Basic of Electrical Circuits
EHB 211E

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Lecture 10

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- Loop Current Method (Mesh Current Method)
- Generalized Mesh Current Method

Loop Current Method (Mesh Current Method)

The number of equations to be solved are equal to the number of independent loops. There exists a tree such that the meshes are Fundamental loops*. In this method KVL equations for the meshes (Mesh equations) will be used and the mesh currents will be the unknown variables !

Mesh Current Method

1. Draw the circuit graph and assign the loop currents.
2. Write mesh equations are obtained from KVL which is applied to each of the loop: $Bv = 0$ It can also be written such as

$$Bv = B_1 v_R + B_2 v_k = 0$$

where v_k and v_R voltages of independent voltage sources and resistors, respectively.

3. Write the ohm law for the resistors: $v_R = \mathbf{R} i_R$
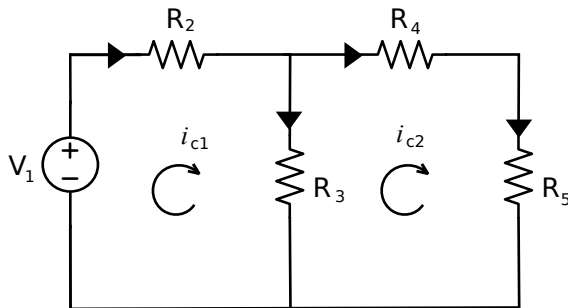4. Substitute the equations in Step 3 into the equations in Step 2.

$$[B_1 \ B_2]v = B_1 \mathbf{R} i_R + B_2 v_k = 0$$

5. Resistors's currents are written in terms of the mesh currents:
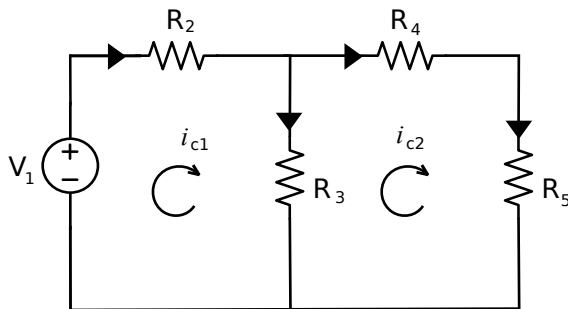
$$i_R = B_1^T i_c$$

6. Substitute the equation in step 5 into step 4:

1. The loop currents are assigned.
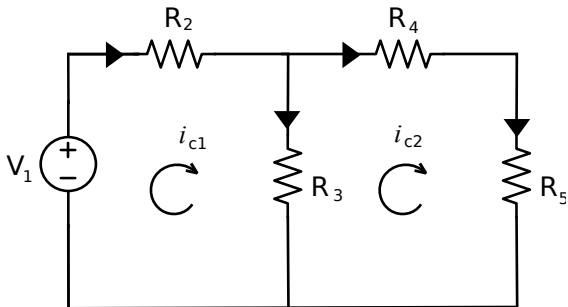
2. Mesh equations:

$$V_2 + V_3 - V_1 = 0$$
$$V_4 + V_5 - V_3 = 0$$

3. Substitute the $v_R = R i_R$ equations into the equations in Step 2:

$$R_2 i_2 + R_3 i_3 - V_1 = 0$$
$$R_4 i_4 + R_5 i_5 - R_3 i_3 = 0$$

4. Resistors's currents are written in terms of the mesh currents:

$$i_2 = i_{c1}$$
$$i_3 = i_{c1} - i_{c2}$$
$$i_4 = i_{c2}$$
$$i_5 = i_{c2}$$

5. Substitute the equation in step 4 into step 3:

$$\begin{aligned} R_2 i_{c1} + R_3(i_{c1} - i_{c2}) - V_1 &= 0 \\ R_4 i_{c2} + R_5 i_{c2} - R_3(i_{c1} - i_{c2}) &= 0 \end{aligned}$$

6. In matrix form:

$$\left[\begin{array}{cc} R_2 + R_3 & -R_3 \\ -R_3 & R_3 + R_4 + R_5 \end{array} \right] \left[\begin{array}{c} i_{c1} \\ i_{c2} \end{array} \right] + \left[\begin{array}{c} 1 \\ 0 \end{array} \right] V_1 = 0$$

Generalized Mesh Current Method

1. Draw the circuit graph and assign the loop currents.
2. Write mesh equations.

$$Bv = B_1 v_R + B_2 v_2 + B_3 v_3 = 0$$

where v_1 resistors, v_2 voltage sources and v_3 other elements's voltages.

3. Write the ohm law for the resistors: $v_1 = \mathbf{R} i_1$
4. Substitute the equations in Step 3 into the equations in Step 2.

$$B_1 \mathbf{R} i_1 + B_2 v_1 + B_3 v_3 = 0$$

5. Resistors's currents are written in terms of the mesh currents:

$$i_1 = B_1^T i_c$$

6. Substitute the equation in step 5 into step 4:

Generalized Mesh Current Method

Srep 6: Write $v - i$ relations of the other elements

$$M_3 i_3 + N_3 v_3 = T i_3$$

and i_3 is written in the term of the mesh currents
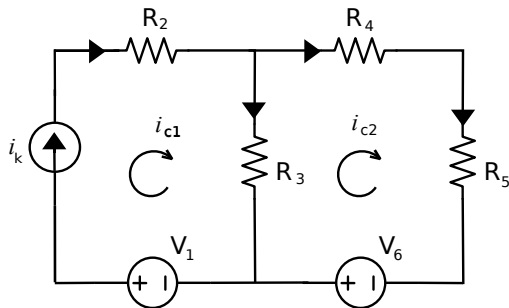
$$M_3 B_3^T i_c + N_3 v_3 = T i_3$$

together with

$$B_1 \mathbf{R} B_1^T i_c + B_2 v_2 + B_3 v_3 = 0$$

Equations can be presented in matrix form:

$$\left[\begin{array}{cc} B_1 \mathbf{R} B_1^T & B_3 \\ M_3 B_3^T & N_3 \end{array} \right] \left[\begin{array}{c} i_c \\ v_3 \end{array} \right] = - \left[\begin{array}{c} B_2 \\ 0 \end{array} \right] v_2 - \left[\begin{array}{c} 0 \\ T i_3 \end{array} \right]$$

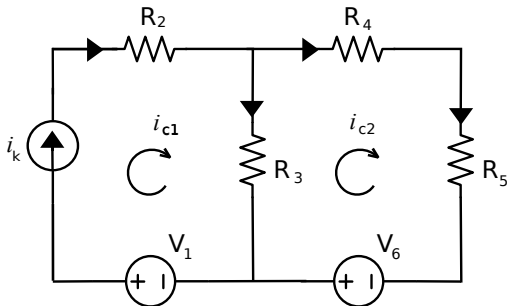The unknown variables : mesh currents and voltages of the current sources!

1. The loop currents are assigned.
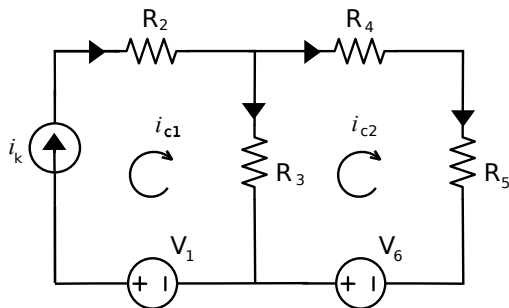
2. Mesh equations:

$$V_2 + V_3 - V_1 + V_k = 0$$
$$V_4 + V_5 - V_6 - V_3 = 0$$

3. Substitute the $v_R = Ri_R$ equations into the equations in Step 2:

$$R_2 i_2 + R_3 i_3 - V_1 + V_k = 0$$
$$R_4 i_4 + R_5 i_5 - V_6 - R_3 i_3 = 0$$

4. Resistors's currents are written in terms of the mesh currents:

$$i_2 = i_{c1}$$
$$i_3 = i_{c1} - i_{c2}$$
$$i_4 = i_{c2}$$
$$i_5 = i_{c2}$$

5. Substitute the equation in step 5 into step 4:

$$\begin{aligned}
R_2 i_{c1} + R_3(i_{c1} - i_{c2}) - V_1 + V_k &= 0 \\
R_4 i_{c2} + R_5 i_{c2} - V_6 - R_3(i_{c1} - i_{c2}) &= 0
\end{aligned}$$

Additional equation:

$$i_k = i_{c1}$$

6. Unknown mesh current:

$$(R_4 + R_5 + R_3)i_{c2} - R_3 i_k - V_6 = 0$$