1.

(3.2-34) (a) $E[X] = \int_0^1 x \frac{5}{4}(1-x^4)\,dx = \frac{5}{4}\left(\frac{x^2}{2} - \frac{x^6}{6}\right)\Big|_0^1 = 5/12.$

(b) $E[4X+2] = 4\bar{X} + 2 = 4(5/12) + 2 = 11/3.$

(c) $E[X^2] = \int_0^1 x^2 \frac{5}{4}(1-x^4)\,dx = \frac{5}{4}\left(\frac{x^3}{3} - \frac{x^7}{7}\right)\Big|_0^1 = 5/21.$

2. $\Phi_{X}(w) = \int_{-\infty}^{\infty} f_x(x) e^{j\omega x} dx = \int_{-\infty}^{\infty} e^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} \delta(x-k) e^{j\omega x} dx = \mathrm{e}^{-b} \sum_{k=0}^{\infty} \frac{b^k}{k!} \delta(x-k) e^{j\omega x} dx =$

$\mathrm{e}^{-b} \sum_{k=0}^{\infty} \frac{(be^{j\omega})^k}{k!} = \mathrm{e}^{-b+b\exp(j\omega)} = e^{-b(1-e^{j\omega})}$

3. E[g(x)]= $\int_{-\infty}^{\infty} g(x) F_x(x) dx = \int_{-\infty}^{x_0} 0.F_x(x)dx + \int_{x_0}^{\infty} 1.F_x(x)dx$

$= 1 - \int_{-\infty}^{x_0} F_x(x)dx = 1 - F_x(x_0)$

4. Y=a/X, X=a/Y.

a>0 için, y<0 a/y < x <0.

$$F_y(y) = \int_{a/y}^{0} f_x(x)dx = F_x(0) - F_X(a/y),\ y < 0. \qquad (1)$$

y ≥ 0 ise -∞ < x < 0 ve a/y < x < ∞

$$F_y(y) = \int_{-\infty}^{0} f_x(x)dx + \int_{a/y}^{\infty} f_x(x)dx = F_x(0) - F_x(-\infty) + F_x(\infty) - F_x(a/y) \qquad (2)$$

$= 1 + F_x(0) - F_x(a/y),\ y \geq 0.$

(1) ile (2) nin türevini alırsak,

$$f_y(y) = \frac{a}{y^2} f_x(a/y) \text{ , } -\infty < y < \infty \text{ ve } a>0. \qquad (3)$$

a < 0 için, y<0 0 < x < a/y.

$$F_y(y) = \int_{0}^{a/y} f_x(x)dx = F_x(a/y) - F_x(0),\ y < 0. \qquad (4)$$

y ≥ 0 0 < x < ∞ ve - ∞ < x < a/y.

$$F_y(y) = \int_{0}^{\infty} f_x(x)dx + \int_{-\infty}^{a/y} f_x(x)dx = F_x(\infty) - F_x(0) + F_x(a/y) - F_x(-\infty) \qquad (5)$$

$= 1 - F_x(0) + F_x(a/y),\ y \geq 0.$

(4) ile (5) in türevini alırsak,

$$f_y(y) = -\frac{a}{y^2} f_x(a/y)\text{ , -∞ < y < ∞ ve a<0.} \qquad (6)$$

(2) ile (6) yı birleştirirsek,

$$f_y(y) = \frac{|a|}{y^2} f_x(a/y)\text{ , -∞ < y < ∞.}$$

5.

5. a) $\Pr\{Y=1 \mid X=-1\} \stackrel{?}{=} \Pr\{Y=1\}$

$1 \neq 0.2$

Dolayısıyla X&Y bağımsız olamaz.

b) $E[XY] = \sum_i x(\zeta_i) y(\zeta_i) \Pr\{\zeta_i\}$

$= \frac{1}{5}\left((-1)(1) + (-\frac{1}{2})(\frac{1}{2})^2 + \frac{1}{2}(\frac{1}{2})^2 + 1(1)^2\right)$

$= \emptyset$

$E[X] = \frac{1}{5}(-1 + (-\frac{1}{2}) + 0 + (\frac{1}{2}) + 1) = \emptyset \Rightarrow C_{XY} = E[XY] - E[X]E[Y]$

6. a) $F_{X,Y}(x,y) = \int_{-\infty}^{y}\int_{-\infty}^{x} f_{X,Y}(u,v)\,du\,dv$

= xy/ab , $\quad 0 \le x < a$ ve $0 \le y < b$

= x/a , $\quad 0 \le x < a$ ve $b \le y$

= y/b , $\quad a \le x$ ve $0 \le y < b$

=1 , $\quad a \le x$ ve $b \le y$

= 0 , $\quad$ x < 0 veya y<0

b) $P\{X+Y\} \le 3a/4 = \int_{y=0}^{3a/4}\int_{x=0}^{(3a/4)-y} \frac{dxdy}{ab} = \int_{y=0}^{3a74} \frac{(3a/4) - ydy = 9a/32b}{ab}$

$P\{Y \le 2bx/a\} = \int_{y=0}^{b}\int_{x=ay/2b}^{a} \frac{dxdy}{ab} = \int_{0}^{b} \frac{a-(ay/2b)}{ab} dy = 3/4.$

7. a)

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx = \int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} \frac{dx}{\Pi r^2}$$

$\frac{2}{\Pi}\frac{\sqrt{r^2-y^2}}{r^2}$, $\quad |y| < r$

0 , $\quad |y| > r$

b) $f_X(X \mid Y=y)=\dfrac{f_{X,Y}(x,y)}{f_Y(y)}=\dfrac{(u(x)-u(x-4))u(y)y^2e^{-(x+1)y^2}}{(e^{-y^2}-e^{-5y^2})}$

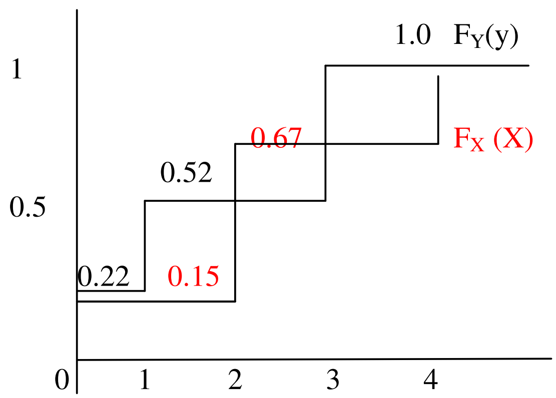
$$f_y(Y \mid X=x)=\frac{f_{X,Y}(x,y)}{f_X(x)}=2(u(x)-u(x-4))u(y)(x+1)^2y^3e^{-(x+1)y^2}$$

8.

$$F_{X,Y}(x,y)=\int_{-\infty}^{y}\int_{-\infty}^{x} f_{X,Y}(\xi_1,\xi_2)d\xi_1 d\xi_2=0.1u(x)u(y)+0.12u(x-4)u(y)+0.05u(x)u(y-1)$$
$$+0.25u(x-2)u(y-1)+0.3u(x-2)u(y-3)+0.18u(x-4)u(y-3).$$

$$F_X(x)=F_{X,Y}(x,\infty)=0.1u(x)+0.12u(x-4)+0.25u(x-2)+0.05u(x)+0.18u(x-4)+0.3u(x-2)$$
$$=0.15u(x)+0.55u(x-2)+0.30u(x-4).$$

$$F_Y(y)=F_{X,Y}(\infty,y)=0.22u(y)+0.30u(y-1)+0.48u(y-3).$$



9. $\sigma_X^2=\displaystyle\int_{-\infty}^{\infty}\frac{(x-m)^2}{2b}e^{-|x-m|/b}dx$

$\xi=(x-m)/b$ ise, $d\xi=dx/b$

$$\sigma_X^2=\frac{b^2}{2}\int_{-\infty}^{\infty}\xi^2e^{-|\xi|}d\xi=b^2\int_0^{\infty}\xi^2e^{-\xi}d\xi$$

$$\sigma_X^2=b^2\{e^{-\xi}(-\xi^2-2\xi-2)\,|_0^{\infty}\}=2b^2$$

10.

(3.4-8.) Y is a discrete random variable. Its values are $y_1 = 3(-4)^3 = -192$, $y_2 = 3(-1)^3 = -3$, $y_3 = 3(2)^3 = 24$, $y_4 = 3(3)^3 = 81$, and $y_5 = 3(4)^3 = 192$. All values occur with probability 1/5 because mapping from X to Y is one-to-one.

(a) Use (2.3-5):

$$f_Y(y) = \frac{1}{5}\delta(y+192) + \frac{1}{5}\delta(y+3) + \frac{1}{5}\delta(y-24) + \frac{1}{5}\delta(y-81) + \frac{1}{5}\delta(y-192).$$

(3.4-8.) (Continued)

(b) Use (3.1-4):

$$E[Y] = \frac{1}{5}[-192 - 3 + 24 + 81 + 192] = 20.4.$$

(c) Use (3.1-7) with $g(X) = X^2$ to first find $E[Y^2]$. $E[Y^2] = \frac{1}{5}\left[(-192)^2 + (-3)^2 + (24)^2 + (81)^2 + (192)^2\right]$

$$= 16174.8.$$

Thus,

$$\sigma_Y^2 = E[Y^2] - (E[Y])^2 = 16174.8 - (20.8)^2$$

$$= 15758.64$$

$$\sigma_Y \approx 125.53.$$

11. $$F_Y(y) = \Pr\{Y \le y\} = \begin{cases} \Pr\{X \le y^2\} & y \ge 0 \\ P(\emptyset) & y < 0 \end{cases}$$

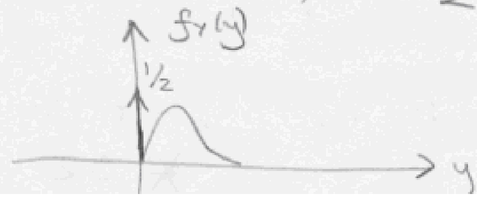
$$= \begin{cases} F_X(y^2) & y \ge 0 \\ 0 & y < 0 \end{cases}$$

$$= F_X(y^2)\, u(y)$$

$$f_Y(y) = \frac{d}{dy}\left(F_X(y^2)\, u(y)\right)$$

$$= 2y f_X(y^2)\, u(y) + \underbrace{F_X(y^2)\,\delta(y)}_{F_X(0)\,\delta(y)}$$

$$= \frac{2y}{\sqrt{2\pi}} \exp\left(-\frac{y^4}{2}\right) + \frac{\delta(y)}{2}$$



12.

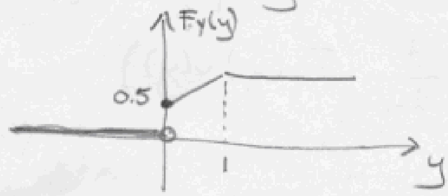
12. $$F_Y(y) = \Pr\{Y \le y\} = \begin{cases} \Pr\{-\infty < X < \infty\}, & y > 1 \\ \Pr\{-\infty < X \le 0\} + \Pr\{\frac{1-y}{2} < X \le \frac{1+y}{2}\} + \Pr\{X > 1\}, & 0 \le y \le 1 \\ P(\emptyset), & y < 0 \end{cases}$$

$$= \begin{cases} 1, & y > 1 \\ F_X(0) + F_X(\frac{1+y}{2}) - F_X(\frac{1-y}{2}) + 1 - F_X(1), & 0 \le y \le 1 \\ 0 & y < 0 \end{cases}$$

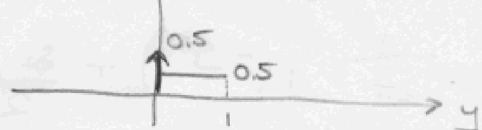
$$F_X(x) = \begin{cases} 0 & x \le 0 \\ x/2 & 0 < x \le 2 \\ 1 & x > 2 \end{cases}$$

$$F_Y(y) = \begin{cases} 1 & y > 1 \\ \frac{1+y}{4} - \left(\frac{1-y}{4}\right) + 1 - \frac{1}{2} & 0 \le y < 1 \\ 0 & y < 0 \end{cases}$$

$$= \begin{cases} 1 & y > 1 \\ 1/2 + y/2 & 0 \le y \le 1 \\ 0 & y < 0 \end{cases}$$



$$f_Y(y) = \frac{d}{dy} F_Y(y) = 0.5\,\delta(y) + \frac{u(y-1) - u(y)}{2}$$



13.

★3.1-16 It was found in the solution of Problem 2.6-3 that

$$f_X(x|X>20) = \frac{e\,x\,u(x-20)}{200}\,e^{-x^2/400}.$$

Thus, $E[X|X>20] = \int_{20}^{\infty} \frac{e}{200}\,x^2 e^{-x^2/400}\,dx$. Let $\xi = x/20$, $d\xi = dx/20$ so $E[X|X>20] = 40e\int_1^{\infty}\xi^2 e^{-\xi^2}\,d\xi$.

This integral is not known in closed form. It is approximated as follows by using a series expansion for $e^{-\xi^2}$. By using (C-52):

$$E[X|X>20] = 40e\left[\frac{\sqrt{\pi}}{4} - \int_0^1 \xi^2 e^{-\xi^2}\,d\xi\right]$$

$$\int_0^1 \xi^2 e^{-\xi^2}\,d\xi = \int_0^1 \xi^2 \sum_{n=0}^{\infty}\frac{(-\xi^2)^n}{n!}\,d\xi = \sum_{n=0}^{\infty}\frac{(-1)^n\,\xi^{2n+3}}{n!\,(2n+3)}\Bigg|_0^1$$

$$= \sum_{n=0}^{\infty}\frac{(-1)^n}{n!\,(2n+3)} = \frac{1}{3} - \frac{1}{5} + \frac{1}{14} - \frac{1}{54} + \frac{1}{264} - \cdots$$

$$\approx 0.1895.$$

$$E[X|X>20] \approx 40e\left[(\sqrt{\pi}/4) - 0.1895\right] \approx 27.58 \text{ weeks.}$$