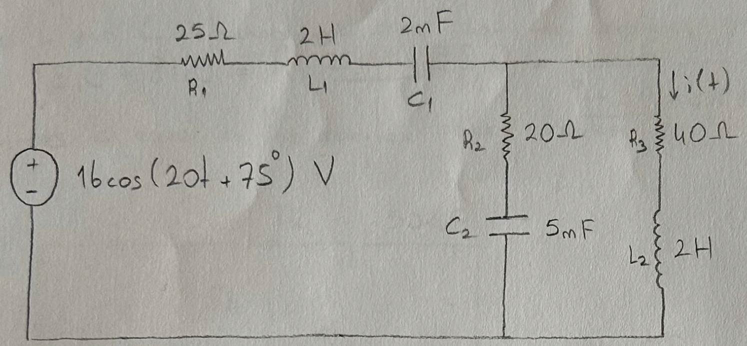
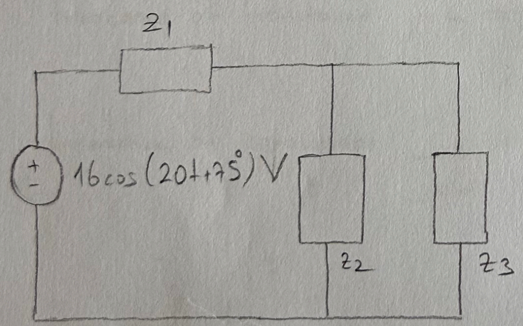
P 10.5 - 23

Determine the steady-state current i(t) for the circuit in Figure P 10.5-23.



$Z_{L_1} = j\omega L = j \cdot 20 \cdot 2 = 40j\ \Omega$ $\quad Z_{C_1} = \dfrac{1}{j\omega C} = \dfrac{10^3}{j \cdot 20 \cdot 2} = -25j\ \Omega$

$Z_{L2} = j\omega L = j \cdot 20 \cdot 2 = 40j\ \Omega$ $\quad Z_{C2} = \dfrac{1}{j\omega C} = \dfrac{10^3}{j \cdot 20 \cdot 5} = -10j\ \Omega$



$Z_1 = 25 + 15j\ \Omega$

$Z_2 = 20 - 10j\ \Omega$

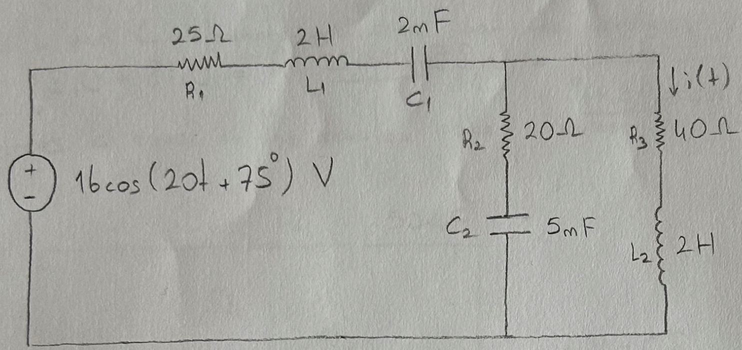
$Z_3 = 40 + 40j\ \Omega$

$Y_2 = \dfrac{1}{20-10j}\ S$, $Y_3 = \dfrac{1}{40+40j}\ S$ $\Longrightarrow Y_2 + Y_3 = 0{,}525 + 7{,}5\times10^{-3}j\ S$

$\dfrac{1}{Y_2 + Y_3} = Z_{23} = 18{,}67 - 2{,}67j$

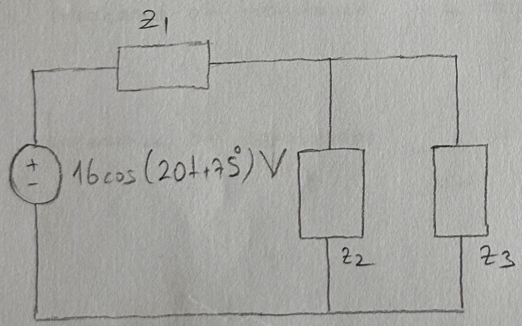
P 10.5-23

Determine the steady-state current i(t) for the circuit in Figure P 10.5-23.



$Z_{L_1} = j\omega L = j\cdot 20\cdot 2 = 40j\ \Omega$ $\quad Z_{C_1} = \frac{1}{j\omega C} = \frac{10^3}{j\cdot 20\cdot 2} = -25j\ \Omega$

$Z_{L2} = j\omega L = j\cdot 20\cdot 2 = 40j\ \Omega$ $\quad Z_{C2} = \frac{1}{j\omega C} = \frac{10^3}{j\cdot 20\cdot 5} = -10j\ \Omega$



$Z_1 = 25 + 15j\ \Omega$

$Z_2 = 20 - 10j\ \Omega$

$Z_3 = 40 + 40j\ \Omega$

$Y_2 = \frac{1}{20-10j}\ S\ ,\ Y_3 = \frac{1}{40+40j}\ S \implies Y_2 + Y_3 = 0{,}525 + 7{,}5\times10^{-3} j\ S$

$\frac{1}{Y_2 + Y_3} = Z_{23} = 18{,}67 - 2{,}67j$

$$Z_{eq} = 25 + 15j + 18,67 - 2,67j = 43,67 + 12,33j \ \Omega$$

$$Z_{eq} = 45,38 \angle 15,77^\circ$$

$$I = \frac{V}{Z_{eq}} = \frac{16 \angle 75^\circ}{45,38 \angle 15,77^\circ} = 0,35 \angle 59,23^\circ \ A = 0,18 + 0,3j$$

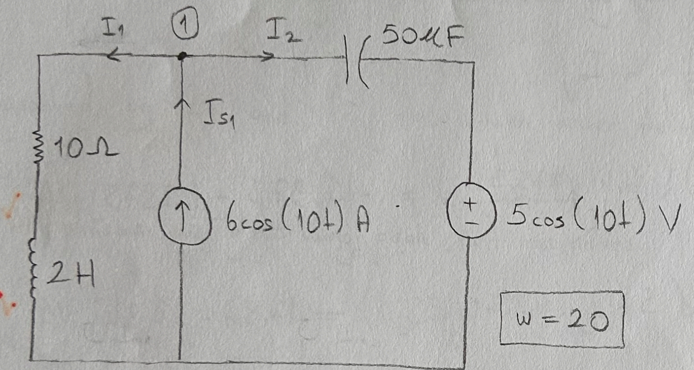
From current divider:

$$(0,18 + 0,3j) \cdot \frac{20 - 10j}{(20 - 10j) + (40 + 40j)} = 0,116 + 0,012j = i(t)$$

$$i(t) \cong 0,12 \angle 6.10^\circ \ A = 0,12 \cos(20t + 6,10^\circ) \ A$$

P 11.5 - 6

For the circuit of Figure P 11.5 - 6, determine the complex power of the R, L and C elements and show that the complex power delivered by the sources is equal to the complex power absorbed by the R, L and C elements.



$I_{S1} = 6\cos(10t)$ A

$V_{S2} = 5\cos(10t)$ V

$\boxed{w = 20}$

Impedance of inductance : $j\omega L \Rightarrow 20j$ ✓

Impedance of capacitance : $\dfrac{1}{j\omega C} \Rightarrow -2j$ ✓

From node 1 :

$$I_{S1} = I_1 + I_2 \Rightarrow 6\angle 0^\circ = I_1 + I_2 \Rightarrow I_1 = 6\angle 0^\circ - I_2$$

From KVL :

$$(10 + 20j)\, I_1 - 5\angle 0^\circ + (2j)\, I_2 = 0$$

$$\Longrightarrow (10+20j)(6-I_2) - 5 + (2j)I_2 = 0$$

$$\Longrightarrow 60 - 10I_2 + 120j - 20jI_2 - 5 + 2jI_2 = 0$$

$$\Longrightarrow I_2(-10-20j+2j) = -55 - 120j \Longrightarrow I_2 = 6,39 + 0,495j \text{ A} \longrightarrow \boxed{I_2 = 6,41 \angle 4,47^\circ \text{ A}}$$

$$I_1 = -0,39 - 0,5j \text{ A} \longrightarrow \boxed{I_1 = 0,63 \angle 232^\circ \text{ A}}$$

<u>The delivered powers:</u> $\left(\frac{1}{2} \cdot V_m \cdot I_m\right)$

$$S_{5\angle 0^\circ} = \frac{1}{2} \cdot (5\angle 0^\circ)(6,41 \angle 175,53^\circ) = -15,975 + 1,2375j \text{ VA}$$

$$S_{6\angle 0^\circ} = \frac{1}{2} (6\angle 0^\circ)(5 - 2j(6,39+0,495j)) = 17,97 - 38,34j \text{ VA}$$

Total delivered power $\longrightarrow (-16+1,1j) + (18-38,3j) = 2 - 37,1j$ VA

<u>The absorbed powers:</u>

+12,5

$$S_{20j\Omega} = (\underbrace{-0,39 - 0,5j}_{I_1})^2 \cdot \left(\frac{20j}{2}\right) = 4j \text{ VA}$$

$$S_{10\Omega} = (\underbrace{-0,39 - 0,5j}_{I_1})^2 \cdot \left(\frac{10}{2}\right) = 2 \text{ VA}$$

$$S_{-2j\Omega} = (\underbrace{6,39 + 0,495j}_{I_2})^2 \left(\frac{-2j}{2}\right) = -41,1j \text{ VA}$$

Total absorbed power $\longrightarrow 2 - 37,1j$ VA

total absorbed power = total delivered power

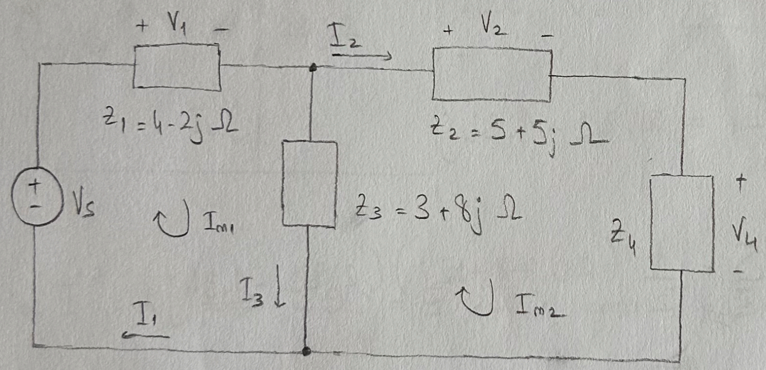
P 11.5 - 14

The source voltage in the circuit shown in Figure P 11.5 - 14 is $V_s = 24\angle 30^\circ$ V.

Consequently, $I_1 = 3,13\angle 25,4^\circ$ A, $I_2 = 1,99\angle 52,9^\circ$ A and $V_4 = 8,88\angle -10,6^\circ$ V.

Determine (a) the average power absorbed by Z_4, (b) the average power absorbed by Z_1, and (c) the complex power delivered by the voltage source.

(All phasors are given using peak, not rms values.)



$I_{m1} = I_1 = 2,82 + 1,34j$

$V_s = 20,78 + 12j$

$I_{m2} = I_2 = 1,2 + 1,58j$

Mesh 1

$$(4-2j)\,I_{m1} + (3+8j)(I_{m1} - I_{m2}) - \underbrace{(20,78+12j)}_{V_s} = 0$$

Mesh 2

$$(5+5j)\,I_{m2} + Z_4 . I_{m2} - (3+8j)(I_{m1} - I_{m2}) = 0$$

$$\Longrightarrow (5+5j)(1,2+1,58j) + Z_4(1,2+1,58j) = (3+8j)(1,62-0,24j)$$

$$Z_4(1,2+1,58j) = 8,68 - 1,66j \Longrightarrow Z_4 = 2-4j\ \Omega$$

$$S_{Z_4} = \frac{1}{2} . (1,2+1,58j)^2 . (2-4j) = 6,52 + 5,9j \text{ VA}$$ → the average power absorbed by Z_4

$$S_{Z_1} = \frac{1}{2} . (2,82+1,34j)^2 . (4-2j) = 0,23 + 1,06j \text{ VA}$$ → the average power absorbed by Z_1

$$V_s = 20,78 + 12j \quad , \quad I_1 = 2,82 + 1,34j$$

$$S = \frac{1}{2} . V . \overset{*}{I} \Longrightarrow S = \frac{1}{2}(20,78+12j)(2,82-1,34j)$$

$$S = 37,33 + 3j \text{ VA}$$ → the complex power delivered by the voltage source

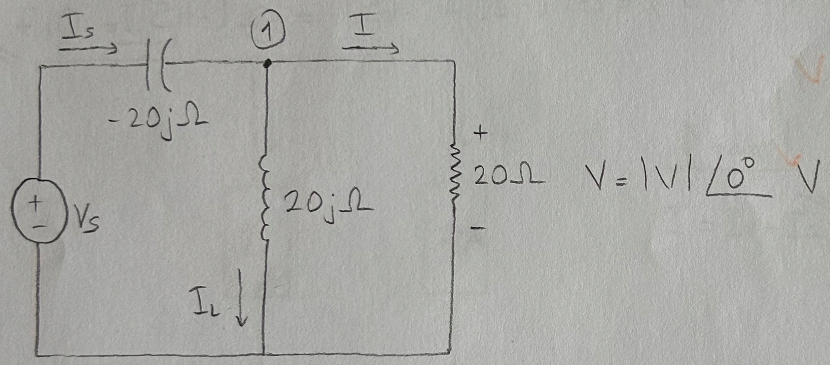
P 11.6-4

Consider the model of one communication circuit, shown in Figure P 11.6-4.

If an average power of 500 W is dissipated in the 20Ω resistor,

find (a) V_{rms}, (b) I_{rms}, (c) the power factor seen by the source, and

(d) $|V_s|$.



$$P = I^2 . R \implies 500 = I^2 . 20 \implies I = 5A$$

$$V = I . R \implies 5\angle 0^\circ \times 20\angle 0^\circ = 100\angle 0^\circ \ V$$

$$I_L = \frac{100\angle 0^\circ}{20j} = 5\angle -90^\circ \ A$$

From node $1 \rightarrow I_s = I + I_L = 5\sqrt{2}\angle -45^\circ \ A$

We can find V_s with : $V_s = V + I_s . (-20j)$

$$V_s = 100\angle 0^\circ + (5\sqrt{2}\angle -45^\circ . (-20j))\ V$$

$$V_s = 100\angle -90^\circ\ V$$

$\Longrightarrow V_{rms} = 100\,V$ 2

(+12,5)

$\Longrightarrow I_{rms} = 7{,}07\,V$ ✓

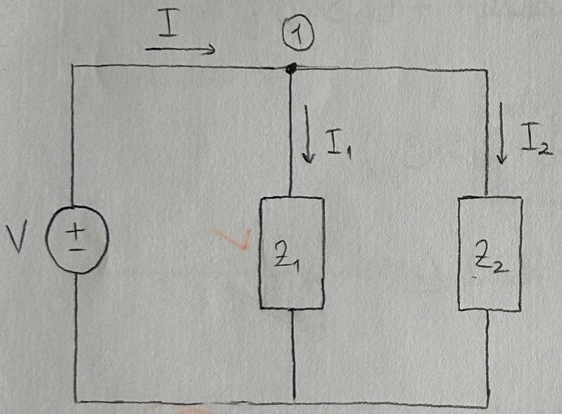
$\Longrightarrow$ power factor $= \cos(\theta_v - \theta_i) = \cos(-90^\circ - (-45^\circ)) = \cos(-45^\circ) = 0{,}7$

↳ lagging ✓

$\Longrightarrow |V_s| = 100V$ ✓

P 11.6 - 5

Two impedances are supplied by $V = 100 \angle 160^\circ$ V_{rms}, as shown in Figure P 11.6-5, where $I = 2 \angle 130^\circ$ A_{rms}. The first load draws $P_1 = 23,2$ W, and $Q_1 = 50$ VAR. Calculate I_1, I_2, the power factor of each impedance, and the total power factor of the circuit.



Node 1

$I = I_1 + I_2$

$S = P + j \cdot Q \implies S_1 = P_1 + j \cdot Q_1$

S → visual power

P → real power

Q → reactive power

$P_1 = 23,2$ W , $Q_1 = 50$ VAR

$S_1 = 23,2 + 50j \implies (23,2)^2 + (50)^2 = (55,12)^2$

$S_1 = 55,12 \angle 65,1^\circ$ VA $\implies pf_1 = 65,1^\circ = 0,422 \rightarrow$ lagging 2

$$S_1 = I_1^* . V \Longrightarrow I_1^* = \frac{S_1}{V} = \frac{55,12\angle 65,1^\circ}{100\angle 160^\circ} = 0,551\angle -94,9^\circ$$

$I_1 = 0,551\angle 94,9^\circ$ A

I is given as $2\angle 190^\circ$ A. From Node 1, $I = I_1 + I_2$.

$I_2 = I - I_1 \longrightarrow$ We have to write components of I and I_1.

$I = -1,97 - 0,348j$ A , $I_1 = -0,047 + 0,549j$ A

$I_2 = (-1,97 - 0,348j) - (-0,047 + 0,549j)$

$I_2 = -1,923 - 0,897j \Longrightarrow I_2 = 2,12\angle -155^\circ$ A

$S_2 = I_2^* . V = (2,12\angle 155^\circ)(100\angle 160^\circ) = 212\angle -45^\circ$ +12,5

$\Longrightarrow pf_2 = \cos(-45^\circ) = 0,707 \longrightarrow$ leading

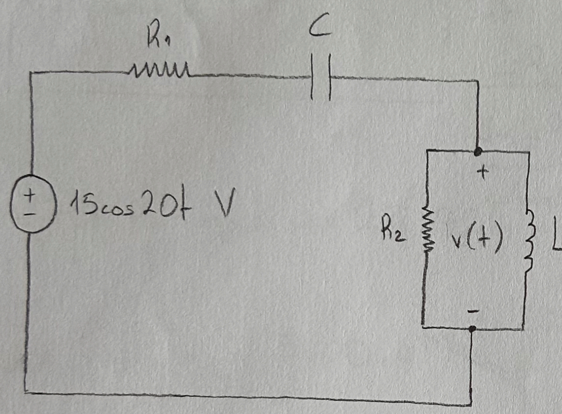
Total power of the circuit:

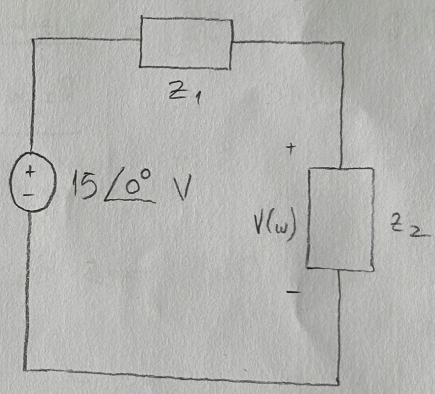
$S = S_1 + S_2 \Longrightarrow (23,2 + 50j) + (150 - 150j) = 173,2 - 100j$

$S = 200\angle -30^\circ$ VA $\Longrightarrow pf = \cos(-30^\circ) = 0,866 \longrightarrow$ leading

P 10.5 - 9

Figure P 10.5 - 9 shows an ac circuit represented in both the time domain and the frequency domain. Suppose $Z_1 = 15{,}3 \angle -24{,}1^\circ \ \Omega$ and $Z_2 = 14{,}4 \angle 53{,}1^\circ \ \Omega$. Determine the voltage $v(t)$ and the values of R_1, R_2, L and C.





$Z_1 = 15{,}3 \angle -24{,}1^\circ \ \Omega \Longrightarrow Z_1 = 13{,}97 - 6{,}25j \ \Omega$

$$\underbrace{Z_1 = R_1}_{\text{real part}} + \underbrace{\frac{1}{j\omega C}}_{\text{complex part}} \longrightarrow 13{,}97 - 6{,}25j = R_1 + \underbrace{\frac{1}{j\omega C}}_{\text{complex part}}$$

$R_1 = 13{,}97 \ \Omega$

$$-6{,}25j = \frac{1}{j\omega C} \Longrightarrow 6{,}25 = \frac{1}{20 \cdot C} \Longrightarrow C = 8 \times 10^{-3} F$$

$Z_2 = 14,4 \angle 53,1^\circ \, \Omega \Longrightarrow Z_2 = 8,64 + 11,51j$

$Y_2 = \dfrac{1}{8,64 + 11,51j} = 0,041 - 0,055j \; S$

$0,041 - 0,055j = \dfrac{1}{R_2} + \dfrac{1}{j\omega L}$

$0,041 - 0,055j = \underbrace{\dfrac{j\omega L}{R_2 j\omega L}}_{\text{real}} + \underbrace{\dfrac{R_2}{R_2 j\omega L}}_{\text{complex}}$

$\dfrac{1}{R_2} = 0,041 \Longrightarrow R_2 \cong 24,03 \, \Omega$

$-0,055j = \dfrac{j}{-\omega L} \Longrightarrow 0,055 \, . \, 20 \, . \, L = 1$

$\Longrightarrow L \cong 0,90 \, H$

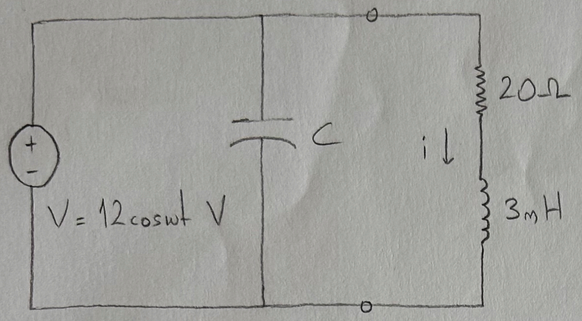
$i(t) = \dfrac{V}{Z_1 + Z_2} = \dfrac{15 + 0j}{22,61 + 5,26j} = 0,62 - 0,14j = 0,6356 \angle 12,72^\circ$

$v(t) = (0,62 - 0,14j)(8,64 + 11,51j) = 6,9682 + 5,9266j$

$\Longrightarrow v(t) \cong 9,3 \, . \cos(20t + 40^\circ) \, V$

P 10.5-13

Determine the current $i(t)$ for a tone at 796 Hz when $C = 10 \mu F$.



$f = 796\ Hz \Longrightarrow \omega = 2\pi \cdot 796 = 5001,41\ rad/s$

$Z_L = j\omega L = j \cdot 5000,41 \cdot 3 \cdot 10^{-3} = 15j$

$Z = 20 + 15j$

$$i(t) = \frac{V}{Z} = \frac{12+0j}{20+15j} = \frac{12\angle 0^\circ}{25\angle 36,87^\circ} = 0,48\angle -36,87^\circ$$

$i(t) = 0,48 \cdot \cos(5001,41t - 36,87^\circ)$

+12,5