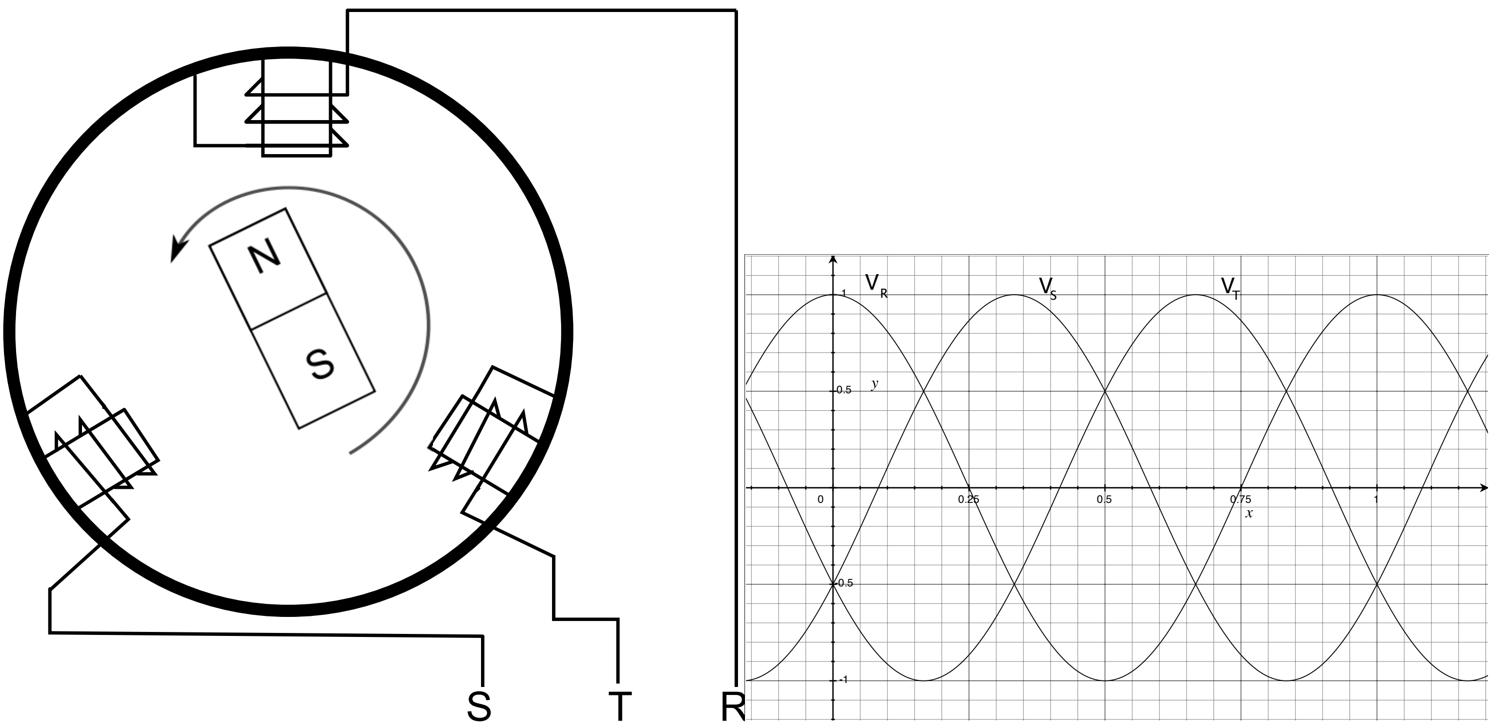
Outline I

1. Three-Phase Systems
   - Y-connected source
   - Line-to-line voltage
   - Delta-connected source
   - Load in Three Phase Circuit
   - Δ-to-Y
   - Power calculation in balanced three-phase circuits

Three-Phase Systems

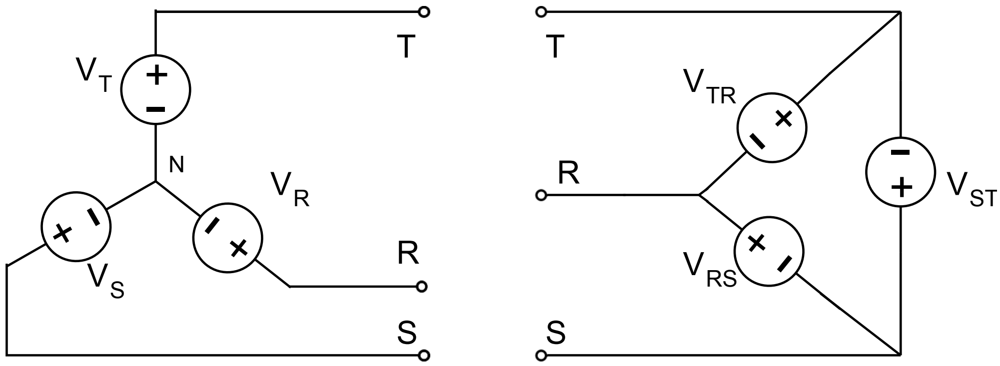


Three-Phase Sources



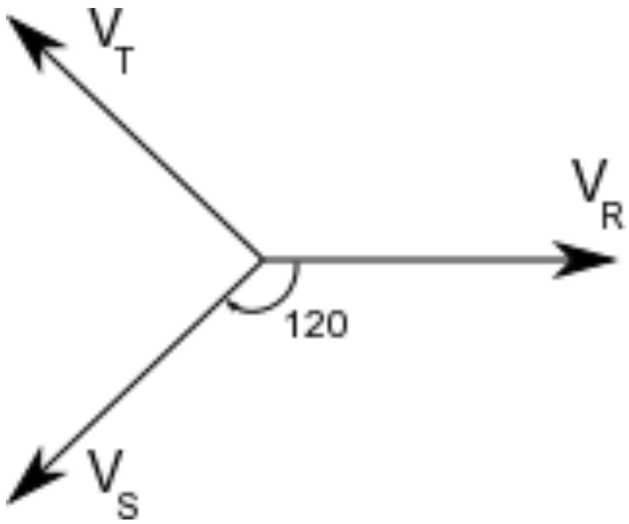
Three-Phase Sources

There are two ways of interconnecting the separate phase windings to form a three phase source in either a wye (Y) or a delta (Δ) configuration.



Phase Voltages of Y-connected source are the line-to-neutral voltages.
Phase Voltages of Δ-connected source are the line-to-line voltages.

Y-connected source



Phase Voltage of Y-connected source (The line-to-neutral voltages)

$$v_R(t) = V_{RM}\cos(wt - \theta_R)$$
$$v_S(t) = V_{SM}\cos(wt - \theta_S)$$
$$v_T(t) = V_{TM}\cos(wt - \theta_T)$$

Y-connected source

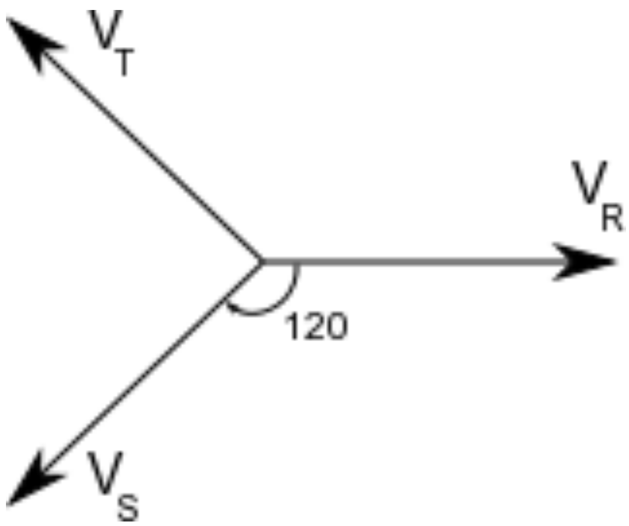
Balanced Three-Phase Voltage Sources

- $$V_{RM} = V_{SM} = V_{TM}$$
- $$\theta_S = \theta_R - 120^\circ$$
  $$\theta_T = \theta_S - 120^\circ$$

$$V_R = V_m e^{j\theta_R}$$
$$V_S = V_m e^{j\theta_S}$$
$$V_T = V_m e^{j\theta_T}$$

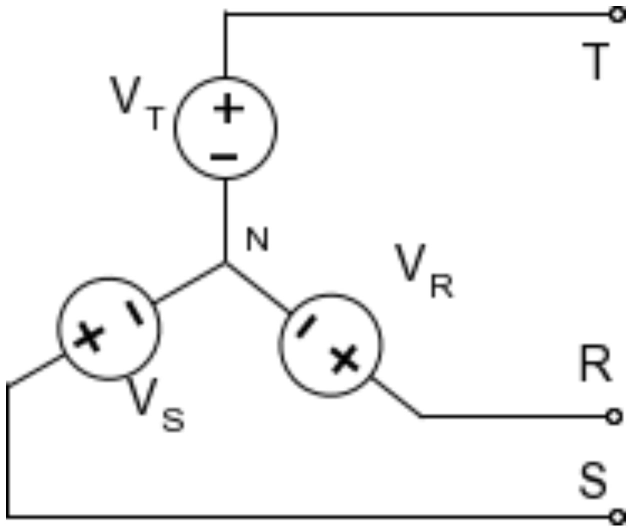
Phasor Diagrams

RST or abc phase sequence !



$$V_R + V_S + V_T = ?$$

Line-to-line voltage

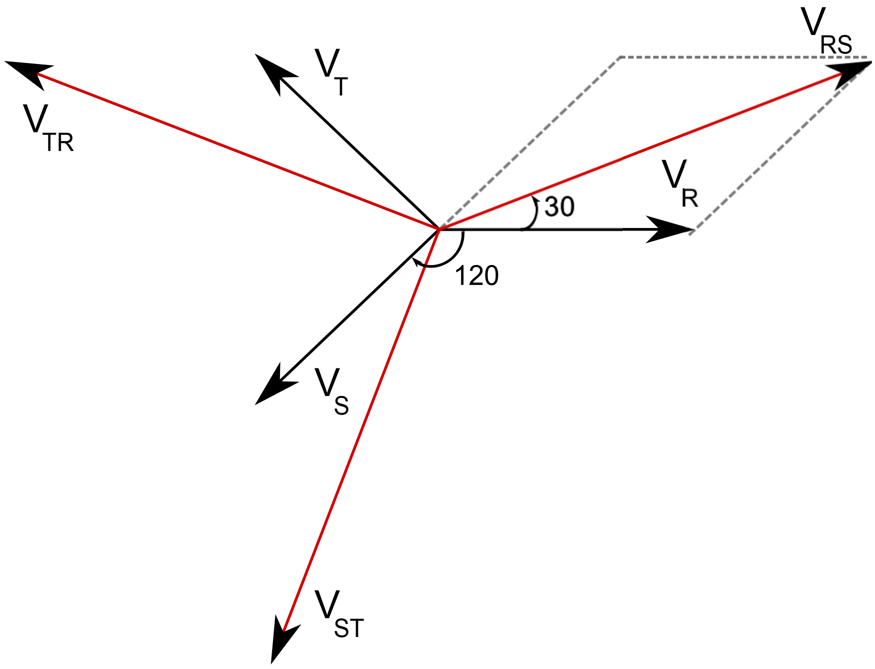


Line-to-line voltages:

$$V_{RS} = V_R - V_S = V_m e^{j\theta_R}(1 - e^{-j120}) = V_m\sqrt{3}e^{j(\theta_R+30)}$$
$$V_{ST} = V_S - V_T = V_m\sqrt{3}e^{j(\theta_R-90)}$$
$$V_{TR} = V_T - V_R = V_m\sqrt{3}e^{j(\theta_R-210)}$$

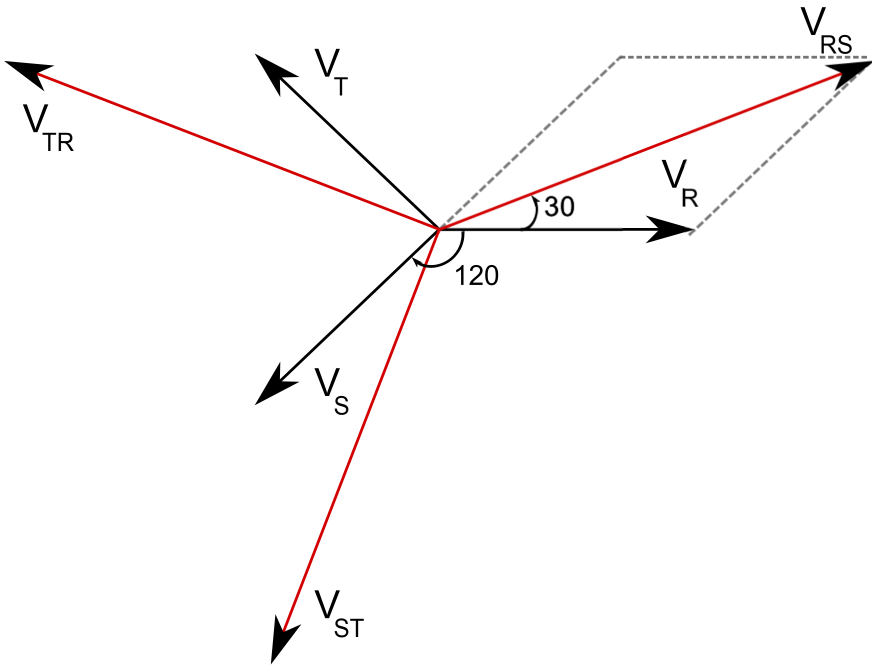
Line voltage refers to the voltage across any pair of lines.

Phasor Diagrams



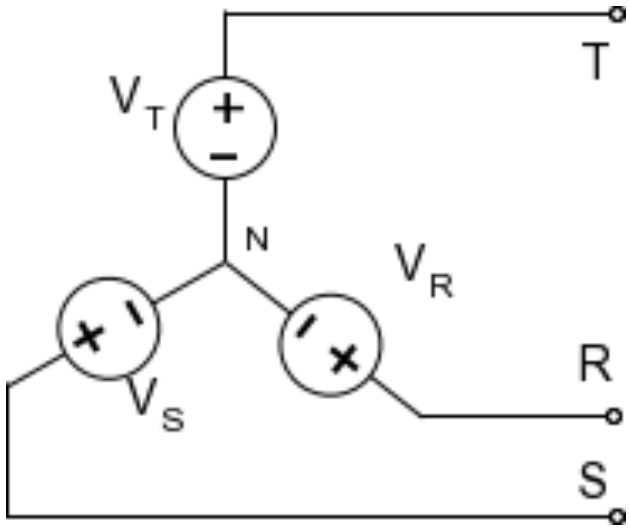
$$|V_{RS}| = \sqrt{3}|V_R|$$
$$|V_{ST}| = \sqrt{3}|V_S|$$
$$|V_{TR}| = \sqrt{3}|V_T|$$

Phasor Diagrams



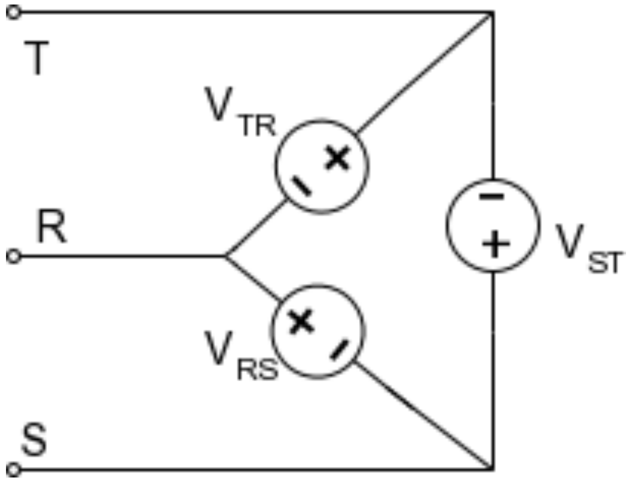
$$V_{RS} = \sqrt{3}V_R e^{30j}$$
$$V_{ST} = \sqrt{3}V_S e^{30j}$$
$$V_{TR} = \sqrt{3}V_T e^{30j}$$

Line-to-line voltage



In Y-connection: line voltage and phase voltage are not identical !

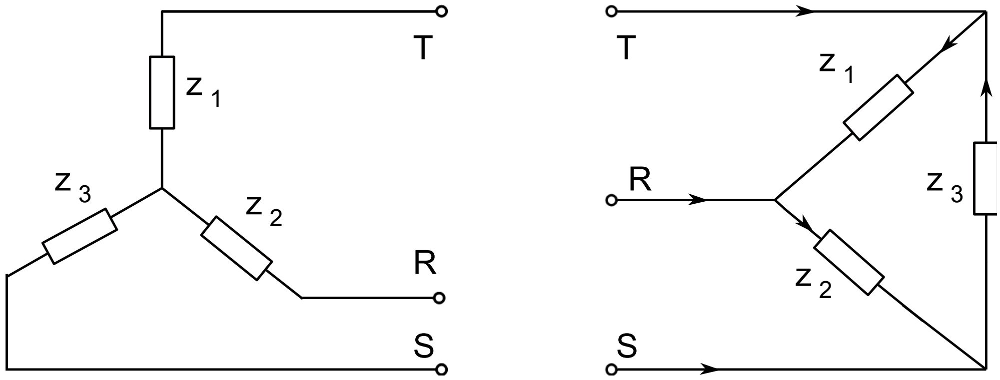
Δ-connected source



$$V_{RS} = V_M e^{\theta}$$
$$V_{ST} = V_M e^{\theta - 120}$$
$$V_{TR} = V_M e^{\theta - 240}$$

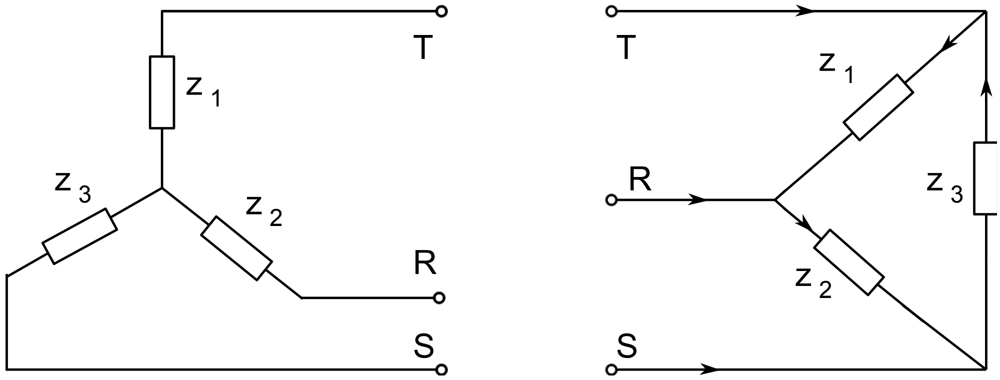
In Δ-connection: line voltage and phase voltage are identical !

Load in Three Phase Circuit



Balanced load : $Z_1 = Z_2 = Z_3$
In Y-connected Load: Phase currents and line current are identical. In Δ-connected they are not!

Y-connected Load



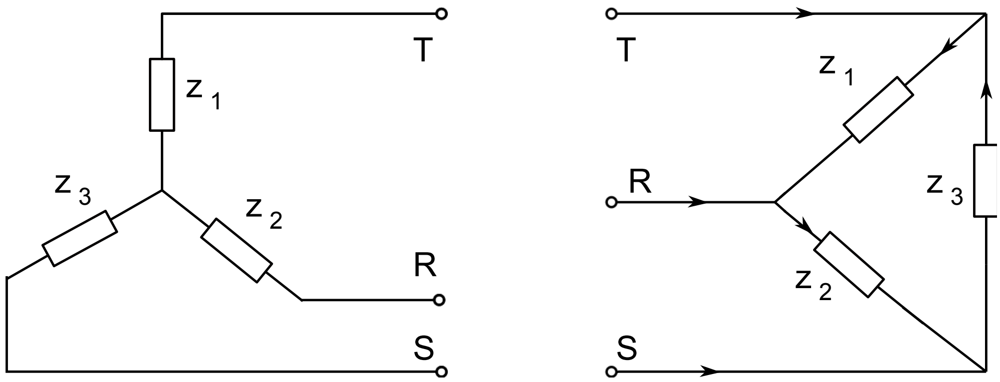
$|V_R| = |V_S| = |V_T|$ and

$$Z_1 = Z_2 = Z_3 = Z_Y = |Z_Y|e^{j\theta_Y}$$

Phase currents (line current);

$$I_R = \frac{V_R}{Z_Y}, \quad I_S = \frac{V_R}{Z_Y}e^{-120j}, \quad I_T = \frac{V_R}{Z_Y}e^{-240j}$$

Δ-connected Load



Phase currents !:

$$I_2 = \frac{V_{RS}}{Z_2} = \frac{V_{RS}}{Z_\triangle}, \quad I_3 = \frac{V_{ST}}{Z_3} = \frac{V_{ST}}{Z_\triangle}, \quad I_1 = \frac{V_{TR}}{Z_1} = \frac{V_{TR}}{Z_\triangle}$$

Line currents:

$$I_R = I_2 - I_1 = \frac{V_{RS} - V_{TR}}{Z_\triangle}, \quad I_S = I_3 - I_2 = \frac{V_{ST} - V_{RS}}{Z_\triangle},$$

$$I_T = I_1 - I_3 = \frac{V_{TR} - V_{ST}}{Z_\triangle}$$

Δ-connected Load

$$
\begin{aligned}
V_{RS} - V_{TR} &= \sqrt{3}V_R e^{30j} - \sqrt{3}V_T e^{30j} \\
&= \sqrt{3}e^{30j}(V_R - V_T) \\
&= \sqrt{3}e^{30j}(\sqrt{3}V_R e^{-30j}) \\
&= 3V_R
\end{aligned}
$$

Line currents;

$$
I_R = \frac{3V_R}{Z_\triangle}, I_S = \frac{3V_S}{Z_\triangle}, I_T = \frac{3V_T}{Z_\triangle}.
$$

$|I_R| = |I_S| = |I_T| = |I_L|$ and $|I_1| = |I_2| = |I_3| = |I_\triangle|$ where

$$
|I_L| = \sqrt{3}|I_\triangle|
$$

Example

In a balanced δ-connected load, line-to-line voltage is $600V$ at @50Hz.(a) Calculate phase current and (b) line current.

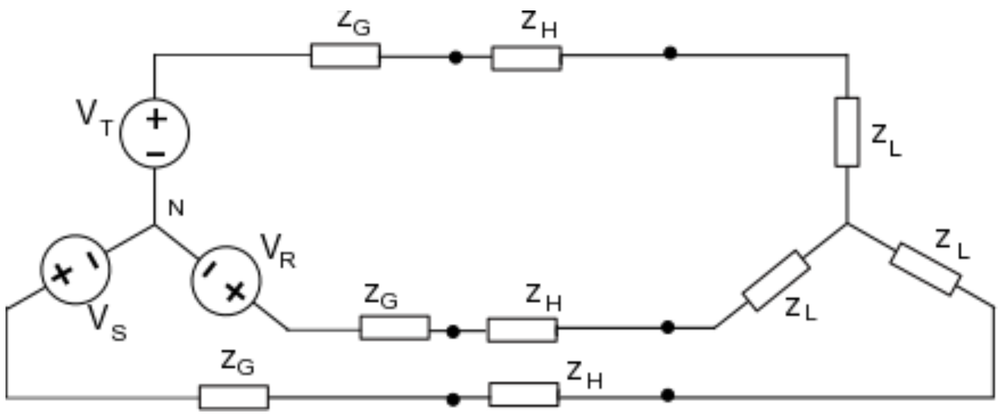
Phase current :

$$I_\Delta = \frac{600}{120} = 5A$$

Line current:

$$I_L = \sqrt{3}I_\Delta = 8.66A$$

Example

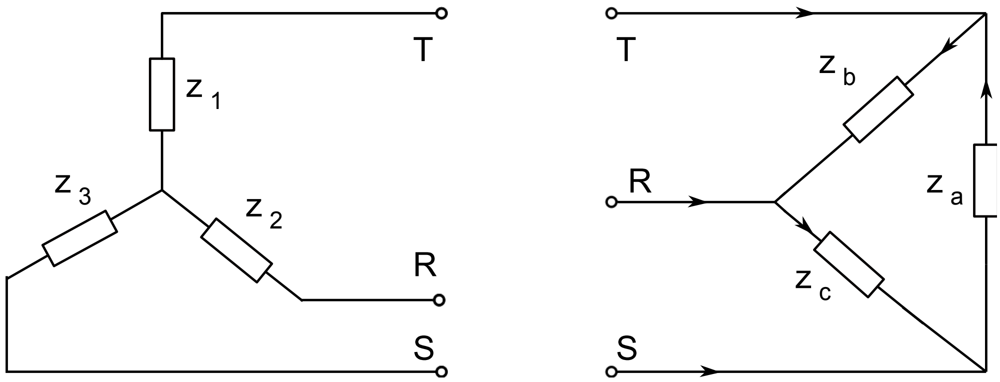


Line current (T):

$$I_L = \frac{V_T}{Z_G + Z_H + Z_L}$$

The phase voltage at the load (T): $V_{TL} = I_L Z_L$ The line voltage at the load (T): $V_{TR} = \sqrt{3} V_{TL} e^{-30j}$

Δ-to-Y



Y to Δ;

$Z_a = \frac{Z_1Z_2+Z_2Z_3+Z_1Z_3}{Z_2}$, $Z_b = \frac{Z_1Z_2+Z_2Z_3+Z_1Z_3}{Z_3}$, $Z_c = \frac{Z_1Z_2+Z_2Z_3+Z_1Z_3}{Z_1}$

Δ to Y;

$Z_1 = \frac{Z_aZ_b}{Z_a+Z_b+Z_c}$, $Z_2 = \frac{Z_bZ_c}{Z_a+Z_b+Z_c}$, $Z_3 = \frac{Z_aZ_c}{Z_a+Z_b+Z_c}$

PS: if $Z_1 = Z_2 = Z_3 = Z_Y$ and $Z_a = Z_b = Z_c = Z_\triangle$ then

$$Z_Y = \frac{Z_\triangle}{3}$$

Power calculation

Complex power in Y-connected load

$$N_Y = N_R + N_S + N_T$$

the complex power associate the R-phase

$$N_R = \frac{1}{2} V_R \overline{I}_R = \frac{1}{2} V_R \overline{V}_R \overline{Y}_Y$$
$$= \frac{1}{2} |V_R|^2 \overline{Y}_Y$$

$|V_R| = |V_S| = |V_T| = |V_P|$ and $V_{P\mathrm{rms}} = \frac{|V_P|}{\sqrt{2}}$

$$N_Y = \frac{3}{2} \overline{Y}_Y |V_R|^2 = 3 \overline{Y}_Y V_{P\mathrm{rms}}^2$$

or using phase current;

$$N_Y = 3 Z_Y I_{L\mathrm{rms}}^2.$$

Power calculation

Line-to-line voltage for balanced load

$$V_{RS} = \sqrt{3} V_R e^{-30j}$$

$|V_{RS}| = |V_{ST}| = |V_{TR}| = |V_{LL}|$ and $V_{LL\mathrm{rms}} = \frac{|V_{LL}|}{\sqrt{2}}$.
Power ;

$$N_Y = \overline{Y}_Y V_{LL\mathrm{rms}}^2$$

Complex power of Y-connected $Z_Y = |Z_Y| e^{j\phi_Y}$ $(\phi_Y = \phi_V - \phi_I)$

$$N_Y = \frac{3}{2} |I_L||V_P| (\cos\phi_Y + j\sin\phi_Y)$$

instead of V_P lets use line-to-line voltage

$$N_Y = \frac{3}{2} |I_L| \frac{|V_{LL}|}{\sqrt{3}} (\cos\phi_Y + j\sin\phi_Y)$$

$$= \frac{\sqrt{3}}{2} |I_L||V_{LL}| (\cos\phi_Y + j\sin\phi_Y)$$

$$N_Y = \sqrt{3} I_{L\mathrm{rms}} V_{LL\mathrm{rms}} (\cos\phi_Y + j\sin\phi_Y)$$

Power calculation in Δ-connected load

$$N_{\triangle} = N_1 + N_2 + N_3$$

For Z_1

$$N_1 = \frac{1}{2} Z_1 |I_1|^2 = \frac{1}{2} Z_{\triangle} |I_1|^2$$

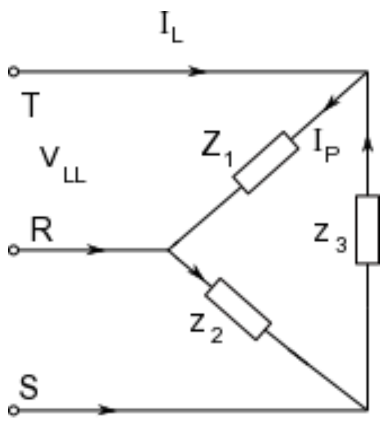
using $|I_1| = |I_3| = |I_3| = |I_P|$

$$N_{\triangle} = 3 Z_{\triangle} |I_{P\mathrm{rms}}|^2$$

we have $|I_L| = \sqrt{3} |I_P|$ hence

$$N_{\triangle} = Z_{\triangle} |I_{L\mathrm{rms}}|^2$$

Power calculation



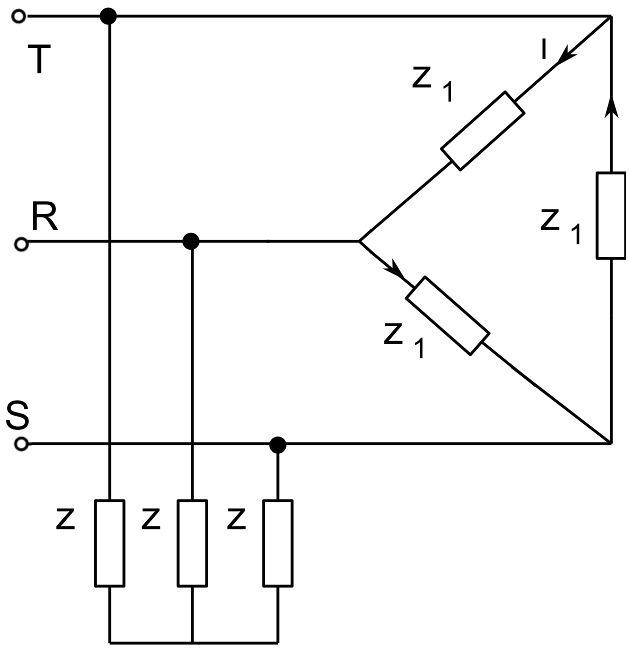
$Z_\triangle = |Z_\triangle| e^{j\phi_\triangle}$ $(\phi_\triangle = \phi_V - \phi_I)$

$$N_\triangle = \frac{3}{2}|I_P||V_{LL}|(\cos\phi_\triangle + j\sin\phi_\triangle)$$

$$N_\triangle = \frac{3}{2}|V_{LL}|\frac{|I_L|}{\sqrt{3}}(\cos\phi_\triangle + j\sin\phi_\triangle) = \frac{\sqrt{3}}{2}|V_{LL}||I_L|(\cos\phi_\triangle + j\sin\phi_\triangle)$$
$$= \sqrt{3} I_{L\mathrm{rms}} V_{LL\mathrm{rms}}(\cos\phi_\triangle + j\sin\phi_\triangle).$$

Examples

A balance three-phase source with line-to-line voltage $220\sqrt{6}$ is supplying $25992W$ active and $0VAR$ reactive power to Y and Δ-connected two balanced loads. Δ-connected balanced load is absorbing $12966W$ active and $4332VAR$ reactive power. Calculate Y-connected load.



Examples

Total Power delivered to loads

$$N = N_Y + N_\triangle = 25992$$

hence

$$N_Y = 12996 - j43432.$$

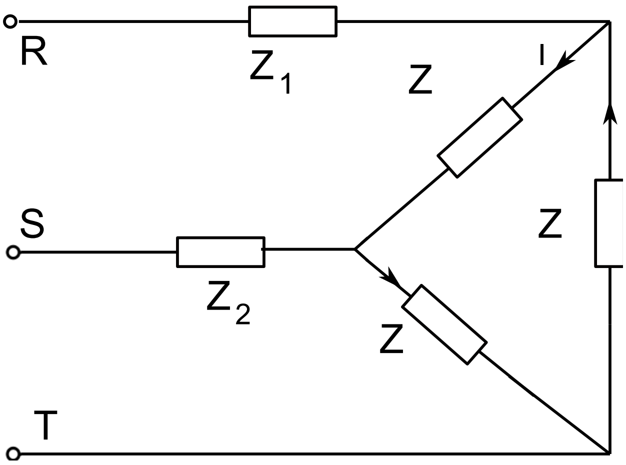
$V_R = \frac{1}{\sqrt{3}} V_{RS} e^{-j30}$ then $V_R = 220\sqrt{2} e^{-j30}$.

$$N_Y = 3V_{L\mathrm{rms}}^2 \overline{Y}_Y$$

using $V_{L\mathrm{rms}} = 220$ we obtaine

$$Y_Y = 0.0895 + j0.02982$$

Examples



In a balanced 3-phase system, $Z = 4 + j2$, $Z_1 = \frac{2}{3}Z$ and $Z_2 = \frac{1}{6}Z$ are given. For $V_{RS} = V_{ST} = 20 + j10$, calculate the currents I_R, I_S, I_T.

Examples

Use Δ to Y conversion, then obtaine the empedance connected to each line : $2Z/3 + Z/3$, $Z/6 + Z/3$ and $Z/3$. Then KVL:

$$(Z)I_R - I_S Z/2 = V_{RS}$$

and

$$Z/2 I_R - I_T Z/3 = V_{ST}$$

then obtaine $I_T = -10$ and $I_R = 20/3$ (2 Equ.s and 3 unknown !!).