# Basic of Electrical Circuits
## EHB 211E

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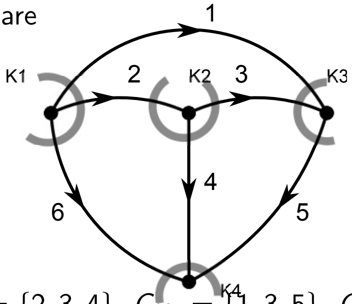
Lecture 4.

# Contents I

# Independent KCL Equations

Cut sets for the nodes are




$G_{K1} = \{1, 2, 6\}$, $G_{K2} = \{2, 3, 4\}$, $G_{K3} = \{1, 3, 5\}$, $G_{K4} = \{4, 5, 6\}$.
If we apply KCL to the cut sets, we obtain

$$
\begin{aligned}
i_1 + i_2 + i_6 &= 0 \\
i_4 + i_3 - i_2 &= 0 \\
i_5 - i_1 - i_3 &= 0 \\
-i_5 - i_4 - i_6 &= 0
\end{aligned}
$$

which are node equations. current reference direction [+ if branch leaves the node]

# Independent KCL Equations

In matrix form

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = 0$$

For an $n_n$-node $n_b$-branch connected graph $G$, node equations are given by

$$A_a i = 0$$

where $i = [i_1\ i_2 \ldots i_{n_b}]^T$ is called the branch current vector. $A_a$ is called incidence matrix of the graph $G$ and $A_a \in \{-1, 0, 1\}^{n_n \times n_b}$.

# Independent KCL Equations

$$
\begin{aligned}
-i_1 - i_2 - i_6 &= 0 \\
-i_4 - i_3 + i_2 &= 0 \\
-i_5 + i_1 + i_3 &= 0 \\
+ - - - -- & \\
-i_5 - i_4 - i_6 &= 0
\end{aligned}
$$

The four equations are linearly dependent. Any three of the four equations are linearly independent.

### Independence property of KCL equations

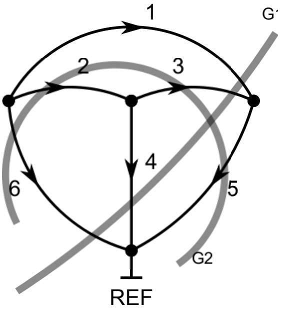
For any connected grahp with $n_n$ nodes, the KCL equations for any $n_n - 1$ of these nodes form a set of $n_n - 1$ linearly independent equations.

# Independent KCL Equations

If in $A_a$, we delete the row corresponding to the datum node (reference node), we obtain the *reduced incidence matrix A which is of dimension* $n_n - 1 \times n_e$*. The KCL equations is given by*

$$A\,i = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = 0$$

# Independent KCL Equations

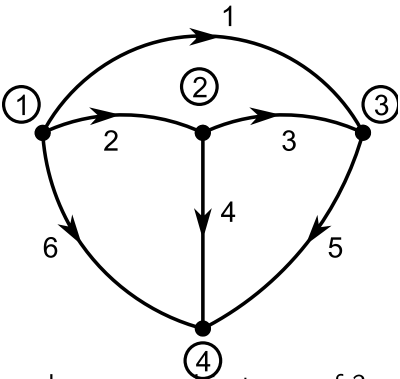



$$i_1 + i_3 + i_4 + i_6 = 0$$

The first one is obtained by first and second node equations

$$\begin{aligned} i_1 + i_2 + i_6 &= 0 \\ i_4 + i_3 - i_2 &= 0 \\ + - - - - - & \\ i_1 + i_3 + i_4 + i_6 &= 0 \end{aligned}$$

# Independent KVL Equations




All 6 edges voltages can be expressed in terms of 3 node-to-datum voltages as follows:

$$\begin{aligned} V_1 &= V_{n1} - V_{n3} \\ V_2 &= V_{n1} - V_{n2} \\ V_3 &= V_{n2} - V_{n3} \\ V_4 &= V_{n2} \\ V_5 &= V_{n3} \\ V_6 &= V_{n1} \end{aligned}$$

# Independent KVL Equations

$$
\begin{aligned}
V_1 &= V_{n1} - V_{n3} \\
V_2 &= V_{n1} - V_{n2} \\
V_3 &= V_{n2} - V_{n3} \\
V_4 &= V_{n2} \\
V_5 &= V_{n3} \\
V_6 &= V_{n1}
\end{aligned}
$$

$$
V = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} V_n
$$

# Independent KVL Equations

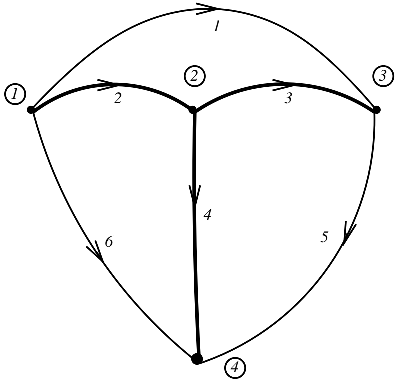
## Independent KVL Equations

For an $n_n$-node $n_b$-branch connected graph G, independent KVL equations are given by

$$V = MV_n$$

where $V = [V_1\ V_2 \dots V_{n_e}]^T$ and $V_n = [V_{n1}\ V_{n2} \dots V_{nn_n-1}]^T$ are called the branch voltage vector and node-to-datum voltage vector, respectively. $M$ is a $n_e \times n_n - 1$ matrix.

# Independent KVL Equations




KVL equation for the closed node sequence 1-2-3-1

$$V_2 + V_3 - V_1 = 0$$

This equation can be obtained from the first three equations.

# Independent KVL and KCL Equations

Comparing the independent KVL and KCL equations, we conclude that

$$M = A^T$$

- current reference directions is chosen,
- a datum node is chosen and the reduced incidence matrix $A$ is defined,
- KCL is dritten as $Ai = 0$
- then we use associated reference directions to find that KVL ($V = A^T V_n$).

## Tellegen's Theorem

Consider an arbitrary circuit. Let the graph have $n_b$ branches. Let us use associated reference directions. Let $i = [i_1 \; i_2 \ldots i_{n_b}]^T$ be any set of branch currents satisfying KCL for $G$ and let $V = [V_1 \; V_2 \ldots V_{n_b}]^T$ be any set of branch voltages satisfying KVL for $G$, then

$$\sum_{k=1}^{n_b} v_k i_k = 0.$$

**Proof** Since $i$ satisfies KCL, we have

$$A\,i = 0$$

and since $V$ satisfies KVL, we have

$$V = A^T\,V_n.$$

Using these two equations, we obtain

$$V^T i = V_n^T A i = V_n^T (A i) = 0.$$