Basic of Electrical Circuits
EHB 211E

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Lecture 11

Contents I

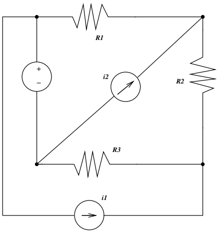
Superposition Theorem

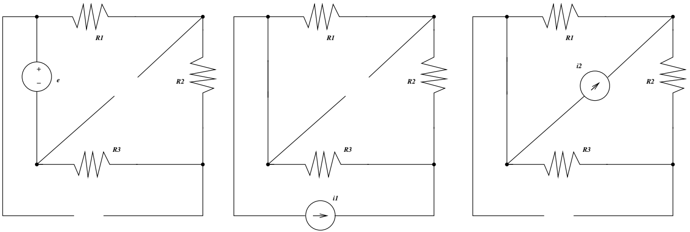
The current through, or voltage across, an element in a linear time-invariant circuit is equal to the algebraic sum of the currents $(i_{k1}, i_{k2}, ...i_{km})$ or voltages $(V_{k1}, V_{k2}, ...V_{kn})$ produced independently by each source. In generally, The current through, or voltage across, an element can be given

$$y = H_1 V_{k1} + ... + H_n V_{kn} + K_1 i_{k1} + ... + K_m i_{km}$$

When there exist multiple energy sources, the currents and voltages in the circuit can be found as the algebraic sum of the corresponding values obtained by assuming only one source at a time, with all other sources turned off.

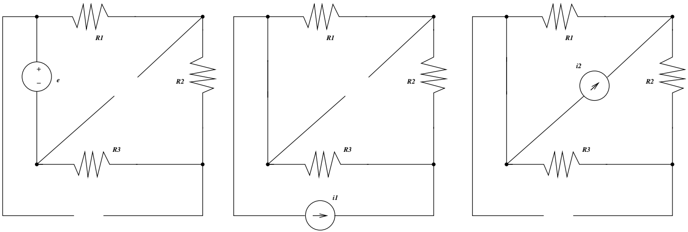
A voltage source is treated as short circuit so that $V = 0$. A current source is treated as open circuit so that $i = 0$.





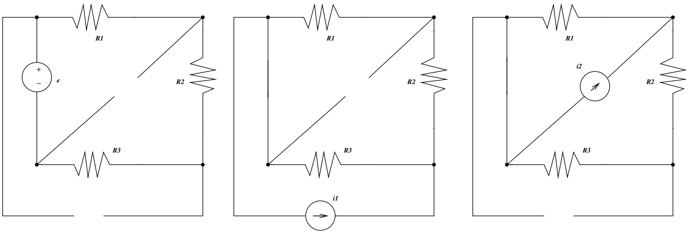
Proposed solution

$$y(e, i_1, i_2) = y(e, 0, 0) + y(0, i_1, 0) + y(0, 0, i_2)$$



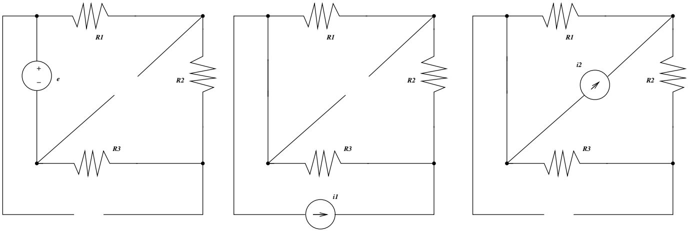
$y(e, 0, 0)$ is equal to the response of y when all independent sources in the circuit except e are set to zero.

$$V_1 = R_1 \frac{e}{R_1 + R_2 + R_3}$$



$y(0, i_1, 0)$ is equal to the response of y when all independent sources in the circuit except i_1 are set to zero.

$$V_1 = \frac{R_1}{R_1 + R_2} \frac{i_1}{\frac{1}{R_1+R_2} + G_3}$$



$y(0,0,i_2)$ is equal to the response of y when all independent sources in the circuit except i_2 are set to zero.

$$V_1 = \frac{i_2}{\frac{1}{R_3+R_2} + G_1}$$

Adding the respective contributions, we obtain:

$$V_1 = R_1 \frac{e}{R_1 + R_2 + R_3} + \frac{R_1}{R_1 + R_2} \frac{i_1}{\frac{1}{R_1+R_2} + G_3} + \frac{i_2}{\frac{1}{R_3+R_2} + G_1}$$

Thevenin Equivalent Circuit

A one-port N is said to be well-defined iff it does not contain any circuit element which is coupled to some physical variable outside of N.

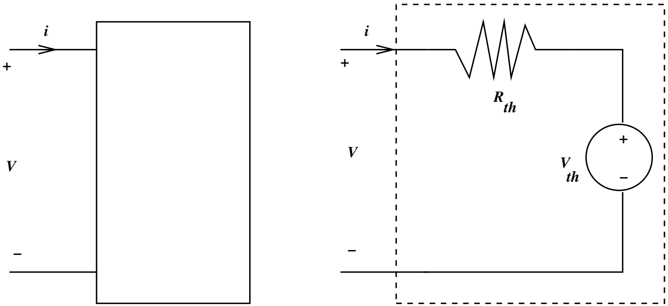
Thevenin Equivalent Circuit

Any well-defined linear time-invariant resistive one-port N can be replaced by one-port N_{eq} which consists of a single voltage source and a single series resistor. Its driving-point characteristic at time t is defined by

$$V = R_{th} i + V_{th}$$

R_{th}: Thevenin equivalent resistance.
V_{th}: Open-circuit voltage.

Thevenin Equivalent Circuit



Calculating the Thévenin equivalent

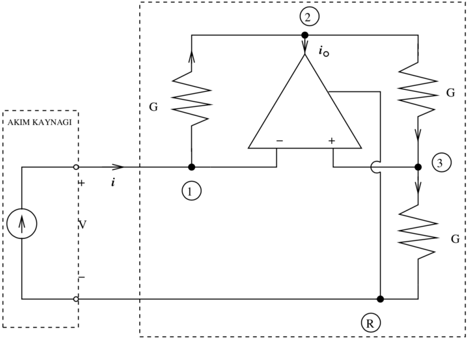
I. Method

- One-port N is driven by an ideal current source.
- Find the terminal voltage in terms of the internal energy sources inside the network and the external current source.

Then the terminal voltage is obtained such as

$$V = R_{th} i + V_{th}$$

Calculating the Thévenin Equivalent



Node equations;

$$
\begin{aligned}
i - G(V_{d1} - V_{d2}) - i_- &= 0 \\
i_0 - G(V_{d1} - V_{d2}) + G(V_{d2} - V_{d3}) &= 0 \\
i_+ - GV_{d3} - G(V_{d2} + V_{d3}) &= 0
\end{aligned}
$$

Thevenin Equivalent Circuit:

From the definition of op amp, we have

$$\begin{aligned} V_- - V_+ &= 0 \\ i_+ = i_- &= 0 \end{aligned}$$

which implies that $V_{d1} = V_{d3}$. Using above equations, V will be represents in the term of the independent sources in circuit and the current of source.

$$\begin{aligned} i - G(V_{d1} - V_{d2}) &= 0 \\ -GV_{d1} + G(V_{d2} - V_{d1}) &= 0 \end{aligned}$$

We have $V_{d2} = 2V_{d1}$. Substituting this eqn. into the first eqn. we will have $i + GV_{d1} = 0$. Then Thevenin Equivalent is

$$V = -Ri$$

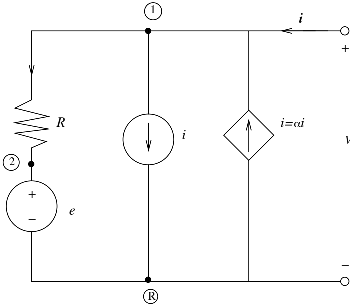
What are the Thevenin equivalent resistance and Open-circuit voltage?

Thevenin Equivalent Circuit:

II. Method

- The terminals be open-circuited, which has the effect of setting the current i to zero. Open-circuit voltage V_{th} which equals the Thévenin equivalent voltage is obtained.
- Setting all independent sources inside the circuit to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resistance (R_{th}) between the two marked terminals.

Thevenin Equivalent Circuit



KCL for the Node ①

$$G(V_{d1} - V_{d2}) + i - i_b = 0$$

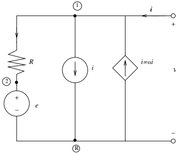
Substituting $V_{d2} = e$ and $i_b = \alpha G(V_{d1} - G_{d2})$ into above eqn.

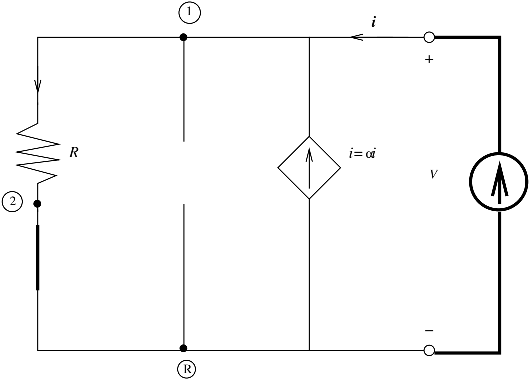
$$G(V_{d1} - e) + i - \alpha G(V_{d1} - e) = 0$$

then

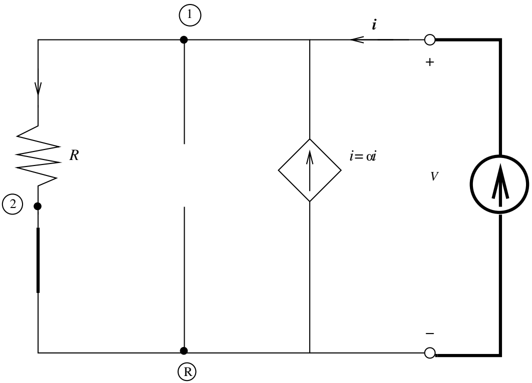
$$V_{d1} = V = \frac{Ri}{\alpha - 1} + e$$

Thevenin Equivalent Circuit





Thevenin Equivalent Circuit



KCL for the node ①; $-i + GV_{d1} - i_b = 0$. Substitute $i_b = \alpha G(V_{d1})$ into the KCL eqn. $-i + GV_{d1} - \alpha GV_{d1} = 0$. Then we obtain

$$i = (\alpha - 1)GV_{d1}$$

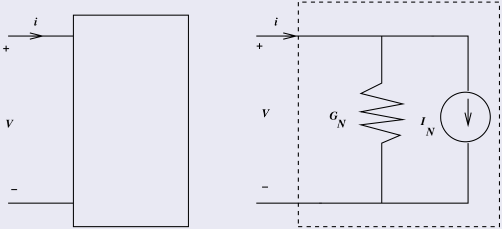
we know that $V = V_{d1}$ hence $R_{th} = \frac{R}{\alpha - 1}$.

Norton Equivalent Circuit

Any well-defined linear time-invariant resistive one-port N can be replaced by one-port N_{eq} which consists of a a single current source and a single parallel resistor. Its driving-point characteristic at time t is defined by

$$i = G_N V + I_N$$

which G_N Norton equivalent canductance, I_N short-circuit current



Norton Equivalent Circuit

- I. Method
  - One-port N is driven by an ideal voltage source.
  - Find the terminal current in terms of the internal energy sources inside the network and the external voltage source.

  Then the terminal current is obtained such as

$$i = G_N V + I_N$$

- II. Method
  - The terminals be short-circuited, which has the effect of setting the voltage V to zero. Short-circuit current I_N is obtained.
  - Setting all independent sources inside the circuit to zero and then finding the conductance (G_N) between the two marked terminals.