Circuit and System Analysis
EHB 232E

Prof. Dr. Müştak E. Yalçın

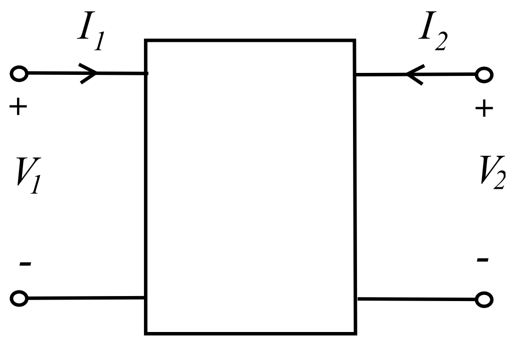
Istanbul Technical University
Faculty of Electrical and Electronic Engineering

mustak.yalcin@itu.edu.tr

Outline I

1. Laplace Transform in Circuit Analysis-Cont.
   - Network Parameters
   - Combinations of two-port networks
   - Reciprocal Network
   - Thevenin - Norton Equivalent Circuits

Network Parameters



Impedance matrix (z-parameter) The two currents I_1 and I_2 are assumed to be known, and the voltages V_1 and V_2 can be found by:

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

where

$$z_{11} = \left.\frac{V_1(s)}{I_1(s)}\right|_{I_2=0} \qquad z_{12} = \left.\frac{V_1(s)}{I_2(s)}\right|_{I_1=0}$$

$$z_{21} = \left.\frac{V_2(s)}{I_1(s)}\right|_{I_2=0} \qquad z_{22} = \left.\frac{V_2(s)}{I_2(s)}\right|_{I_1=0}$$

Network Parameters

Admittance matrix (y-parameters) The two voltages V_1 and V_2 are assumed to be known, and the currents I_1 and I_2 can be found by:

$$\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix}$$

where

$$y_{11} = \left.\frac{I_1(s)}{V_1(s)}\right|_{V_2=0} \qquad y_{12} = \left.\frac{I_1(s)}{V_2(s)}\right|_{V_1=0}$$
$$y_{21} = \left.\frac{I_2(s)}{V_1(s)}\right|_{V_2=0} \qquad y_{22} = \left.\frac{I_2(s)}{V_2(s)}\right|_{V_1=0}$$

y_{21} and y_{12} are transfer admittances

g-parameters

Inverse hybrid model, we assume V_1 and I_2 are known, and find V_2 and I_1 by :

$$\begin{bmatrix} I_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1(s) \\ I_2(s) \end{bmatrix}$$

where

$$g_{11} = \left.\frac{I_1(s)}{V_1(s)}\right|_{I_2=0} \qquad g_{12} = \left.\frac{I_1(s)}{I_2(s)}\right|_{V_1=0}$$

$$g_{21} = \left.\frac{V_2(s)}{V_1(s)}\right|_{I_2=0} \qquad g_{22} = \left.\frac{V_2(s)}{I_2(s)}\right|_{V_1=0}$$

h-parameters

Hybrid model, we assume V_2 and I_1 are known, and find V_1 and I_2 by:

$$\begin{bmatrix} V_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1(s) \\ V_2(s) \end{bmatrix}$$

where

$$h_{11} = \left. \frac{V_1(s)}{I_1(s)} \right|_{V_2=0} \qquad h_{12} = \left. \frac{V_1(s)}{V_2(s)} \right|_{I_1=0}$$

$$h_{21} = \left. \frac{I_2(s)}{I_1(s)} \right|_{V_2=0} \qquad h_{22} = \left. \frac{I_2(s)}{V_2(s)} \right|_{I_1=0}$$

ABCD-parameters

Transmission model, we assume V_1 and I_1 are known, and find V_2 and I_2 by:

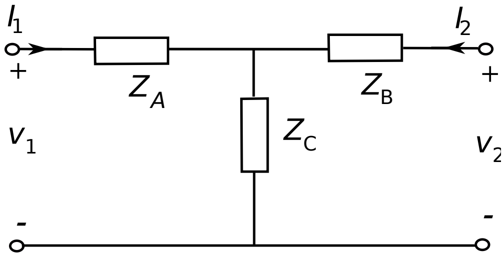
$$\begin{bmatrix} V_1(s) \\ I_1(s) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2(s) \\ -I_2(s) \end{bmatrix}$$

where

$$A = \left.\frac{V_1(s)}{V_2(s)}\right|_{I_2=0} \qquad B = \left.\frac{V_1(s)}{-I_2(s)}\right|_{V_2=0}$$

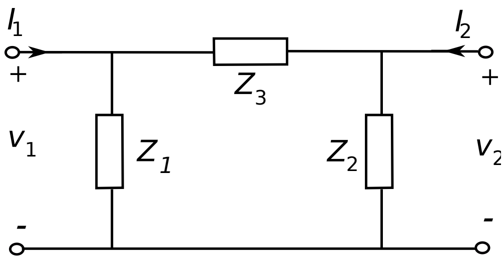
$$C = \left.\frac{I_1(s)}{V_2(s)}\right|_{I_2=0} \qquad D = \left.\frac{-I_1(s)}{I_2(s)}\right|_{V_2=0}$$

Example



$$z_{11} = \left.\frac{V_1(s)}{I_1(s)}\right|_{I_2=0} = Z_A + Z_C \qquad z_{12} = \left.\frac{V_1(s)}{I_2(s)}\right|_{I_1=0} = Z_C$$

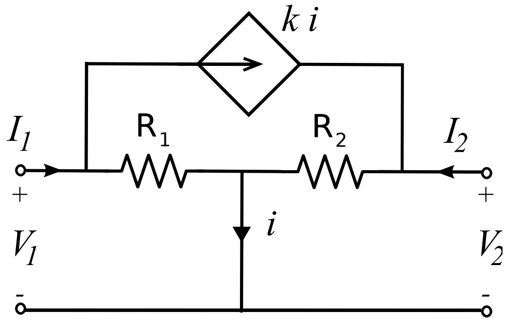
$$z_{21} = \left.\frac{V_2(s)}{I_1(s)}\right|_{I_2=0} = Z_C \qquad z_{22} = \left.\frac{V_2(s)}{I_2(s)}\right|_{I_1=0} = Z_B + Z_C$$



ABCD

$$1+\frac{Z_3}{Z_2} \quad Z_3 \quad \frac{1}{Z_1}+\frac{1}{Z_2}+\frac{Z_3}{Z_1 Z_2} \quad 1+\frac{Z_3}{Z_1}$$

Example

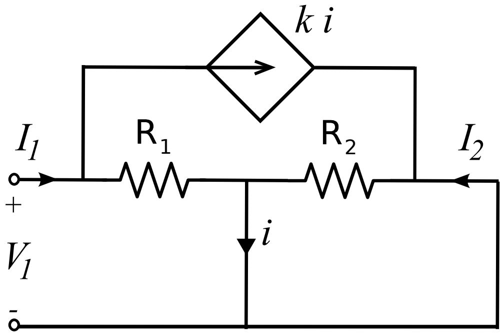


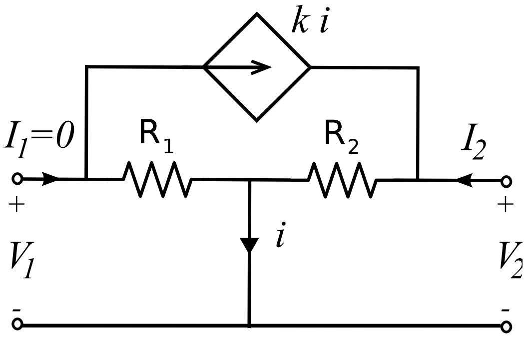
For h-parameters

$$h_{11} = \left.\frac{V_1(s)}{I_1(s)}\right|_{V_2=0} \quad \text{Port 2 } \textit{short circuit}$$

$$h_{21} = \left.\frac{I_2(s)}{I_1(s)}\right|_{V_2=0} \quad \text{Port 2 } \textit{short circuit}$$

$$h_{11} = \left.\frac{V_1(s)}{I_1(s)}\right|_{V_2=0} \qquad h_{21} = \left.\frac{I_2(s)}{I_1(s)}\right|_{V_2=0}$$





Input current ($i = I_1$)

$$I_1 = k\frac{V_1}{R_1} + \frac{V_1}{R_1}$$

then

$$h_{11} = \frac{R_1}{1+k}$$

for h_{21}

$$I_2 = -ki$$

Substituing $I_1 = (k+1)i$ into the above equ

$$h_{21} = -\frac{k}{1+k}$$

Problems: Richard C. Dorf, James A. Svoboda-Introduction to Electric Circuits-Wiley (2013) page: 850

The rest of the parameters

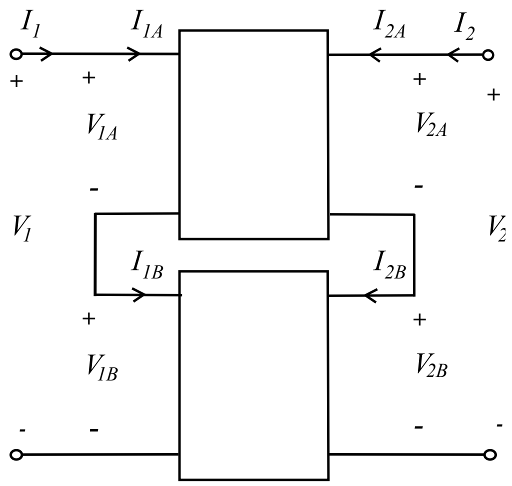
$$h_{12} = \left.\frac{V_1(s)}{V_2(s)}\right|_{I_1=0} \qquad h_{22} = \left.\frac{I_2(s)}{V_2(s)}\right|_{I_1=0}$$

For $I_1 = 0$, the circuit is given

$$h_{12} = -\frac{R_1 k}{(1+k)R_2} \qquad h_{22} = \frac{1}{(1+k)R_2}$$

Combinations of two-port networks

Series-series connection



From KCL and KVL

$$I_1 = I_{1A} = I_{1B} \qquad I_2 = I_{2A} = I_{2B}$$

and

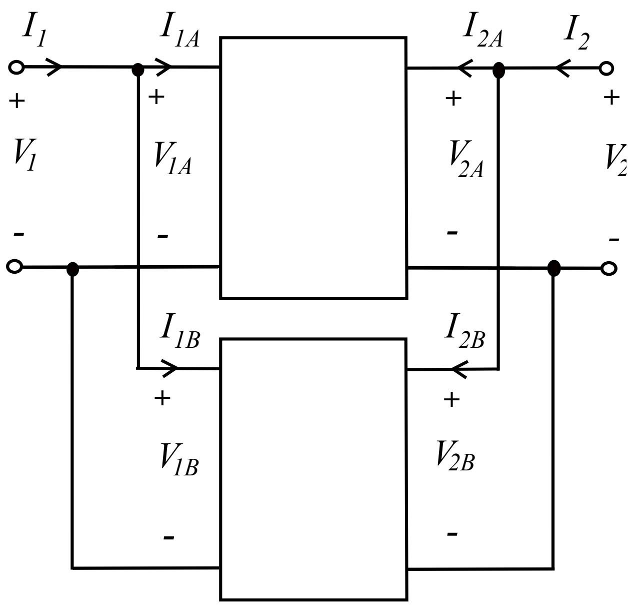
$$V_1 = V_{1A} + V_{1B} \qquad V_2 = V_{2A} + V_{2B}$$

$$\begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} V_{1A}(s) \\ V_{2A}(s) \end{bmatrix} + \begin{bmatrix} V_{1B}(s) \\ V_{2B}(s) \end{bmatrix} = \begin{bmatrix} z_{11A} & z_{12A} \\ z_{21A} & z_{22A} \end{bmatrix} \begin{bmatrix} I_{1A}(s) \\ I_{2A}(s) \end{bmatrix} + \begin{bmatrix} z_{11B} & z_{12B} \\ z_{21B} & z_{22B} \end{bmatrix} \begin{bmatrix} I_{1B}(s) \\ I_{2B}(s) \end{bmatrix}$$

$$= \left\{ \begin{bmatrix} z_{11A} & z_{12A} \\ z_{21A} & z_{22A} \end{bmatrix} + \begin{bmatrix} z_{11B} & z_{12B} \\ z_{21B} & z_{22B} \end{bmatrix} \right\} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}$$

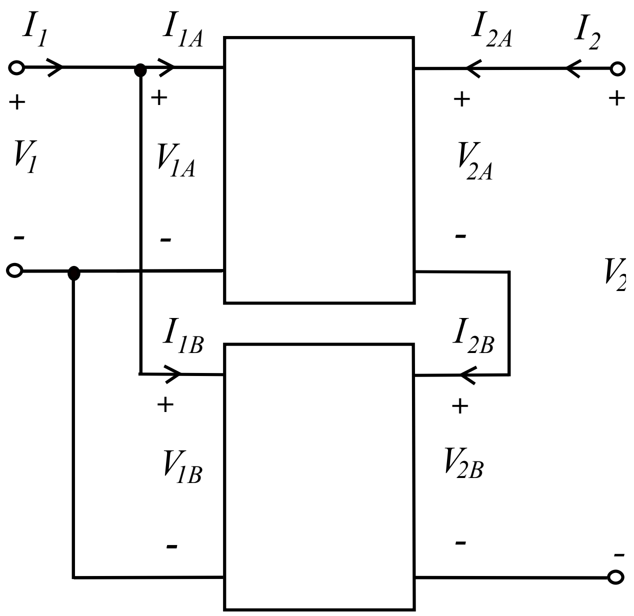
$$Z = Z_A + Z_B$$

Parallel-parallel connection



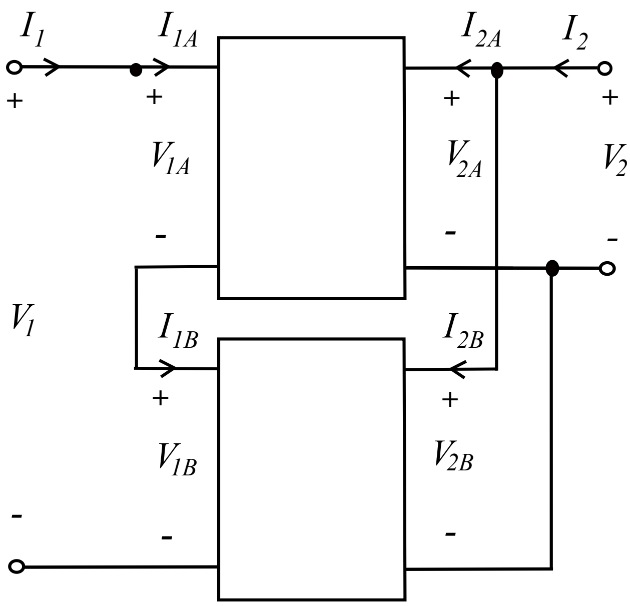
$$Y = Y_A + Y_B$$

Parallel-series connection



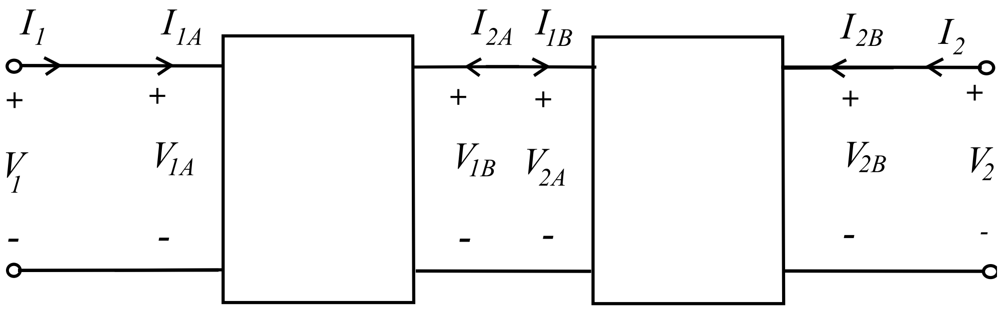
$$g = g_A + g_B$$

Series-parallel connection



$$h = h_A + h_B$$

Cascade connection



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}_A + \begin{bmatrix} A & B \\ C & D \end{bmatrix}_B$$

Good to read : `http://en.wikipedia.org/wiki/Two-port_network`

Common emittor

$$v_{be} = f(i_b, v_{ce}) \ \ i_c = g(i_b, v_{ce}$$

small signal analysis:

$$v_{be} = \frac{v_{be}}{i_b} i_b + \frac{v_{be}}{v_{ce}} v_{ce} = h_i i_b + h_r i_b$$

$$i_c = \frac{i_c}{i_b} i_b + \frac{i_c}{v_{ce}} v_{ce} = h_f i_b + h_o v_{ce}$$

- h_i input impedance with $v_{ce} = 0$. This is AC resistance between base and emitter, the reciprocal of the slope of the current-voltage curve of the input characteristics.
- h_r reverse transfer voltage ratio with $i_b = 0$. In general is small and can be ignored.
- h_f forward transfer current ratio or current amplification factor with $v_{ce} = 0$.
- h_o output admittance with $i_b = 0$. It is slope of the current-voltage curve in the output characteristics. In general is small and can be ignored.

Reciprocal Network

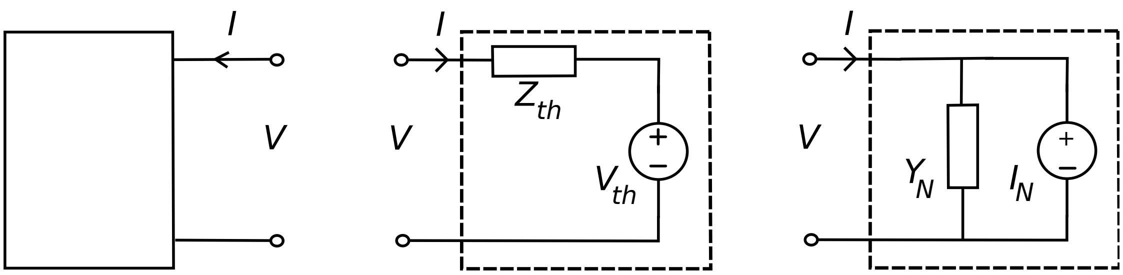
Reciprocity theorem For a reciprocal two-port N, the following relationship holds for each associated two-port representation which exists:

$$
\begin{aligned}
z_{12} &= z_{21} \\
y_{12} &= y_{21} \\
h_{12} &= -h_{21} \\
g_{12} &= -g_{21}
\end{aligned}
$$

READ : PROOF OF THE RECIPROCITY THEOREM (Chua's book page: 776)

- A gyrator ($i_1 = Gv_2$ and $i_2 = -Gv_1$) is not reciprocal two-port
- An ideal transformer ($v_1 = nv_2$ and $i_2 = ni_1$) is a reciprocal two-port.

Thevenin - Norton Equivalent Circuits



Driving-point characteristic of Thevenin equivalent circuit is defined by

$$V = Z_{\mathrm{th}}(s)I + V_{\mathrm{th}}(s)$$