Circuit and System Analysis
EHB 232E

Prof. Dr. Müştak E. Yalçın

Istanbul Technical University
Faculty of Electrical and Electronic Engineering

mustak.yalcin@itu.edu.tr

Outline I

Bode plot

A Bode plot is a graph of the transfer function $H(jw)$ of a linear time-invariant system versus frequency. Bode diagam consists of two separate plots. One shows how the amplitude $|H(jw)|$ of $H(jw)$ varies with frequency, and the other shows how the phase angle $\angle H(jw)$ of $|H(jw)|$ varies with frequency.
The amplitude of $H(jw)$ in terms of a logarithmic value: the decibel (dB).

$$20 \log_{10} |H(jw)|$$

Decibel (dB)

When referring to measurements of power quantities, a ratio can be expressed as a level in decibels by evaluating ten times the base-10 logarithm of the ratio of the measured quantity to reference value.

$$L_p = 10 \log(\frac{P}{P_0})$$

When referring to measurements of voltage or current ;

$$L_v = 20 \log(\frac{v}{v_0})$$

(power is typically proportional to the square of voltage or current) Ref: Wikipedia.

Straight-Line Bode diagram Plots

A transfer function

$$H(s) = k\frac{P(s)}{Q(s)} = k\frac{(s-s_{01})(s-s_{02})...(s-s_{0m})}{(s-s_{p1})(s-s_{p2})...(s-s_{pn})}$$

$$20\log_{10}|H(jw)| = 20\log_{10}|k| + \sum_{i=1}^{m} 20\log_{10}|jw - s_{0i}| - \sum_{i=1}^{n} 20\log_{10}|jw - s_{pi}|$$

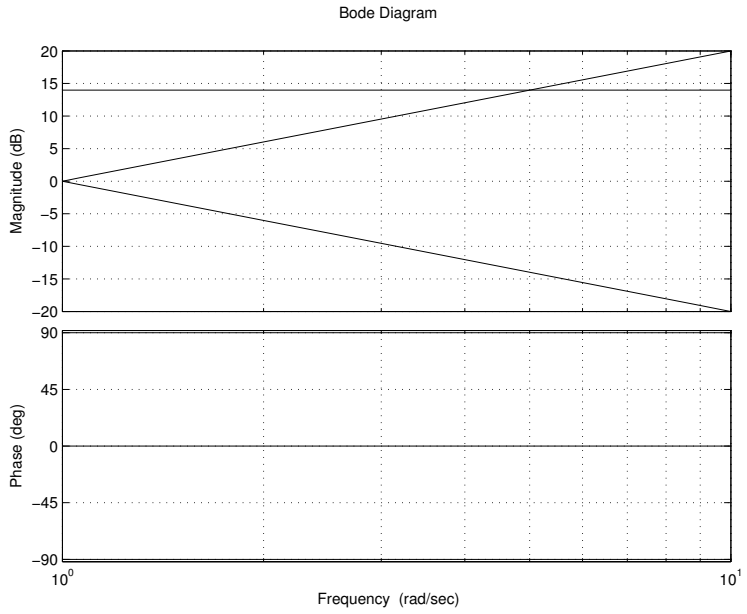
and phase

$$\angle H(jw) = \angle k + \sum_{i=1}^{m} \angle(jw - s_{0i}) - \sum_{i=1}^{n} \angle(jw - s_{pi})$$

Straight-Line Bode diagram Plots

- for s^l
  - the amplitude is a straight line with a slope of $20l\ dB/decade$.
  - the phase angle is $90l\ ^\circ$.
  - the amplitude is $0\ dB$ wher $w = 1$.

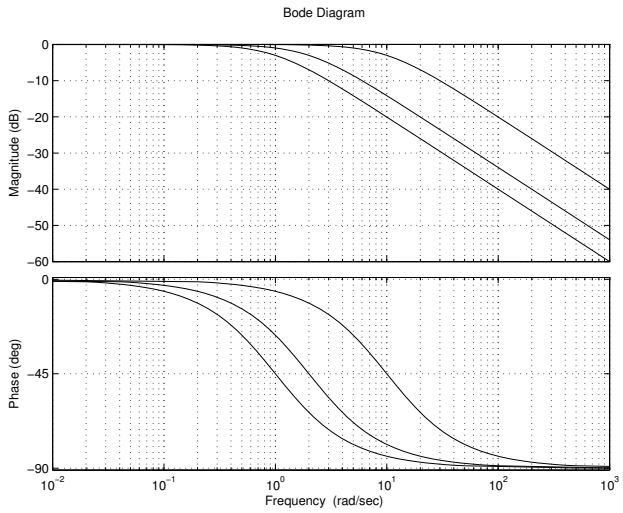
$H(s) = s$, $H(s) = \frac{1}{s}$ and $H(s) = k$



Straight-Line Bode diagram Plots

- $H(s) = \frac{p}{s+p}$ ve $p \in R$.
  - $|H(jw)| \approx 0$ for $w < |p|$.
  - $|H(jw)|$ is a straight line having a slope of $-20\,dB/decade$ for $w > |p|$.
  - $\angle H(jw) \approx 0\,^\circ$ for $w < 0.1|p|$.
  - $\angle H(jw) \approx -90\,^\circ$ for $w > 10|p|$.
  - $-45\,^\circ/decade$ where $0.1|p| < w < 10|p|$

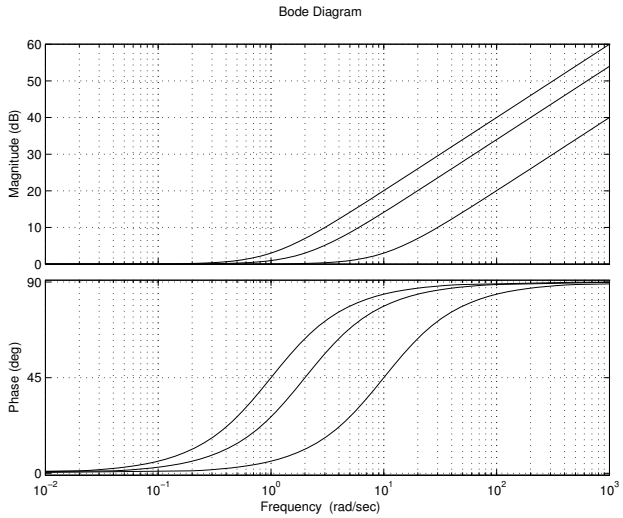
$H(s) = \frac{1}{s+1}, \frac{2}{s+2}, \frac{10}{s+10}$ (`H1 = tf([1 ],[1 1]); bode(H1)`)



Straight-Line Bode diagram Plots

- $H(s) = \frac{s+z}{z}$ ve $p \in R$.
  - $|H(jw)| \approx 0$ for $w < |z|$.
  - $|H(jw)|$ is a straight line having a slope of $20\,dB/decade$ for $w > |z|$.
  - $\angle H(jw) \approx 0\,^\circ$ for $w < 0.1|p|$.
  - $\angle H(jw) \approx 90\,^\circ$ for $w > 10|p|$.
  - $45\,^\circ/decade$ where $0.1|p| < w < 10|p|$.

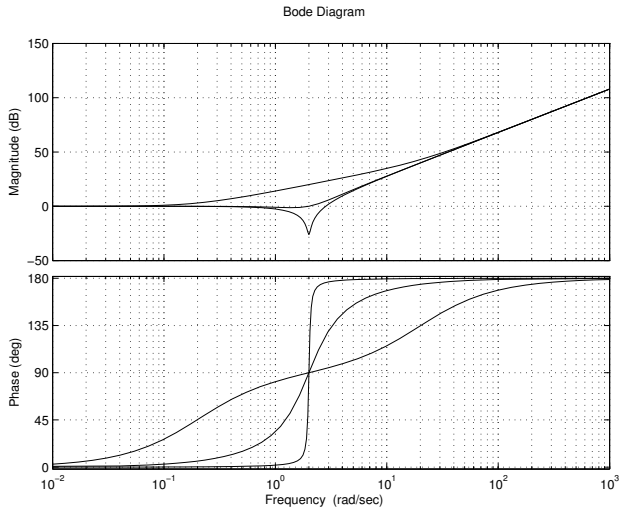
$H(s) = \frac{s+1}{1}$, $H(s) = \frac{s+2}{2}$ and $H(s) = \frac{s+10}{10}$



Straight-Line Bode diagram Plots

- $H(s) = \frac{s^2+as+b}{b}$.
  - $|H(jw)| \approx 0$ for $w < w_0$.
  - $|H(jw)|$ is a straight line having a slope of $40\,dB/decade$ for $w > w_0$.
  - $\angle H(jw) \approx 0\,^\circ$ for $w < 0.1|p|$.
  - $\angle H(jw) \approx 180\,^\circ$ for $w > 10|p|$.
  - $90\,^\circ/decade$ where $0.1|p| < w < 10|p|$.

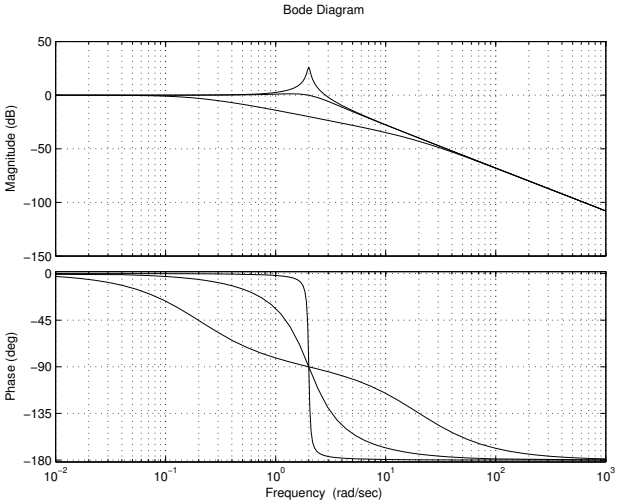
$H(s) = \frac{s^2+2s+4}{4}$, $H(s) = \frac{s^2+20s+4}{4}$, $H(s) = \frac{s^2+0.1s+4}{4}$



Straight-Line Bode diagram Plots

- $H(s) = \frac{b}{s^2+as+b}$.
  - $|H(jw)| \approx 0$ for $w < w_0$.
  - $|H(jw)|$ is a straight line having a slope of $-40\,dB/decade$ for $w > w_0$.
  - $\angle H(jw) \approx 0\,^\circ$ for $w < 0.1|p|$.
  - $\angle H(jw) \approx -180\,^\circ$ for $w > 10|p|$.
  - $-90\,^\circ/decade$ where $0.1|p| < w < 10|p|$.

$H(s) = \frac{4}{s^2+2s+4}$, $H(s) = \frac{4}{s^2+20s+4}$ and $H(s) = \frac{4}{s^2+0.1s+4}$ i



Example

$$H(s) = \frac{1000(s+10)}{(s+1)(s+1000)}$$