Circuit and System Analysis
EHB 232E

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Outline I

1. Sinusoidal Steady-State Analysis-Cont.
   - Sinusoidal Steady-State Power Calculation
   - Average Power
   - Complex, Real and Reactive Powers
   - Tellegen Theorem
   - Maximum Power Transfer

Sinusoidal Steady-State Power Calculation

v and i are steady-state sinusoidal signals
$v(t) = V_m \cos(wt + \theta_v)$ and $i(t) = I_m \cos(wt + \theta_i)$

Instantaneous Power

$$P = V_m \cos(wt + \theta_v) I_m \cos(wt + \theta_i)$$
$$= \frac{1}{2} V_m I_m \{\cos(\theta_v - \theta_i) + \cos(2wt + \theta_v + \theta_i)\}$$

Power factor angle

$$\phi = \theta_v - \theta_i$$

Power factor

$$\mathrm{pf} = \cos(\theta_v - \theta_i)$$

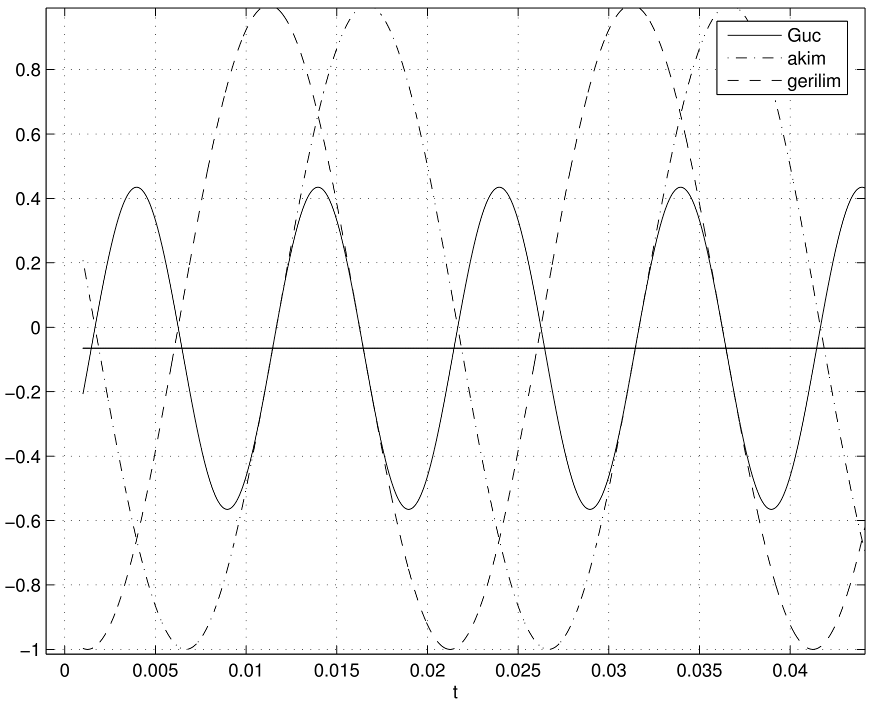
Reactive factor

$$\mathrm{pf} = \sin(\theta_v - \theta_i)$$

Example

$v(t) = \cos\left(2\pi 50 t + \frac{\pi}{3}\right)$ and $i(t) = \cos\left(2\pi 50 t + \frac{7\pi}{8}\right)$ Instantaneous Power

$$P(t) = -0.0653 + \cos(2\pi 100 t + \frac{\pi}{3} + \frac{7\pi}{8})$$



Average Power

The average power associated with sinusoidal signals is the average of the instantaneous power over one period

$$P_{\text{avr}} = \int_0^T p(t)dt$$

$$\begin{aligned} P_{\text{avr}} &= \int_0^T p(t)dt \\ &= \frac{1}{2}V_m I_m \{\cos(\theta_v - \theta_i) \\ &= \frac{1}{2}V_m I_m \cos\phi \end{aligned}$$

Complex Power

$$S = \frac{1}{2}V\bar{I} = \frac{1}{2}V_m e^{j\theta_v} I_m e^{-j\theta_i}$$
$$= \frac{1}{2}V_m I_m e^{j(\theta_v - \theta_i)}$$
$$= \frac{1}{2}V_m I_m e^{j\phi}$$

units volt-amps (VA)

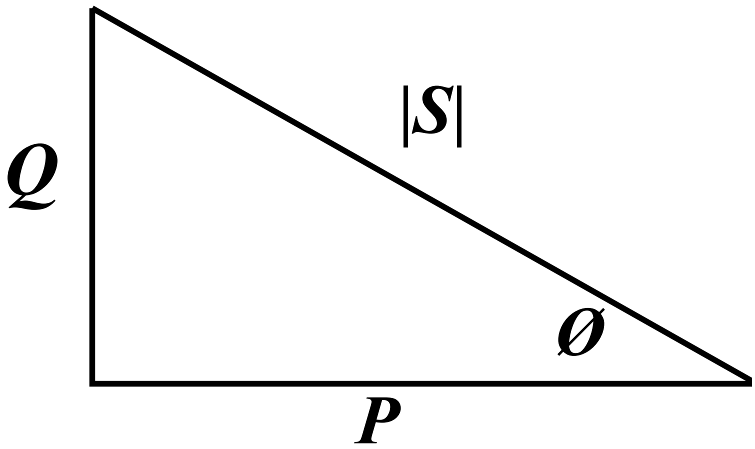
$$S = P + jQ$$

P is **Active Power** (Watt) and Q is **Reactive Power** (VAR),
$|S| = \sqrt{P^2 + Q^2}$ Apparent pover (volt-amps)

$$P = \frac{1}{2}V_m I_m \cos\phi = P_{\mathrm{avr}}$$

and

$$Q = \frac{1}{2}V_m I_m \sin(\phi)$$



$$\tan\phi = \frac{Q}{P}$$

	Active P	Reactive Q
Resistor	$\frac{1}{2}R{I_m}^2$	0
Capacitor	0	$-\frac{1}{2wC}{I_m}^2$
Inductor	0	$\frac{wL}{2}{I_m}^2$

- Power for Purely Resistive Circuits : Power can not be extracted from a purely resistive network. In a purely inductive and capacitive circuits, the average power are zero.
- In a purely inductive circuit, energy is being stored the magnetic field, and then it is being extracted from the magnetic fields.
- In a purely capacitive circuit, the power is continually exchanged between the source driving the circuit and the electric field associate with the capacitive element.

Lagging power factor : $Q > 0$ inductive load.
Leading power factor : $Q < 0$ capacitive load.

$$P = \frac{1}{2} V_m I_m \cos\phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi = V_{\mathrm{rms}} I_{\mathrm{rms}} \cos\phi$$

and

$$Q = V_{\mathrm{rms}} I_{\mathrm{rms}} \sin\phi$$

Example: $220V$ 100 W lamp has a resistance of $\frac{220^2}{100} = 484\Omega$ and $I_{\mathrm{rms}} = \frac{220}{484} = 0.45A$.
Example 10.4 (page 402) Example 10.5 (page 406), Example 10.6 (page 407), Electric Circuits, James W. Nilsson and Susan A. Riedel

Example: A load is connected in parallel across a $120V$ (rms) voltage source. The load is deliveding a reactive power of 1800VAR at leading power factor $pf = \frac{\sqrt{3}}{2}$. The frequency of the voltage source is $80 rad/sn$. (a) Calculate the admittance of the load. (b) compute the value of element that would correct the power factor to 1 if placed in parallel with the load.

Power factor is described as leading therefore the load is capacitive, furthermore the laod is delivering a reactive power so $Q < 0$ which means that again load is capacitive.

$$\cos\theta = \frac{\sqrt{3}}{2}$$

then $\theta = -30^\circ$

$$Q = |S|\sin(-30) = |V||I|\sin(-30) = -1800$$

$I = 30e^{30j}$. In order to find admittance $Y = V/I = 0.25e^{30j}$.

To obtain power factor to 1, let Y_x placed in parallel with the load. The load is deliveding a reactive power of 1800VAR therefore Y_x must be absorb 1800VAR in order to get 0 total reactive power! The Y_x must be inductive and absorb a reactive power of 1800VAR. From $S = V\bar{I} = |V|^2\bar{Y}$ we obtain $1800 = 120^2 \frac{1}{L80}$ equation and $L = 0,1H$.

Example: Calculate the average power and the reactive power at the terminal of
a port circuit element if $v = 100\cos(wt+15^\circ)$V and $i = 4\sin(wt-15^\circ)$Amp.

$$
\begin{aligned}
S &= \frac{1}{2} \cdot 100 \cdot e^{j15} \cdot 4 \cdot e^{j(90+15)} \\
&= \frac{1}{2} \cdot 100 \cdot 4 \cdot e^{j(15+105)} = 100 + j173.21 \\
&= \frac{1}{2} \cdot 100 \cdot 4 \cdot (\cos(120) + j\sin(120))
\end{aligned}
$$

Hence $P = -100$W and $Q = 173.21$ VAR. The negative value of -100W means that the one-port is delivering average power and absorbing reactive power.

A blender motor is modelled by a 30Ω resistor (modelling the coil resistance) in series with a $\frac{40}{2\pi 60}H$ inductor (modelling the inductive effects of the coil). What power is dissipated by the motor?

rms current phasor $= I_{\text{rms}} = \frac{120}{30+2\pi 60\frac{40}{2\pi 60}j} = 2.4e^{-j53^\circ}$ Average power dissipated $P = Re\{120 \times 2.4e^{53^\circ}\} = 172.8$watts The motor draws more current $\underline{i(t) = 2.4\sqrt{2}\cos(2\pi 60t - 53^\circ)}$ The voltage and current are 53° out of phase, so the motor draws more current than it should. Hook a capacitor C in parallel with the motor. RMS current phasor: $I_{rms} = 120(\frac{1}{30+40j} + j\pi 60C)$ What value of C makes the phase of I_{rms} zero? You should obtain $C = 42.4\mu F$. Then $I_{rms} = 1.44$ Average power dissipated $P = Re\{120 \times 1.44\} = 172.8$watts. But The current amplitude has dropped from $2.4\sqrt{2}$ to $1.44\sqrt{2}$ amps. We have almost halved the peak current, while maintaining average power$= 172.8$ watts.

Tellegen Theorem

Tellegen's theorem is based on the fundamental law of conservation of energy!

Tellegen Theorem

It states that the algebraic sum of power absorbed by all elements in a circuit is zero at any instant.

Tellegen's theorem asserts that

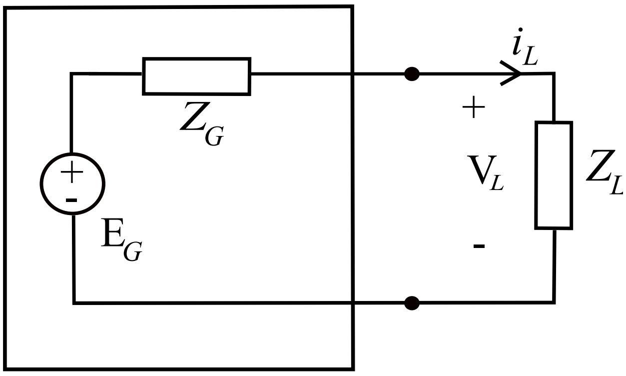
$$P = \sum_{i=1}^{n_e} \frac{1}{2} V_i \bar{I}_i = 0$$

$$\begin{aligned} P &= \sum_{i=1}^{n_e} \frac{1}{2} V_i \bar{I}_i = \frac{1}{2} V_e^T \bar{I}_e \\ &= \frac{1}{2} V_n^T M^T \bar{I}_e = \frac{1}{2} V_e^T M^T \bar{I}_e = 0 \end{aligned}$$

see : EHB211E Slayt Number 73.

Maximum Power Transfer

Maximum amount of power from the source to the load.



We must determine the load impedance that results in the delivery of maximum average power to terminal Z_L.

$$P_L = P_{E_G} - P_{R_G} = \frac{1}{2}E_G|I_L|\cos(\theta_L) - \frac{1}{2}R_G|I_L|e^{\theta_L}|I_L|e^{-\theta_L}$$
$$= \frac{1}{2}E_G|I_L|\cos\theta_L - \frac{1}{2}R_G|I_L|^2$$

$$P_L = \frac{1}{2} E_G |I_L| \cos\theta_L - \frac{1}{2} R_G |I_L|^2$$

First let $\cos\theta_L = 1$ to maximize P_L.
Then we must find the values of Z_L where $\frac{dP_L}{d|I_L|} = 0$.

$$\frac{dP_L}{d|I_L|} = \frac{1}{2} E_G - R_G |I_L|$$

then

$$|I_L| = \frac{1}{2} \frac{E_G}{R_G}$$

From circuit

$$I_L = \frac{E_G}{Z_L + Z_G}$$

If $\cos\theta_L = 1$, $Z_L + Z_G$ must be resistive. Therefore $Z_L = \bar{Z}_G$ and $Z_L + Z_G = 2R_G$.
Then

$$R_L = R_G$$

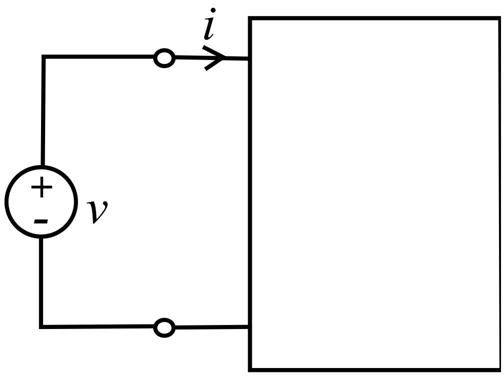
and

$$X_L = -X_G$$

For maximum average power transfer

$$Z_L = \bar{Z}_G$$

Average Power Due to Several Sinusoidal Inputs



We drive a linear time-invariant one-port by a voltage source given by

$$v(t) = V_1 \cos(w_1 t + \theta_{v1}) + V_2 \cos(w_2 t + \theta_{v2})$$

and the port current

$$i(t) = I_1 \cos(w_1 t + \theta_{i1}) + I_2 \cos(w_2 t + \theta_{i2})$$

Using standard formulas, The (instantaneous) power delivered by the voltage source to the one-port

$$
\begin{aligned}
P &= v(t)i(t) \\
&= \frac{1}{2}V_1 I_1\{\cos(\theta_{v1}-\theta_{i1}) + \frac{1}{2}V_2 I_2\{\cos(\theta_{v2}-\theta_{i2}) \\
&+ \frac{1}{2}V_1 I_1 \cos(2wt+\theta_{v1}+\theta_{i1})\} + \frac{1}{2}V_2 I_2 \cos(2wt+\theta_{v2}+\theta_{i2})\} \\
&+ \frac{1}{2}V_1 I_2 \cos((w_1+w_2)t+\theta_{v1}+\theta_{i2})\} + \frac{1}{2}V_2 I_1 \cos((w_1+w_2)t+\theta_{v2}+\theta_{i1}) \\
&+ \frac{1}{2}V_1 I_2 \cos((w_1-w_2)t+\theta_{v1}-\theta_{i2})\} + \frac{1}{2}V_2 I_1 \cos((w_1-w_2)t+\theta_{i1}-\theta_{v2})
\end{aligned}
$$

The average power over $T_c = n_1 T_1 = n_2 T_2$

$$
\begin{aligned}
P_{\text{ort}} &= \frac{1}{T_c}\int_0^{T_c} P(t)dt \\
&= \frac{1}{2}V_1 I_1\{\cos(\theta_{v1}-\theta_{i1}) + \frac{1}{2}V_2 I_2\{\cos(\theta_{v2}-\theta_{i2})
\end{aligned}
$$

The average power delivered by source to the one-port is equal to the sum of the average powers that each individual sinusoidal waveform would deliver to one-port if it were acting alone.

! if $w_1 = w_2$, it does not hold !