Circuit and System Analysis
EEF 232E

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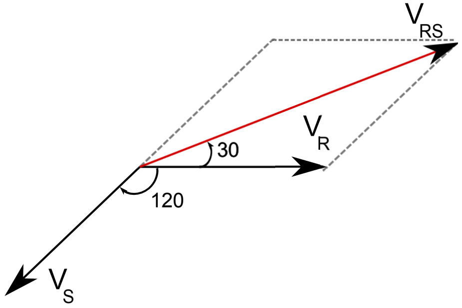
Outline I

- Phasor diagram
- Sinusoidal Steady-State Power Calculation
- Average Power
- Complex, Real and Reactive Powers
- Lagging & Leading power factor
- *pf* Correction
- Tellegen Theorem

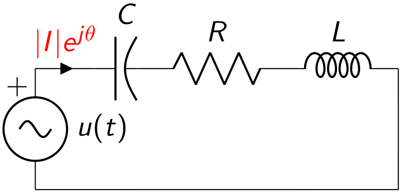
Phasor diagram

A phasor diagram shows the magnitude and phase angle of each phasor quantity in the complex number plane.

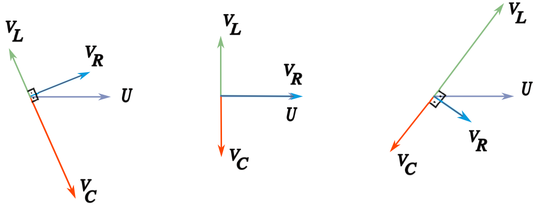


$$V_R = V,\ V_S = Ve^{-j120^\circ},\ V_{RS} = \sqrt{3}Ve^{j30^\circ} = V_R - V_S,$$

Serial Resonance Circuit Analysis on Phasor diagram



$$V_L + V_C + V_R = |U|e^{j0}$$
$$V_L = Ljw\frac{|U|}{Z(w)} = Lw\frac{|U|}{|Z|}e^{j(-\theta+\frac{\pi}{2})}$$
$$V_C = \frac{1}{Cjw}\frac{|U|}{Z(w)} = \frac{|U|}{Cw|Z|}e^{j(-\theta-\frac{\pi}{2})}$$
$$V_R = R\frac{|U|}{Z(w)} = R\frac{|U|}{|Z|}e^{-j\theta}$$



Sinusoidal Steady-State Power Calculation

v and i are steady-state sinusoidal signals
$v(t) = V_m \cos(wt + \theta_v)$ and $i(t) = I_m \cos(wt + \theta_i)$
Instantaneous Power

$$\begin{aligned} P &= V_m \cos(wt + \theta_v) I_m \cos(wt + \theta_i) \\ &= \frac{1}{2} V_m I_m \{\cos(\theta_v - \theta_i) + \cos(2wt + \theta_v + \theta_i)\} \end{aligned}$$

Power factor angle

$$\phi = \theta_v - \theta_i$$

Power factor

$$\mathrm{pf} = \cos(\theta_v - \theta_i)$$

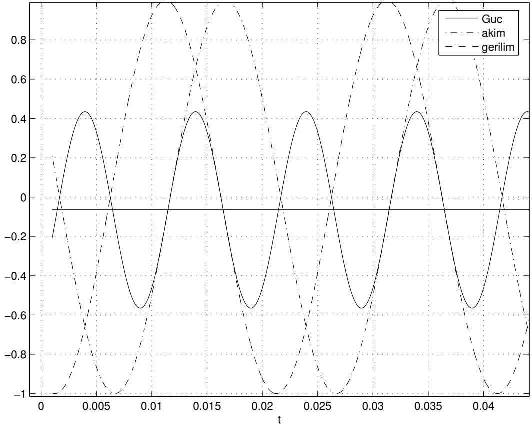
Reactive factor

$$\mathrm{pf} = \sin(\theta_v - \theta_i)$$

Example

$v(t) = \cos\left(2\pi 50t + \frac{\pi}{3}\right)$ and $i(t) = \cos\left(2\pi 50t + \frac{7\pi}{8}\right)$ Instantaneous Power

$$P(t) = -0.0653 + \cos(2\pi 100t + \frac{\pi}{3} + \frac{7\pi}{8})$$



Average Power

The average power associated with sinusoidal signals is the average of the instantaneous power over one period

$$P_{\mathrm{avr}} = \int_0^T p(t)dt$$

$$\begin{aligned} P_{\mathrm{avr}} &= \int_0^T p(t)dt \\ &= \frac{1}{2} V_m I_m \{\cos(\theta_v - \theta_i) \\ &= \frac{1}{2} V_m I_m \cos\phi \end{aligned}$$

Complex Power

$$S = \frac{1}{2} V \bar{I} = \frac{1}{2} V_m e^{j\theta_v} I_m e^{-j\theta_i}$$
$$= \frac{1}{2} V_m I_m e^{j(\theta_v - \theta_i)}$$
$$= \frac{1}{2} V_m I_m e^{j\phi}$$

units volt-amps (VA)

$$S = P + jQ$$

P is **Active Power** (Watt) and Q is **Reactive Power** (VAR),
$|S| = \sqrt{P^2 + Q^2}$ Apparent pover (volt-amps)

$$P = \frac{1}{2} V_m I_m \cos\phi = P_{\mathrm{avr}}, \text{ and } \mathrm{Q} = \frac{1}{2} \mathrm{V_m I_m} \sin(\phi)$$

$$\tan\phi = \frac{Q}{P}$$

	Active P	Reactive Q
Resistor	$\frac{1}{2}R{I_m}^2$	0
Capacitor	0	$-\frac{1}{2wC}{I_m}^2$
Inductor	0	$\frac{wL}{2}{I_m}^2$

- Power for Purely Resistive Circuits : Power can not be extracted from a purely resistive network. In a purely inductive and capacitive circuits, the average power are zero.
- In a purely inductive circuit, energy is being stored the magnetic field, and then it is being extracted from the magnetic fields.
- In a purely capacitive circuit, the power is continually exchanged between the source driving the circuit and the electric field associate with the capacitive element.

Lagging & Leading power factor

Lagging power factor : $Q > 0$ inductive load.
Leading power factor : $Q < 0$ capacitive load.

Remember an inductive load impedance $Z_L = |Z|e^{j\theta_L}$ which means $\theta_L > 0$ $|V_L|e^{j\theta_v} = |Z_L|e^{j\theta_L}|I_L|e^{j\theta_i}$ then $\theta_L = \theta_v - \theta_i > 0$ meaning the current drawn by the circuit lags the supply voltage then

$$Q = \frac{1}{2} V_m I_m \sin(\theta_L) > 0.$$

If an capacitive load impedance which means $\theta_L < 0$ then $\theta_v < \theta_i$ and $\theta_L = \theta_v - \theta_i < 0$ meaning the current leads with the supply voltage then

$$Q = \frac{1}{2} V_m I_m \sin(\theta_L) < 0.$$

Why : sin is an odd function ($\sin(-\theta) = -\sin(\theta)$).

Power Calculation based on RMS Values

$$P = \frac{1}{2} V_m I_m \cos\phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi = V_{\mathrm{rms}} I_{\mathrm{rms}} \cos\phi$$

and

$$Q = V_{\mathrm{rms}} I_{\mathrm{rms}} \sin\phi$$

Example:
$220V$ 100 W lamp has a resistance of $\frac{220^2}{100} = 484\Omega$ and $I_{\mathrm{rms}} = \frac{220}{484} = 0.45A$.

Example: A series-connected load draws a current $i(t) = 4cos(100\pi t + 10^\circ)A$ when the applied voltage is $v(t) = 120cos(100\pi t - 20^\circ)V$. Find the apparent power and the power factor of the load.

$$|S| = \frac{1}{2} 120 \cdot 4 = 240VA$$

The pf is leading because the current leads the voltage

$$pf = \cos\left(-20^\circ - (10^\circ)\right)$$

or

$$Z = \frac{120\angle{-20^\circ}}{4\angle{-20^\circ}} = 30\angle{-30^\circ}$$

then

$$pf = \cos\left(-30^\circ\right)$$

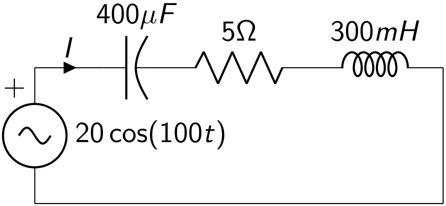
Example: Obtain the power factor and the apparent power of a load whose imped- ance is $Z = 60 + j40$

$$pf = \cos(\arctan(\frac{40}{60})) = 0.832$$

pf is lagging (why load inductive ($Q = 40 > 0$)).

Homeworks: Example 10.4 (page 402), Example 10.5 (page 406), Example 10.6 (page 407), Electric Circuits, James W. Nilsson and Susan A. Riedel

Serial Resonance Circuit



$Z = 5 + 5j\Omega,\ |U| = 20,$
$I = \frac{U}{Z} = (2 - 2j)A$

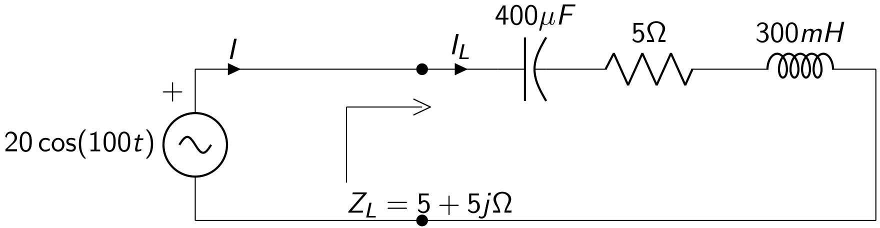
$$V_R = RI = (10 - 10j)V, V_C = (-50 - 50j)V, V_L = (60 + 60j)V$$

$$S_Z = \frac{1}{2}20(2 + 2j) = (20 + 20j), (S_R = 20, S_C = -100j, S_L = 120j)$$

$Q = 20 > 0$ Load is inductive $(Z = (5 + 5j)\Omega)$
$w_0 = \frac{1}{\sqrt{LC}} = 91.2871Hz$ and $Z(w_0) = 5\Omega$ and $I = \frac{U}{Z} = 4A$

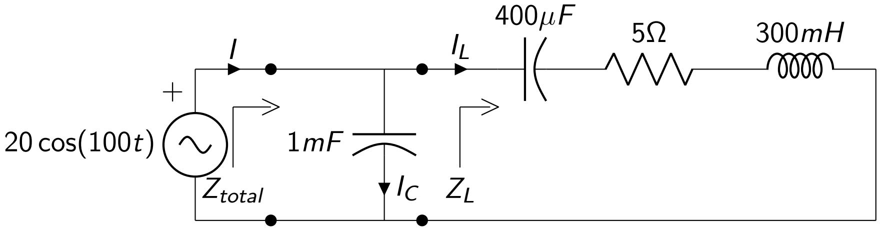
$$S_Z = \frac{1}{2}(20)4 = 40VA(S_R = 24, S_C = -219.09j, S_L = 219.09j)$$

Serial Resonance Circuit



$$I_L = \frac{U}{Z_L} = (2 - 2j)A$$

$$S_U = \frac{1}{2} 20 \overline{(-2 + 2j)} = -(20 + 20j)$$



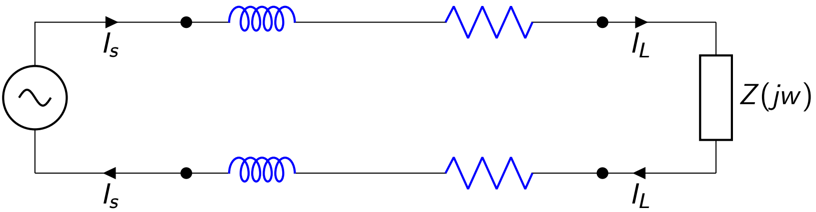
$$I_L = \frac{U}{Z} = (2 - 2j)A$$

Adding $C_c = 1mF$ such as $Y + C_c wj \in R$ and $Z_{total} = 10\Omega$

$$I = \frac{U}{Z_{total}} = 2A, \quad I_C = |U|(1m)j100 = 2jA$$

$$S_U = \frac{1}{2}20(-2) = -20$$

Why : pf Correction



$$\frac{V_L}{I_L} = Z_L = R_L + X_L jw = |Z_L| e^{\phi j}, \quad \phi = \theta_v - \theta_i$$

$$Z_{LINE} = R_H + L_H jw$$

$$S_L = \frac{1}{2} V I_L = \frac{1}{2} |V||I_L| \cos\phi + \frac{1}{2} |V||I_L| \sin\phi$$

$$P_L = \frac{1}{2} |V_L||I_L| \cos\phi = \frac{1}{2} |V_L||I_L| pf$$

$$P_H = \frac{1}{2} R_H |I_L|^2$$

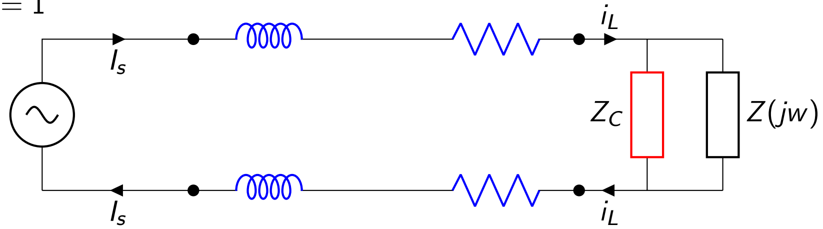
$$P_L = \frac{1}{2}|V_L||I_L|pf$$

$$|I_L| = \frac{2P_L}{|V_L|pf}$$

$$P_H = \frac{1}{2}R_H|I_L|^2 = \frac{1}{2}R_H\frac{4P_L^2}{|V_L|^2pf^2} = R_H\frac{2P_L^2}{|V_L|^2pf^2}$$

If $|pf| = 1$ then P_H is minimum! Idea to have $pf = 1$.

Solution : pf correction such that adding Z_C to Load impedance making $pf = 1$



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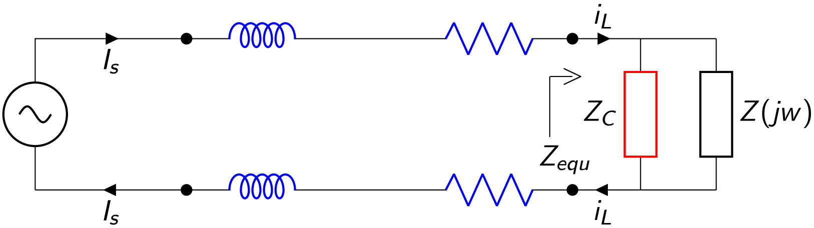


$$Z_{eq} = Z_L // Z_C$$

$$Y_{equ} = (G + Xj) + \frac{1}{Z_C} = G, \;\; \frac{1}{Z_C} = Y_C = -Xj$$

$I_L = \frac{V}{Z_{equ}}$ then $pf = 1$!

Example: A load is connected in parallel across a $120V$ (rms) voltage source. The load is deliveding a reactive power of 1800VAR at leading power factor $pf = \frac{\sqrt{3}}{2}$. The frequency of the voltage source is $80 rad/sn$. (a) Calculate the admittance of the load. (b) compute the value of element that would correct the power factor to 1 if placed in parallel with the load.

Power factor is described as leading therefore the load is capacitive, furthermore the laod is delivering a reactive power so $Q < 0$ which means that again load is capacitive.

$$\cos\theta = \frac{\sqrt{3}}{2}$$

then $\theta = -30^\circ$

$$Q = |S|\sin(-30^\circ) = |V||I|\sin(-30^\circ) = -1800$$

$I = 30e^{30^\circ j}$. In order to find admittance $Y = V/I = 0.25e^{30^\circ j}$.

To obtain power factor to 1, let Y_x placed in parallel with the load. The load is deliveding a reactive power of 1800VAR therefore Y_x must be absorb 1800VAR in order to get 0 total reactive power! The Y_x must be inductive and absorb a reactive power of 1800VAR. From $S = V\bar{I} = |V|^2\bar{Y}$ we obtain $1800 = 120^2 \frac{1}{L80}$ equation and $L = 0,1H$.

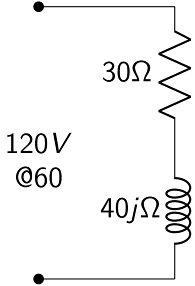
Example: Calculate the average power and the reactive power at the terminal of an one-port circuit element if $v = 100\cos(wt + 15^\circ)$ V and $i = 4\sin(wt - 15^\circ)$ Amp.

$$\begin{aligned} S &= \frac{1}{2} \cdot 100 \cdot e^{j15^\circ} \cdot 4 \cdot e^{j(90+15)^\circ} \\ &= \frac{1}{2} \cdot 100 \cdot 4 \cdot e^{j(15+105)^\circ} = 100 + j173.21 \\ &= \frac{1}{2} \cdot 100 \cdot 4 \cdot (\cos(120^\circ) + j\sin(120^\circ)) \end{aligned}$$

Hence $P = -100$W and $Q = 173.21$ VAR. The negative value of -100W means that the one-port is delivering average power and absorbing reactive power.

Example:

A blender motor is modelled by a 30Ω resistor (modelling the coil resistance) in series with a $\frac{40}{2\pi 60}H$ inductor (modelling the inductive effects of the coil). What power is dissipated by the motor?



$$I_{\rm rms} = \frac{120}{30 + 2\pi 60 \frac{40}{2\pi 60} j} = 2.4 e^{-j53^\circ}$$

Average power dissipated:

$$P = Re\{120 \times 2.4 e^{53^\circ}\} = 172.8 Watts.$$

The motor draws current:

<u>i(t)=2.4 $\sqrt{2}\cos(2\pi 60 t - 53^\circ)$</u>

The voltage and current are 53° out of phase, so the motor draws more current than it should.

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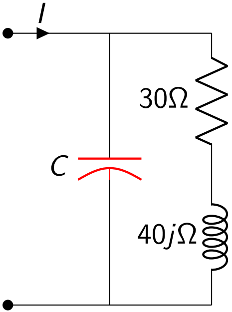


Hook a capacitor C in parallel with the motor.

RMS current phasor: $I_{rms} = 120(\frac{1}{30+40j} + j\pi 60C)$

What value of C makes the phase of I_{rms} zero?

You should obtain $C = 42.4\mu F$. Then $I_{rms} = 1.44$

Average power dissipated $P = Re\{120 \times 1.44\} = 172.8$Watts.
But the current amplitude has dropped from $2.4\sqrt{2}$ to $1.44\sqrt{2}$ Amps.
We have almost halved the peak current, while maintaining average power is 172.8 Watts.

Tellegen Theorem

Tellegen's theorem is based on the fundamental law of conservation of energy!

Tellegen Theorem

It states that the algebraic sum of power absorbed by all elements in a circuit is zero at any instant.

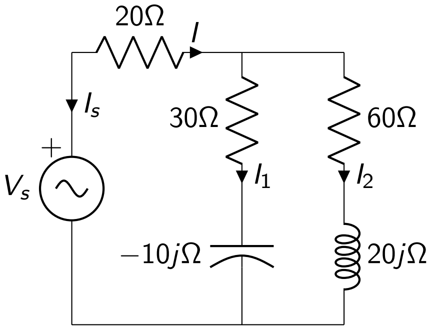
Tellegen's theorem asserts that

$$P = \sum_{i=1}^{n_e} \frac{1}{2} V_i \bar{I}_i = 0$$

$$\begin{aligned} P &= \sum_{i=1}^{n_e} \frac{1}{2} V_i \bar{I}_i = \frac{1}{2} V_e^T \bar{I}_e \\ &= \frac{1}{2} V_n^T M^T \bar{I}_e = \frac{1}{2} V_e^T M^T \bar{I}_e = 0 \end{aligned}$$

see : EHB211E Slayt Number 73.

The 60Ω resistor absorbs an average power of $240W$. Find the complex powers of each circuit elements.




```
Z1=30-10*j; Z2=60+20*j;
Y1=1/Z1;Y2=1/Z2;
I2=sqrt(240*2/60);
V2=I2*Z2; V1=V2;
S1=0.5*V1*conj(V1*Y1)
>>480-160j
S2=0.5*V2*conj(V2*Y2)
>>240+80j
I1=V1*Y1; I=I1+I2;
VR=I*20;
SR=0.5*VR*conj(VR)/20
>>656
Vs=VR+V1; Is=-I;
Ss=0.5*Vs*conj(Is)
>>-1376+80j
```

Verify that $Ss + SR + S1 + S2 = 0$