Basic of Electrical Circuits
EHB 211E

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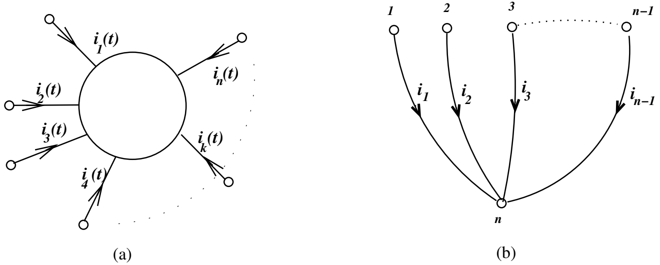
Lecture 2.

Contents I

First Postulate of Circuit Theory

First Postulate of Circuit Theory

All the properties of an n-terminal (or $n-1$-port) electrical element can be described by a mathematical relation between a set of $(n-1)$ voltage and a set of $(n-1)$ current variables.



First Postulate of Circuit Theory

Terminal variables and Terminal equation of n-terminal circuit element:

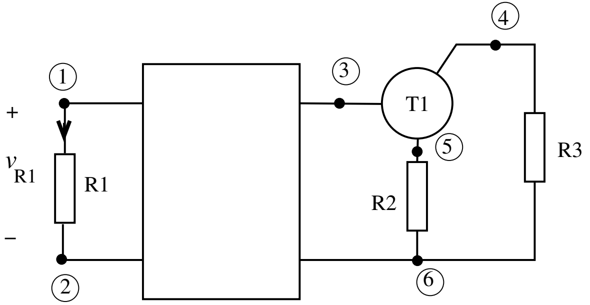
$$v = \begin{bmatrix} V_{1,n} \\ V_{2,n} \\ V_{3,n} \\ . \\ . \\ . \\ V_{n-1,n} \end{bmatrix}, i = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ . \\ . \\ . \\ i_{n-1} \end{bmatrix} \text{ and } f\left(v, i, \frac{dv}{dt}, \frac{di}{dt}, t\right) = 0$$

Power delivered at time t to the n-terminal circuit element:

$$P = \sum_{k=1}^{n} v_k i_k$$

From Circuit to Graph

For a given circuit if we replace each element by its element graph, the result is a directed circuit graph (digraph).



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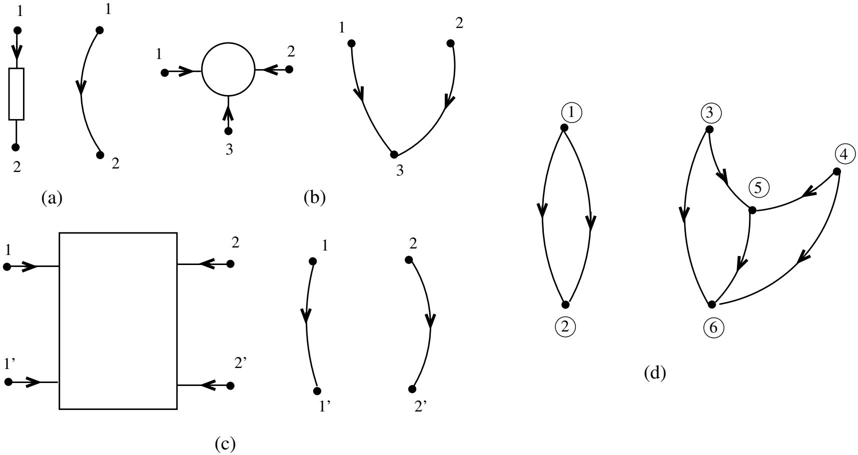
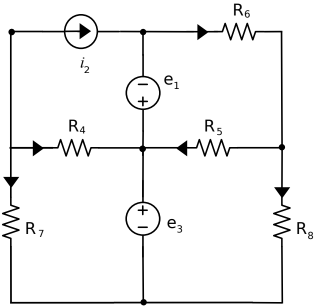
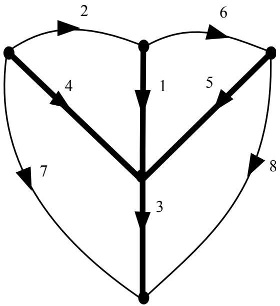
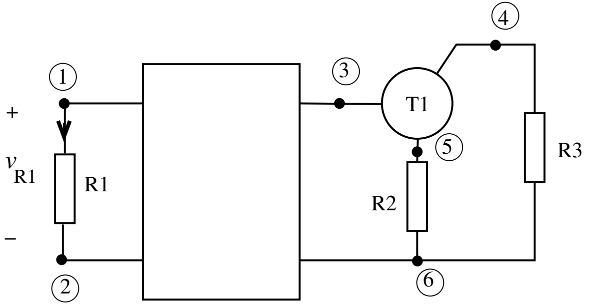


From Circuit to Graph





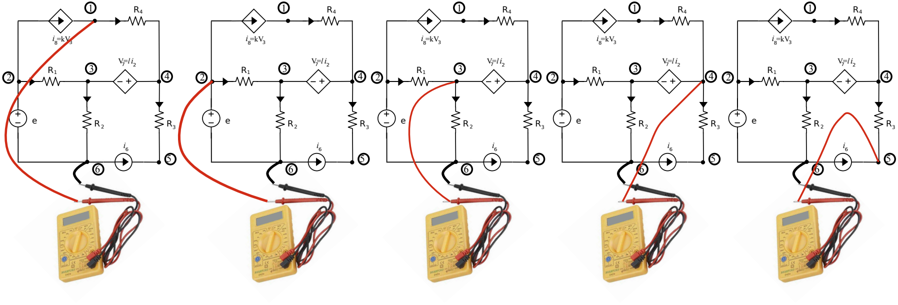
From Circuit to Graph



Node voltages: $e_1, e_2, ...e_n$.

Let $v_{k,l}$denote the voltage difference between node k and node l.

$$v_{k,l} = e_k - e_l$$



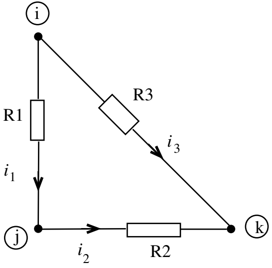
$$e_1, e_2, e_3, e_4, e_5, e_6 = 0$$

$$v_1 = e_2 - e_3; v_2 = e_3 - e_6; v_3 = e_4 - e_5; v_4 = e_1 - e_4; \ldots$$

if we know the nodes voltage, we can calculate all the branch voltages!

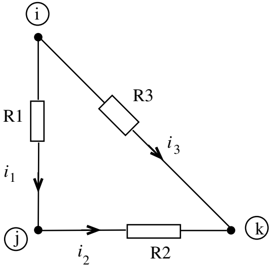
Second Postulate of Circuit Theory: Kirchhoff Voltage Law (KVL)

For all lumped connected circuits, for all closed node sequences, for all times t, the algebraic sum of all node-to-node voltages around the chosen closed node sequence is equal to zero.



Let us consider the closed node sequence $i - j - k - i$.

Kirchhoff Voltage Law (KVL)



Let us consider the closed node sequence $i - j - k - i$.

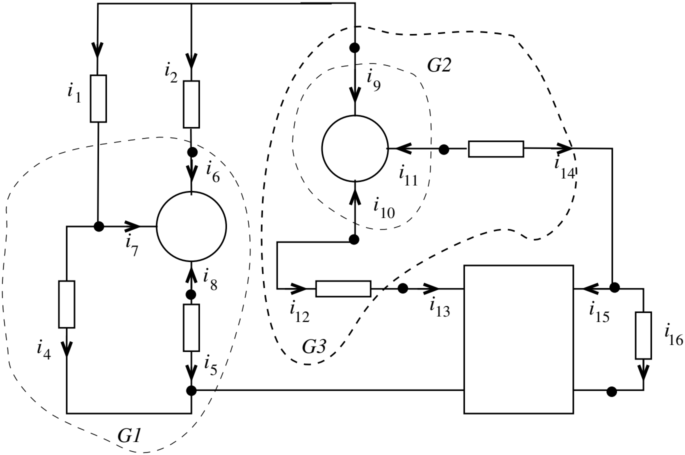
$$V_{i,j} + V_{j,k} + V_{k,i} = 0$$

$$e_i - e_j + e_j - e_k + e_k - e_i = 0$$

Kirchhoff Current Law (KCL)

Gaussian Surface

It is a closed surface such that it cuts only the connecting wires which connect the circuit elements.



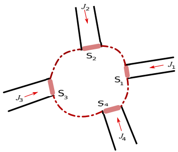
KCL from Electromagnetism Theory

Continuity equation;

Charge is leaving the enclosed volume

$$\int_S J\ da \ = \ -\frac{d}{dt}\int_V \rho\ dv$$

The total charge inside the volume at any instant



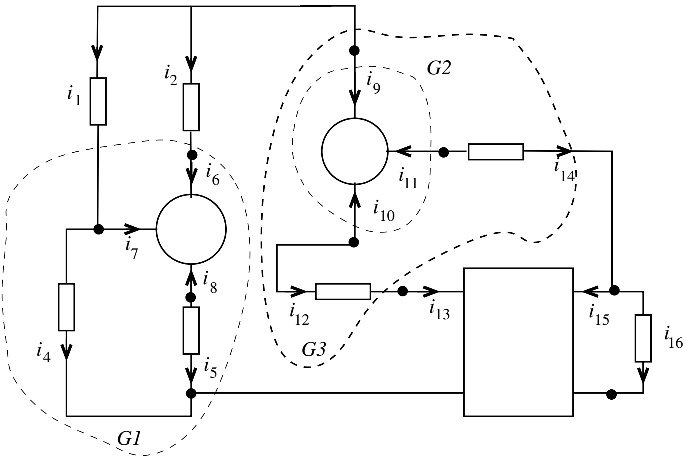
if charge density is constant (ρ); $\int_S J\ da = 0$. Conductive currents are within wires so J_i is non-zero only though S_i,

$$\int_S J\ da = \sum_{i=1}^{4}\int_{S_i} J\ da = \sum_{i=1}^{4} i_i = 0$$

Kirchhoff Current Law (KCL)

Third Postulate of Circuit Theory: Kirchhoff Current Law (KCL)

For all lumped circuits, for all gaussian surfaces G, for all times t, the algebraic sum of all the currents leaving the gaussian surface G at time t is equal to zero.

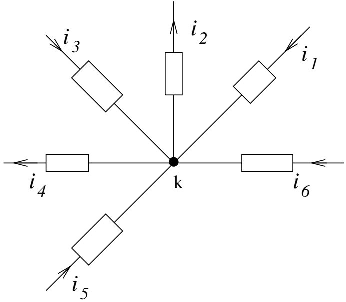


$$
\begin{aligned}
G1 \quad & i_1 + i_2 + i_{13} = 0 \\
G2 \quad & i_9 + i_{10} + i_{11} = 0 \\
G3 \quad & i_9 - i_{12} - i_{14} = 0
\end{aligned}
$$

Kirchhoff Current Law (KCL)

KCL (node law)

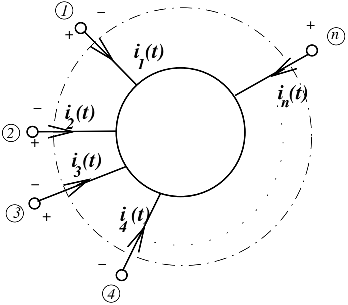
For all lumped circuits, far all times t, the algebraic sum of the currents leaving any node is equal to zero.



For the node k:

$$i_1 - i_2 + i_3 - i_4 + i_5 + i_6 = 0$$

Examples

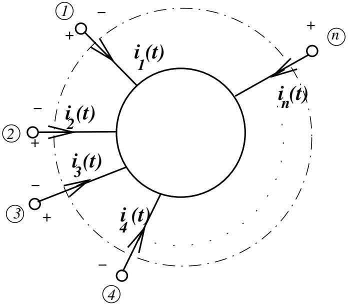


For Gaussian surface;

$$i_1 + i_2 + ...i_{n-1} + i_n = 0$$

$n - 1$ currents can be specified independently! Why ?

$$-i_n = i_1 + i_2 + ...i_{n-1}$$



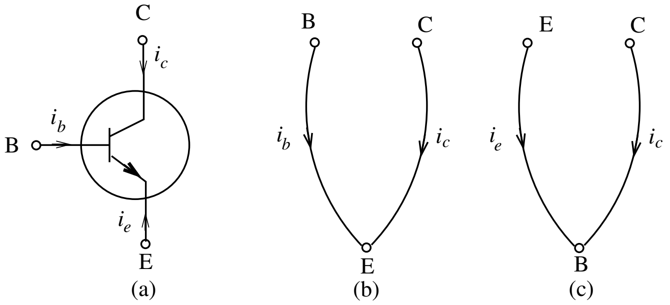
Let us consised the closed node sequence 1-2-3-...-n-1 and apply KVL (the sum of the voltages is equal to zero)

$$V_{1,2} + V_{2,3} + V_{3,4} + .. + V_{n-1,n} + V_{n,1} = 0$$

$n-1$ voltages can be specified independently! Why ?

$$-V_{n,1} = V_{1,2} + V_{2,3} + V3,4 + .. + V_{n-1,n}$$

Remember : First Postulate of Circuit Theory: All the properties of an n-terminal (or $n-1$-port) electrical element can be described by a mathematical relation between a set of $(n-1)$ voltage and a set of $(n-1)$ current variables



Mathematical model is given by the terminal equation

$$\begin{bmatrix} v_{bc} \\ i_c \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_b \\ V_{ce} \end{bmatrix}$$

and terminal graph (b). Find the terminal equation in the form

$$\begin{bmatrix} v_{eb} \\ i_c \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} i_e \\ V_{cb} \end{bmatrix}$$

if (c) is the terminal graph.

Terminal equations

$$\begin{aligned} v_{bc} &= h_{11} i_b + h_{12} v_{ce} \\ i_c &= h_{21} i_b + h_{22} v_{ce} \end{aligned}$$

KCL and KVL for the circuit element

$$\begin{aligned} i_c + i_e + i_b &= 0 \\ v_{ce} + v_{eb} + v_{bc} &= 0. \end{aligned}$$

New terminal variables are i_e and V_{eb} (additional to i_c and V_{cb}).
Substituting KVL and KCL Eqs. into above Eqs. we obtain

$$\begin{aligned} v_{bc} &= h_{11}(-i_c - i_e) + h_{12}(-v_{eb} + v_{cb}) \\ i_c &= h_{21}(-i_c - i_e) + h_{22}(-v_{eb} + v_{cb}) \end{aligned}$$

$$\begin{aligned} h_{12}v_{eb} + h_{11}i_c &= -h_{11}(i_e) + (1 + h_{12})v_{cb} \\ (1 + h_{21})i_c + h_{22}v_{eb} &= -h_{21}i_e + h_{22}v_{cb} \end{aligned}$$

New terminal equations

$$\begin{bmatrix} h_{12} & h_{11} \\ h_{22} & 1 + h_{21} \end{bmatrix} \begin{bmatrix} v_{eb} \\ i_c \end{bmatrix} = \begin{bmatrix} -h_{11} & (1 + h_{12}) \\ -h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_e \\ V_{cb} \end{bmatrix}$$

and terminal graph (c) will be the new mathematical model !

Tellegen Theorem

Tellegen's theorem is based on the fundamental law of conservation of energy!

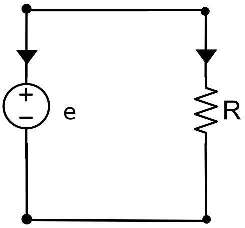
Tellegen Theorem

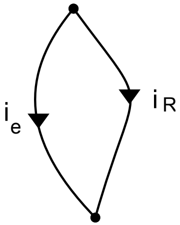
It states that the algebraic sum of power absorbed by all elements in a circuit is zero at any instant.

Tellegen's theorem asserts that

$$\sum_{k=1}^{n_e} v_k i_k = 0$$

Tellegen Theorem





$R = 2\Omega$ and $e = 2V$, from KCL

$$i_e = -i_R = \frac{2}{2} = 1A$$

Lets apply Tellegen Theorem:

$$P = i_e \cdot e + V_R \cdot i_R = 2 \cdot (-1) + 2 \cdot 1$$

Power absorbed by a resistor is always positive, whereas a source may deliver power. Then in this case, the power associated with the source is negative.