STATICS

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3 Equilibrium of a Particle

Chapter Objectives

- Concept of the free-body diagram for a particle
- Solve particle equilibrium problems using the equations of equilibrium

Chapter Outline

1. Condition for the Equilibrium of a Particle
2. The Free-Body Diagram
3. Coplanar Systems
4. Three-Dimensional Force Systems

3.1 Condition for the Equilibrium of a Particle

- Particle at *equilibrium* if
  - At rest
  - Moving at constant a constant velocity

- Newton's first law of motion

$$\sum \mathbf{F} = 0$$

where $\sum \mathbf{F}$ is the vector sum of all the forces acting on the particle

3.1 Condition for the Equilibrium of a Particle

- Newton's second law of motion

$$\sum \mathbf{F} = m\mathbf{a}$$

- When the force fulfill Newton's first law of motion,

$$m\mathbf{a} = 0$$

$$\mathbf{a} = 0$$

therefore, the particle is moving in constant velocity or at rest

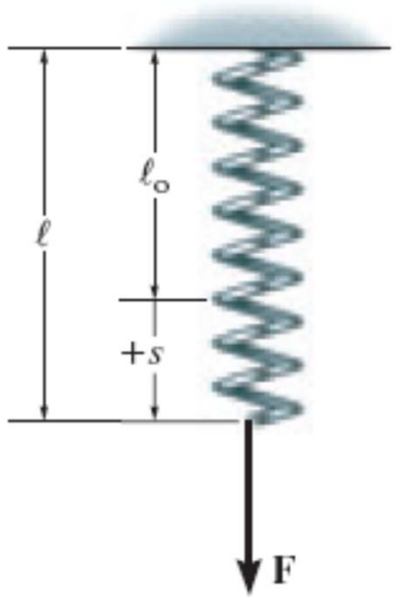
3.2 The Free-Body Diagram

- Best representation of all the unknown forces ($\sum \mathbf{F}$) which acts on a body
- A sketch showing the particle “free” from the surroundings with all the forces acting on it
- Consider two common connections in this subject –
  - Spring
  - Cables and Pulleys

3.2 The Free-Body Diagram

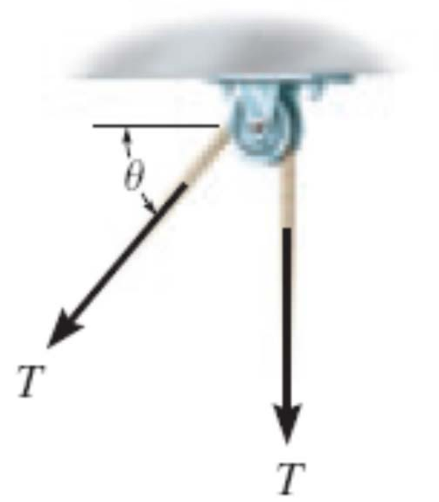
- Spring
  - Linear elastic spring: change in length is directly proportional to the force acting on it
  - *spring constant or stiffness k*: defines the elasticity of the spring
  - Magnitude of force when spring is elongated or compressed

    ➔ $F = ks$



3.2 The Free-Body Diagram

- Cables and Pulley
  - Cables (or cords) are assumed negligible weight and cannot stretch
  - Tension always acts in the direction of the cable
  - Tension force must have a constant magnitude for equilibrium
  - For any angle θ, the cable is subjected to a constant tension *T*



Cable is in tension

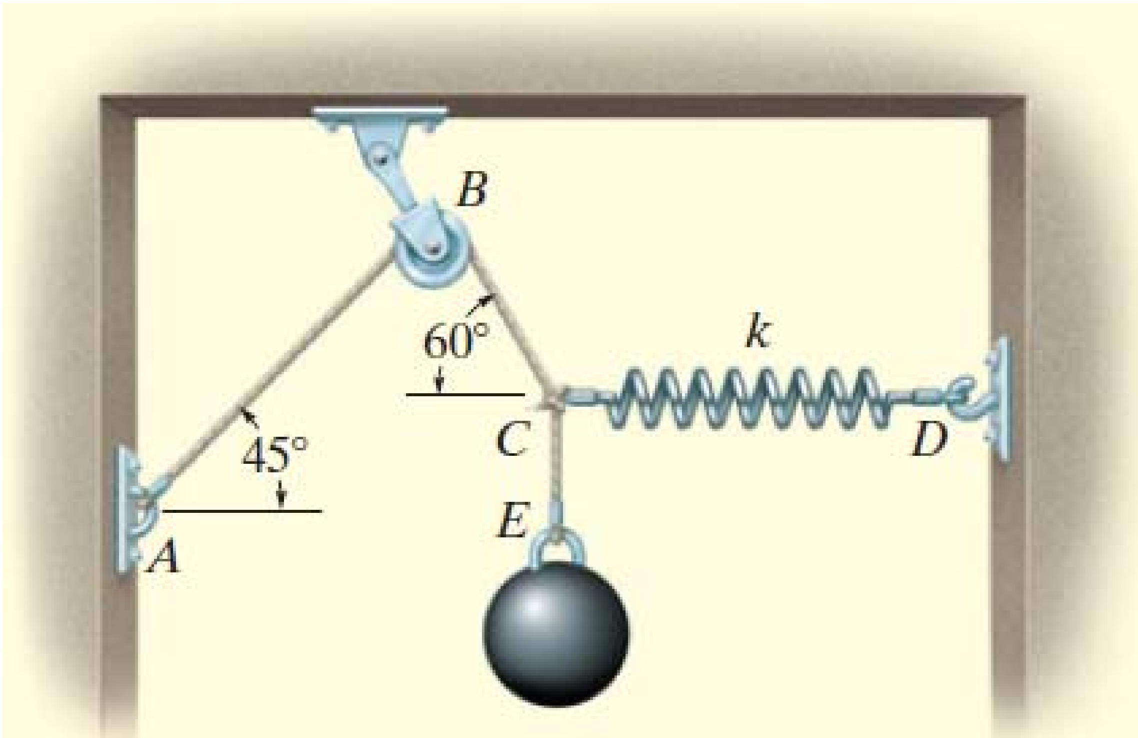
3.2 The Free-Body Diagram

Procedure for Drawing a FBD

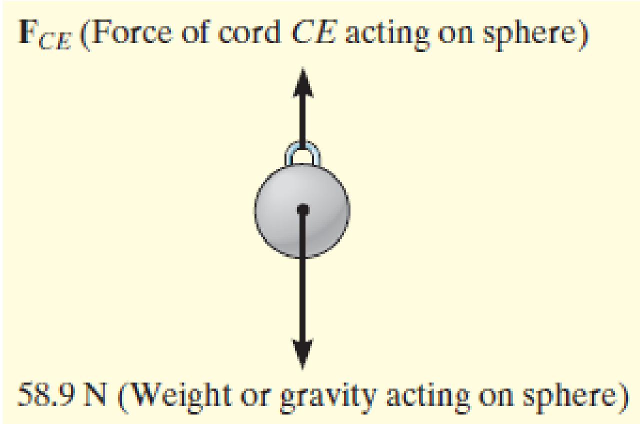
1. Draw outlined shape
2. Show all the forces
   - Active forces: particle in motion
   - Reactive forces: constraints that prevent motion
3. Identify each forces
   - Known forces with proper magnitude and direction
   - Letters used to represent magnitude and directions

Example 3.1

The sphere has a mass of 6kg and is supported. Draw a free-body diagram of the sphere, the cord CE and the knot at C.



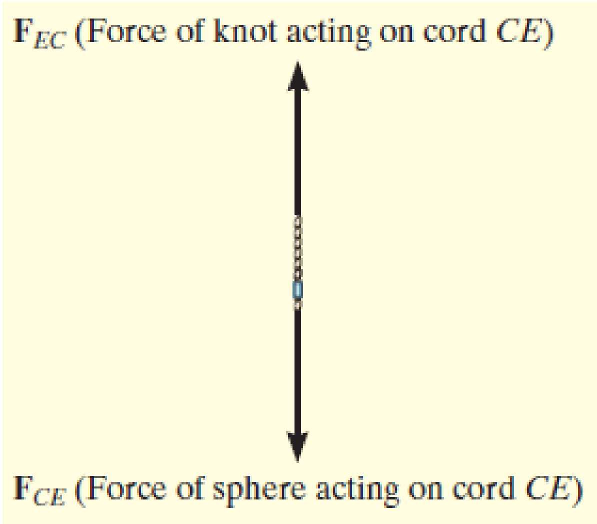
Solution



<u>FBD at Sphere</u>

Two forces acting, weight and the force on cord CE.

Weight of 6kg (9.81m/s^2) = 58.9N



<u>Cord CE</u>

Two forces acting: sphere and knot

Newton's 3rd Law:

$\mathbf{F}_{CE}$ is equal but opposite

$\mathbf{F}_{CE}$ and $\mathbf{F}_{EC}$ pull the cord in tension

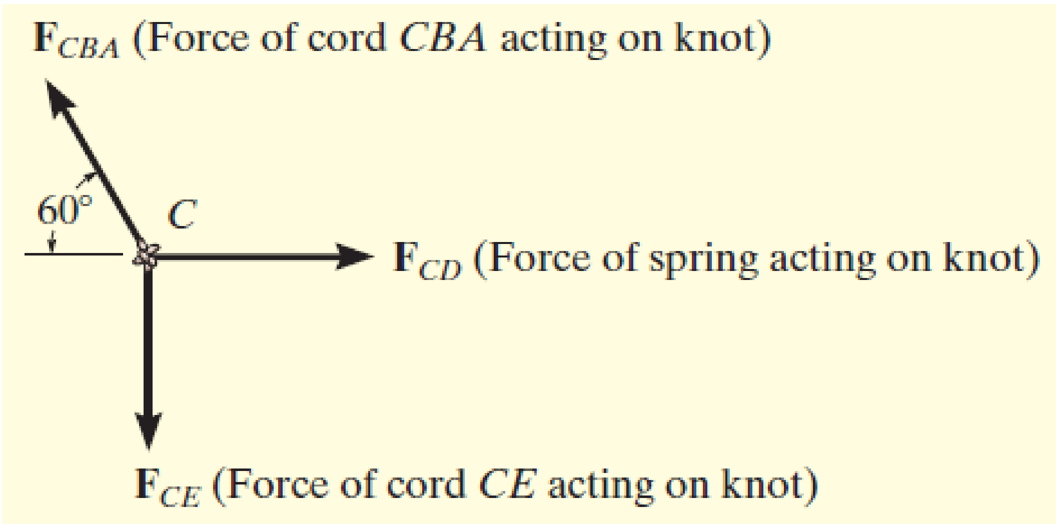
For equilibrium, $\mathbf{F}_{CE} = \mathbf{F}_{EC}$

Solution

<u>FBD at Knot</u>

3 forces acting: cord CBA, cord CE and spring CD

Important to know that the weight of the sphere does not act directly on the knot but subjected to by the cord CE



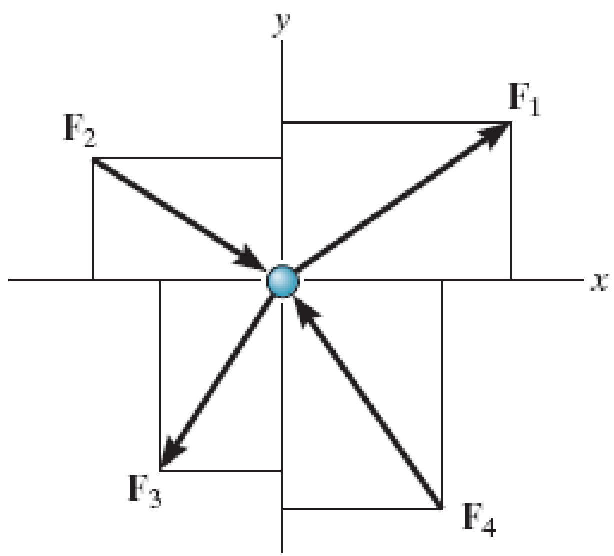
3.3 Coplanar Systems

- A particle is subjected to coplanar forces in the x-y plane
- Resolve into **i** and **j** components for equilibrium

$$\sum \mathbf{F}_x = 0$$

$$\sum \mathbf{F}_y = 0$$

- Scalar equations of equilibrium require that the algebraic sum of the x and y components to equal to zero



3.3 Coplanar Systems

- Procedure for Analysis

1. Free-Body Diagram

   - Establish the x, y axes

   - Label all the unknown and known forces

2. Equations of Equilibrium

   - Apply $F = ks$ to find spring force

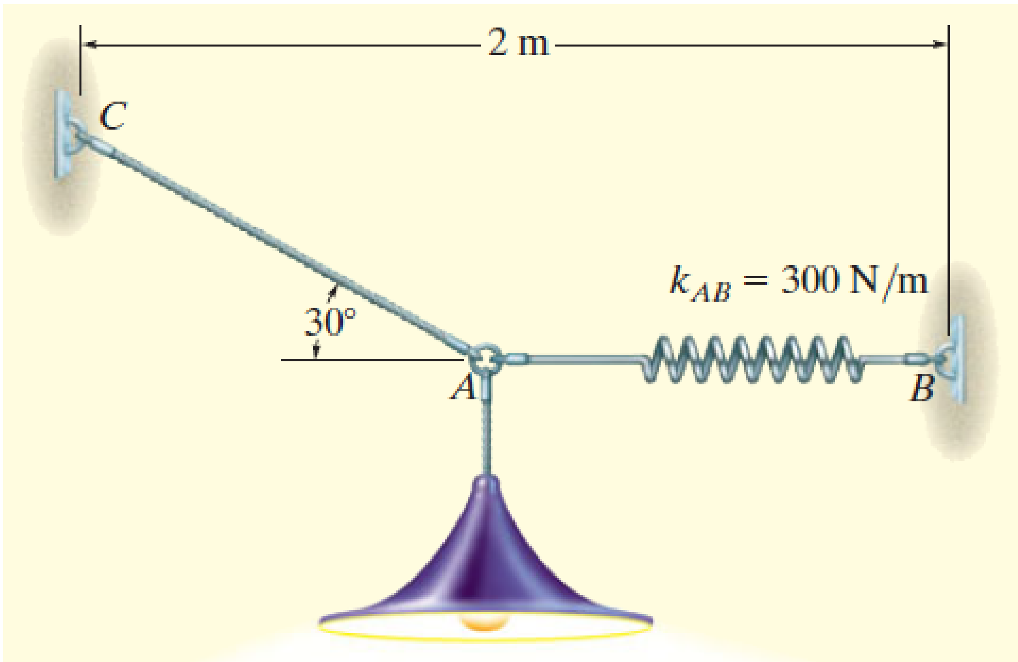
   - When negative result force is the reserve

   - Apply the equations of equilibrium

$$\sum \mathbf{F}_x = 0 \qquad \sum \mathbf{F}_y = 0$$

Example 3.4

Determine the required length of the cord AC so that the 8kg lamp is suspended. The undeformed length of the spring AB is $l'_{AB} = 0.4$m, and the spring has a stiffness of $k_{AB} = 300$N/m.



Solution

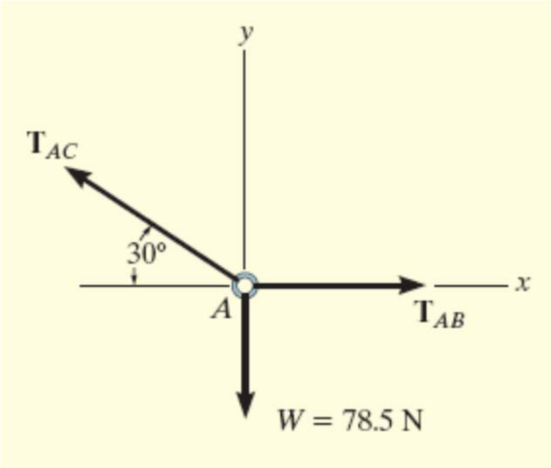
<u>FBD at Point A</u>

Three forces acting, force by cable AC, force in spring AB and weight of the lamp.

If force on cable AB is known, stretch of the spring is found by $F = ks$.

$+\rightarrow \quad \sum \mathbf{F}_x = 0; \quad T_{AB} - T_{AC}\cos 30^o = 0$

$+\uparrow \quad \sum \mathbf{F}_y = 0; \quad T_{AB}\sin 30^o - 78.5N = 0$



Solving,

$T_{AC} = 157.0kN$

$T_{AB} = 136.0kN$

Solution

$T_{AB} = k_{AB}s_{AB}$; $136.0N = 300N/m(s_{AB})$

$s_{AB} = 0.453m$

For stretched length,

$l_{AB} = l'_{AB} + s_{AB}$

$l_{AB} = 0.4m + 0.453m$

$= 0.853m$

For horizontal distance BC,

$2m = l_{AC}\cos 30^\circ + 0.853m$

$l_{AC} = 1.32m$

3.4 Three-Dimensional Force Systems

- For particle equilibrium

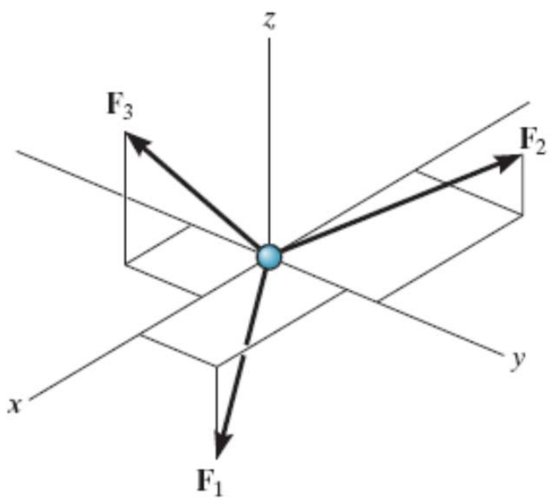
$$\sum \mathbf{F} = 0$$

- Resolving into **i**, **j**, **k** components

$$\sum F_x\mathbf{i} + \sum F_y\mathbf{j} + \sum F_z\mathbf{k} = 0$$

- Three scalar equations representing algebraic sums of the x, y, z forces

$$\sum F_x\mathbf{i} = 0$$
$$\sum F_y\mathbf{j} = 0$$
$$\sum F_z\mathbf{k} = 0$$



3.4 Three-Dimensional Force Systems

- Procedure for Analysis

Free-body Diagram

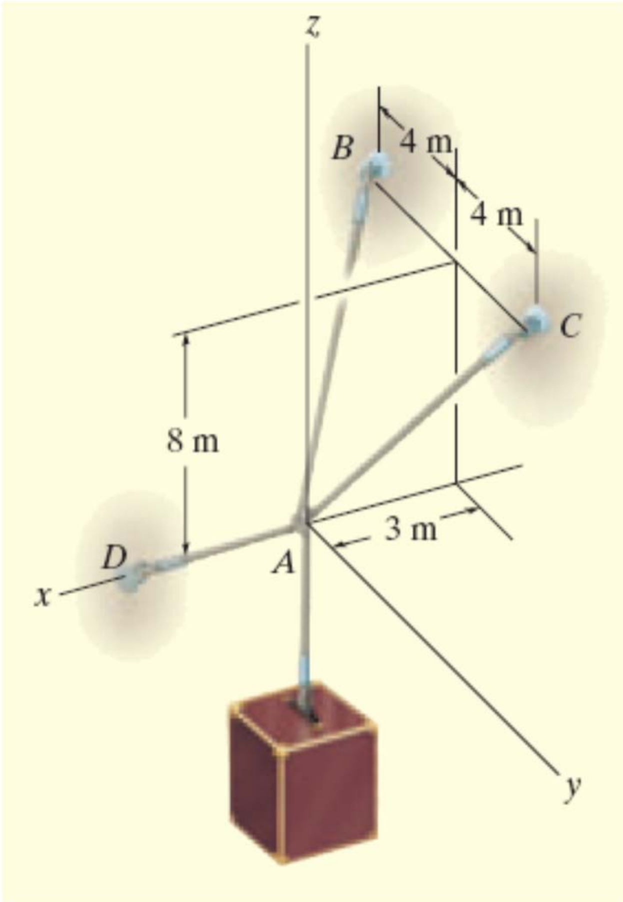
- Establish the z, y, z axes
- Label all known and unknown force

Equations of Equilibrium

- Apply $\sum F_x = 0$, $\sum F_y = 0$ and $\sum F_z = 0$
- Substitute vectors into $\sum \mathbf{F} = 0$ and set **i**, **j**, **k** components = 0
- Negative results indicate that the sense of the force is opposite to that shown in the FBD.

Example 3.7

Determine the force developed in each cable used to support the 40kN crate.



Solution

<u>FBD at Point A</u>

To expose all three unknown forces in the cables.

<u>Equations of Equilibrium</u>

Expressing each forces in Cartesian vectors,

$\mathbf{F}_B = F_B(\mathbf{r}_B / r_B)$

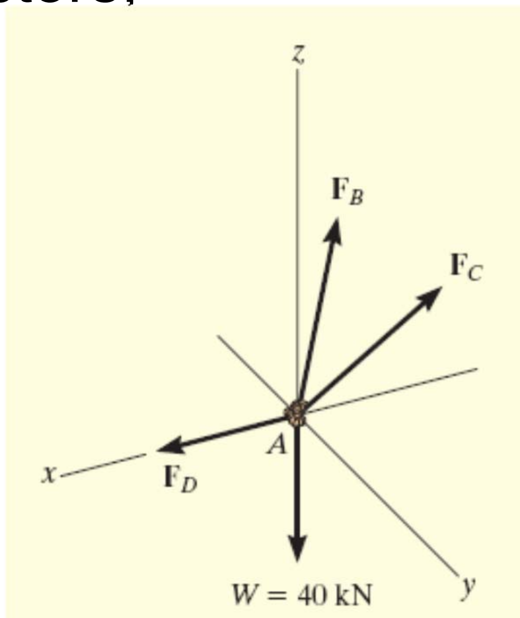
$= -0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k}$

$\mathbf{F}_C = F_C(\mathbf{r}_C / r_C)$

$= -0.318F_C\mathbf{i} - 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k}$

$\mathbf{F}_D = F_D\mathbf{i}$

$\mathbf{W} = -40\mathbf{k}$



Solution

For equilibrium,

$\sum \mathbf{F} = 0; \qquad \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D + \mathbf{W} = 0$

$-0.318F_B\mathbf{i} - 0.424F_B\mathbf{j} + 0.848F_B\mathbf{k} - 0.318F_C\mathbf{i}$

$\qquad - 0.424F_C\mathbf{j} + 0.848F_C\mathbf{k} + F_D\mathbf{i} - 40\mathbf{k} = 0$

$\sum F_x = 0; \qquad -0.318F_B - 0.318F_C + F_D = 0$

$\sum F_y = 0; \qquad -0.424F_B \quad - 0.424F_C = 0$

$\sum F_z = 0; \qquad 0.848F_B + 0.848F_C - 40 = 0$

Solving,

$F_B = F_C = 23.6\text{kN}$

$F_D = 15.0\text{kN}$