(6.26)

$$R_X[m] = 10\, e^{-\lambda_1 |m|} + 5\, e^{-\lambda_2 |m|} \qquad 0 < \lambda_1, \lambda_2 < \infty$$

$$S_X(\omega) = \sum_{m=-\infty}^{+\infty} R_X[m]\, e^{-j\omega m}$$

$$= \sum_{m=-\infty}^{+\infty} 10\, e^{-\lambda_1 |m|} e^{-j\omega m} + \sum_{m=-\infty}^{+\infty} 5\, e^{-\lambda_2 |m|} e^{-j\omega m}$$

$$= 10 \sum_{m=0}^{\infty} e^{-\lambda_1 m - j\omega m} + 10 \sum_{m=-\infty}^{-1} e^{+\lambda_1 m - j\omega m}$$

$$+ 5 \sum_{m=0}^{\infty} e^{-\lambda_2 m - j\omega m} + 5 \sum_{m=-\infty}^{-1} e^{+\lambda_2 m - j\omega m}$$

$$= 10 \sum_{m=0}^{\infty} e^{-(\lambda_1 + j\omega) m} + 10 \sum_{m=0}^{\infty} e^{-(\lambda_1 - j\omega) m} - 10$$

$$+ 5 \sum_{m=0}^{\infty} e^{-(\lambda_2 + j\omega) m} + 5 \sum_{m=0}^{\infty} e^{-(\lambda_2 - j\omega) m} - 5$$

$$= \frac{10}{1 - e^{-(\lambda_1 + j\omega)}} + \frac{10}{1 - e^{-(\lambda_1 - j\omega)}} + \frac{5}{1 - e^{-(\lambda_2 + j\omega)}} + \frac{5}{1 - e^{-(\lambda_2 - j\omega)}} - 15,$$

$$= 10\, \frac{1 - e^{-2\lambda_1}}{1 - 2\cos\omega\, e^{-\lambda_1} + e^{-2\lambda_1}} + 5\, \frac{1 - e^{-2\lambda_2}}{1 - 2\cos\omega\, e^{-\lambda_2} + e^{-2\lambda_2}}$$

$$= 10 \left(\frac{1 - \rho_1^2}{1 + \rho_1^2 - 2\rho_1 \cos\omega} \right) + 5 \left(\frac{1 - \rho_2^2}{1 + \rho_2^2 - 2\rho_2 \cos\omega} \right)$$

with $\rho_i \triangleq e^{-\lambda_i}$ $i = 1, 2.$

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$$W[n] \longrightarrow \boxed{H[w]} \longrightarrow X[n]$$

$\mu_W[n] = 0$, $K_W[m] = \delta[m]$, we know;

$$X[n] = \sum_{K=-\infty}^{+\infty} h[K]W[n-K] = \sum_{K=-\infty}^{+\infty} h[n-K]W[K]$$

and $R_{WW}[n] = E\{W^*[m]W[m+n]\}$

$$R_{XW}[n] = E\{W^*[m]X[m+n]\}$$

$$= E\{W^*[m] \sum_{K=-\infty}^{+\infty} h[K]W[m+n-K]\}$$

If the Σ converges, then

$$R_{XW}[n] = \sum_{K=-\infty}^{+\infty} h[K]\, E\{W^*[m]W[m+n-K]\}$$

$$= \sum_{K=-\infty}^{+\infty} h[K]R_{WW}[n-K]$$

OR:

$$R_{XW}[n] = R_{WW}[n] * h[n]$$

$$= \delta[n] * h[n] = h[n].$$

Therefore similar result holds in the Discrete time case.

6.31

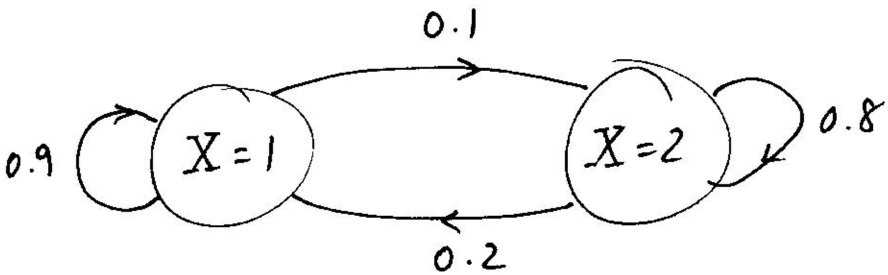
a) Since $\underline{p}[n] = \left(P[X[n]=1],\ P[X[n]=2]\right)$

$$= \left(P[X[n-1]=1]\,P_{11} + P[X[n-1]=2]\,P_{21},\ P[X[n-1]=1]\,P_{12} + P[X[n-1]=2]\,P_{22}\right)$$

$$= \underline{p}[n-1]\ \underline{\underline{P}}$$

$$\Rightarrow \underline{p}[n] = \underline{p}[0]\ \underline{P}^{n}$$

b)



c) Let p be the probability of the <u>first</u> transition to state 2 occuring at time n, given $X[0]=1$.

Then
$$p = P_{11}^{\,n-1} \cdot P_{12}$$
$$= (0.9)^{n-1}(0.1)$$
$$= (0.1)(0.9)^{n-1}$$

(7.3) $B(n)$ is Bernoulli r.s. ; Let $+1$ occur w.p. $\frac{1}{2}$

$X(t) \triangleq \sqrt{P}\, \mathrm{Sin}\,(2\pi f_0 t + B[n]\frac{\pi}{2})$ for $nT \le t < (n+1)T$,

$\forall n$ and $\sqrt{P}, f_0 \in \mathbb{R}$

$$\text{(a)}\ \mu_X(t) \triangleq E[X(t)] = E\{\sqrt{P}\,\mathrm{Sin}\,(2\pi f_0 t + B[n]\tfrac{\pi}{2})\}$$

$$= \sqrt{P}\, E\{\mathrm{Sin}\,(2\pi f_0 t + B[n]\tfrac{\pi}{2})\}$$

$$= \sqrt{P}\,\{\tfrac{1}{2}\mathrm{Sin}(2\pi f_0 t + \tfrac{\pi}{2}) + \tfrac{1}{2}\mathrm{Sin}(2\pi f_0 t - \tfrac{\pi}{2})\}$$

$$= \sqrt{P}\{\tfrac{1}{2}(+\mathrm{Cos}\,2\pi f_0 t) + \tfrac{1}{2}(-\mathrm{Cos}\,2\pi f_0 t)]$$

$$\mu_X(t) = \sqrt{P}\,(\tfrac{1}{2} - \tfrac{1}{2})\,\mathrm{Cos}\,2\pi f_0 t = 0 .$$

(b) $K_X(t,s) = E[X(t)X(s)] - E[X(t)]E[X(s)]$

(i) For $nT \le t,\ s < (n+1)T$, i.e., t and s are in the same interval.

$$
\begin{aligned}
E[X(t)X(s)] &= \tfrac{1}{2}\sqrt{P}\sin\left(2\pi f_0 t + \tfrac{\pi}{2}\right)\sqrt{P}\sin\left(2\pi f_0 s + \tfrac{\pi}{2}\right) \\
&\quad + \left(1-\tfrac{1}{2}\right)P\sin\left(2\pi f_0 t - \tfrac{\pi}{2}\right)\sin\left(2\pi f_0 s - \tfrac{\pi}{2}\right) \\
&= \tfrac{1}{2}P\cos(2\pi f_0 t)\cos(2\pi f_0 s) \\
&\quad + \left(1-\tfrac{1}{2}\right)P\cos(2\pi f_0 t)\cos(2\pi f_0 s) \\
&= P\cos(2\pi f_0 t)\cos(2\pi f_0 s)
\end{aligned}
$$

$$\Longrightarrow K_X(t,s) = P\cos(2\pi f_0 t)\cos(2\pi f_0 s)$$

(ii) For $nT \le t < (n+1)T$, $kT \le s < (k+1)T$, $k \ne n$, $X(t)$ and $X(s)$ are independent.

$$
\begin{aligned}
\Longrightarrow K_X(t,s) &= E[X(t)X(s)] - E[X(t)]E[X(s)] \\
&= E[X(t)]E[X(s)] - E[X(t)]E[X(s)] \\
&= 0.
\end{aligned}
$$

7.6 (a) Use property (3) for $t_1 = 0$ and $t_2 = t$

Then by property (1), $N(0) = 0$. So, (3) becomes:

$$P[N(t) = n] = \frac{\left[\int_0^t \lambda(\gamma)\,d\gamma\right]^n}{n!} e^{-\int_0^t \lambda(\gamma)\,d\gamma}; \; n \geq 0$$

Then, since $N(t)$ is poisson distributed, we recognize the mean as

$$\mu_N(t) = \int_0^t \lambda(\gamma)\,d\gamma$$

(b) Take $t_2 \geq t_1$ and write

$$E[N(t_2)N(t_1)] = E[[N(t_1) + (N(t_2) - N(t_1))]N(t_1)]$$

Then using the linearity of E[ ] and the independent increments of property (2), we have

$$E[N(t_2)N(t_1)] = E[N^2(t_1)] + E[N(t_2) - N(t_1)]\,E[N(t_1)]$$

Now, the first term is the second moment of a possion r.v., so

$$E[N^2(t_1)] = \int_0^{t_1} \lambda(\gamma)\,d\gamma + \left(\int_0^{t_1} \lambda(\gamma)\,d\gamma\right)^2$$

Now

$$E[N(t_2) - N(t_1)] = \int_{t_1}^{t_2} \lambda(\nu)\,d\nu$$

& from a) $E[N(t_1)] = \int_0^{t_1} \lambda(\nu)\,d\nu$

so

$$R_N(t_1,t_2) = E[N(t_1)N(t_2)]$$

$$= \int_0^{t_1} \lambda(\nu)\,d\nu + \left(\int_0^{t_1} \lambda(\nu)\,d\nu\right)^2 + \left(\int_{t_1}^{t_2} \lambda(\nu)\,d\nu\right)\left(\int_0^{t_1} \lambda(\nu)\,d\nu\right)$$

$$= \left(\int_0^{t_1} \lambda(\nu)\,d\nu\right)\left[1 + \int_0^{t_2} \lambda(\nu)\,d\nu\right] \qquad t_2 \geq t_1$$

$$= \int_0^{\min(t_1,t_2)} \lambda(\nu)\,d\nu \left[1 + \int_0^{\max(t_1,t_2)} \lambda(\nu)\,d\nu\right] \qquad \text{for all } t_1,t_2.$$

c)

We have to show (1), (2), & (3).

(1) $N_u(0) \triangleq N(t(0)) = N(0) \triangleq 0$ ✓

(2) Let $\tau_1 \leq \tau_2 \leq \ldots \leq \tau_K$, Then since $t(\tau)$ is monotone increasing (since it is integral of positive $\lambda(\nu)$), we have $t_1 \leq t_2 \leq \ldots \leq t_K$ where $t_i \triangleq t(\tau_i)$. Thus $N_u(\tau_i) \triangleq N(t(\tau_i)) = N(t_i)$. So by def. $N(t_1)$, $N(t_2) - N(t_1)$, ..., $N(t_K) - N(t_{K-1})$ are jointly indep. But $N_u(\tau_i) - N_u(\tau_{i-1}) = N(t_i) - N(t_{i-1})$, so The $N_u(\tau)$ process has independent increments too!

(3) Since $N_u(\tau_2) - N_u(\tau_1) = N(t_2) - N(t_1)$ with mean value $\int_{t_1}^{t_2} \lambda(\nu)\,d\nu = \int_0^{t_2} - \int_0^{t_1} = \tau_2 - \tau_1$.

7.15 Since W_1 & W_2 are independent Wiener processes, their difference is still Gaussian. Since the mean of $W_2(t)$ is zero, $-W_2(t)$ still has correlation function $\alpha_2 \min(t_1, t_2)$.

a) Thus $R_X(t_1, t_2) = (\alpha_1 + \alpha_2) \min(t_1, t_2)$

b) $f_X(x; t) \sim N(0, (\alpha_1 + \alpha_2) t)$.

7.16 a)

$$Y(t) = X'(t)$$

so $\mu_Y(t) = \frac{d}{dt} \mu_X(t) = \frac{d}{dt} 4 = 0.$

b) Since $\mu_X(t) = 4$,

$$R_Y(t_1, t_2) = \frac{\partial^2}{\partial t_1 \partial t_2} \left[5 \min^2(t_1, t_2) + 4^2 \right]$$

$$= \frac{\partial}{\partial t_1} \left[\frac{\partial}{\partial t_2} \left(5 \min^2(t_1, t_2) \right) \right]$$

$$= \frac{\partial}{\partial t_1}\left[10\,^{10}\min(t_1,t_2)\ u(t_1-t_2)\right]$$

$$= 10 \min(t_1,t_2)\ \delta(t_1-t_2)$$

$$+ 10\, u\cancel{(t_2-t_1)\ u(t_1-t_2)} \to 0$$

so $K_Y = R_Y(t_1,t_2) = 10 \min(t_1,t_2)\ \delta(t_1-t_2)$

$$= 10\, t_1\ \delta(t_1-t_2)$$

$$= 10\, t_2\ \delta(t_1-t_2).$$

c) (i) $\mu_Y(t) = 0$ ✓

(i) $R_Y(t+\tau, t) = (t+\tau)\,\delta(\tau)$

$= t\,\delta(\tau)$ not WSS!

d) One way to get This covariance is to multiply together two independent Wiener processes $W_1(t)$ $W_2(t)$ each with covariance $\min(t_1,t_2)$. Then multiply result by $\sqrt{5}$ and add the value 4.

$$X_c(t) = \sqrt{5}\ W_1(t)\ W_2(t)$$

Then $K_X(t_1,t_2) = 5\ E\left[W_1(t_1)\,W_1(t_2)\right]\ E\left[W_2(t_1)W_2(t_2)\right]$

$$= 5 \min(t_1,t_2)\cdot\min(t_1,t_2).$$

a) $P[\underline{\text{remain}} \text{ in state 2 till time } t \mid X(0)=2]$, $t>0$

$$= P[\text{no transition to } 3] \cdot P[\text{no trans. to } 1]$$

$$= e^{-\lambda_2 t} e^{-\mu_2 t} = \exp -(\lambda_2+\mu_2)t$$

b)
$$\begin{bmatrix} P_1(t+\Delta t) \\ P_2(t+\Delta t) \\ P_3(t+\Delta t) \end{bmatrix} \simeq \begin{bmatrix} 1-\lambda_1\Delta t & \mu_2\Delta t & 0 \\ \lambda_1\Delta t & 1-(\lambda_2+\mu_2)\Delta t & \mu_3\Delta t \\ 0 & \lambda_2\Delta t & 1-\mu_3\Delta t \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix}$$

or
$$\underline{\dot{P}} = \begin{bmatrix} -\lambda_1 & \mu_2 & 0 \\ \lambda_1 & -(\lambda_2+\mu_2) & \mu_3 \\ 0 & \lambda_2 & -\mu_3 \end{bmatrix} \underline{P}$$

c) In steady state $\underline{\dot{P}} = 0$

so
$$-\lambda_1 P_1 + \mu_2 P_2 = 0$$

and $\lambda_2 P_2 - \mu_3 P_3 = 0.$

Also we know $P_1 + P_2 + P_3 = 1$

so $P_2 = \frac{\lambda_1}{\mu_2} P_1$ & $P_3 = \frac{\lambda_2}{\mu_3} P_2$

so $P_1 + P_2 + P_3 = P_1 \left[1 + \frac{\lambda_1}{\mu_2} + \frac{\lambda_2}{\mu_3} \cdot \frac{\lambda_1}{\mu_2} \right] = 1.$

or
$$P_1 = \frac{1}{1 + \frac{\lambda_1}{\mu_2}\left(1 + \frac{\lambda_2}{\mu_3}\right)} =$$

$$P_2 = \frac{\lambda_1/\mu_2}{1 + \frac{\lambda_1}{\mu_2}\left(1 + \frac{\lambda_2}{\mu_3}\right)}$$

and
$$P_3 = \frac{\frac{\lambda_1}{\mu_2} \cdot \frac{\lambda_2}{\mu_3}}{1 + \frac{\lambda_1}{\mu_2}\left(1 + \frac{\lambda_2}{\mu_3}\right)}.$$

(7.29) a) $\mu_Y(t) = E[X(t) + 0.3X'(t)]$

$= \mu_X(t) + 0.3\,\mu_X'(t)$

$= 5t + 0.3(5)$

$= 5t + 1.5$ //

b) $K_Y(t_1,t_2) = E[(X_c + .3X_c')(X_c + .3X_c')]$

where $X_c \triangleq X - \mu_X$

so

$$K_X(t_1,t_2) + 0.3\frac{\partial K_X(t_1,t_2)}{\partial t_1} + 0.3\frac{\partial K_X(t_1,t_2)}{\partial t_2} + .09\frac{\partial^2 K_X(t_1,t_2)}{\partial t_1 \partial t_2}$$
$$= K_Y(t_1,t_2)$$

Now $\dfrac{\partial K_X(t_1,t_2)}{\partial t_1} = \dfrac{-\sigma^2 2\alpha(t_1-t_2)}{(1+\alpha(t_1-t_2)^2)^2}$

& $\dfrac{\partial K_X(t_1,t_2)}{\partial t_2} = -\dfrac{\sigma^2 2\alpha(t_2-t_1)}{(1+\alpha(t_1-t_2)^2)^2}$

$= -\dfrac{\partial K_X(t_1,t_2)}{\partial t_1}$

so cross-terms cancel. Thus, with

$$\frac{\partial^2 K_X(t_1,t_2)}{\partial t_1 \partial t_2} = \frac{-6\alpha^2\sigma^2(t_1-t_2)^2 + 2\alpha\sigma^2}{(1+\alpha(t_1-t_2)^2)^3},$$ we have

finally $$K_Y(t_1,t_2) = \frac{\sigma^2}{1+\alpha\tau^2}\left\{1 + \frac{-0.54\alpha^2\tau^2 + .18\alpha}{(1+\alpha\tau^2)^2}\right\} = K_Y(\tau)$$

$\tau = t_1 - t_2$.

7.32 (a) $X(t)$ has constant mean 128.

$$\mu_X(t) = \mu_X H(0) = 128 \times 1 = 128$$

(b) $$K_Y(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |H(\omega)|^2 \cdot S_{X_cX_c}(\omega) e^{j\omega\tau}\, d\omega$$

$$|H(\omega)|^2 = \frac{1}{1+\omega^2}$$

$$S_{X_cX_c}(\omega) = \int_{-\infty}^{\infty} 1000\, e^{-10|\tau|} e^{-j\omega\tau}\, d\tau$$

$$= 1000\int_{-\infty}^{\infty} e^{-10|\tau| - j\omega\tau}\, d\tau$$

(note: $X_c \triangleq X - \mu_X$)

$$= 1000\left[\int_{0}^{\infty} e^{-10\tau - j\omega\tau} d\tau + \int_{-\infty}^{0} e^{10\tau - j\omega\tau} d\tau\right]$$

$$= 1000\left[\frac{1}{10+j\omega} + \frac{1}{10-j\omega}\right]$$

$$= 1000\,\frac{20}{100+\omega^2}$$

$$= \frac{20{,}000}{100+\omega^2}$$

$$K_Y(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{1+\omega^2}\cdot\frac{20{,}000}{100+\omega^2}\, e^{j\omega\tau}\, d\omega$$

$$= \frac{20{,}000}{2\pi\cdot 99}\int_{-\infty}^{\infty}\left[\frac{1}{1+\omega^2} - \frac{1}{100+\omega^2}\right] e^{j\omega\tau}\, d\omega$$

$$= \frac{20000}{99}\left[\frac{1}{2}e^{-|\tau|} - \frac{1}{20}e^{-10|\tau|}\right]$$

$$= (101.01)\, e^{-|\tau|} - (10.10)\, e^{-10|\tau|}$$

(7.33) a) $S_Y(\omega) = |H(\omega)|^2 S_X(\omega)$

$$H(\omega) = \frac{1}{4}\int_{-2}^{+2} e^{-j\omega t}\,dt = \frac{e^{-j\omega 2} - e^{+j\omega 2}}{-j2\,(2\omega)}$$

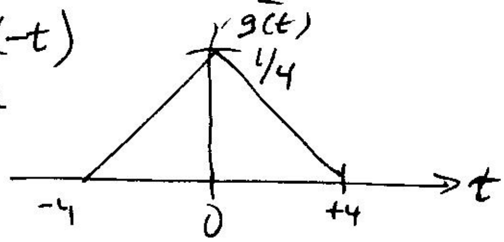
$$= \frac{\sin 2\omega}{2\omega}$$

S_X is given as 2.

So $S_Y(\omega) = 2\left(\frac{\sin 2\omega}{2\omega}\right)^2$.

b) $R_X(\tau) = 2\,\delta(\tau) \qquad h(t) = \frac{1}{4}\left[u(t+2) - u(t-2)\right]$

let $g(t) = h(t) * h^*(-t)$
$= \text{triangle}$



$g(0) = \frac{1}{4} = \int_{-2}^{+2} h^*(-\tau)\,h(-\tau)\,d\tau$

$Y = h * X$

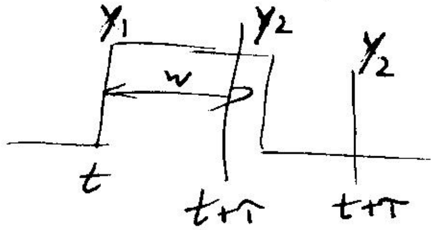
$= \frac{1}{4}\cdot\frac{1}{4}\cdot\int_{-2}^{+2} d\tau$

$= \frac{1}{4}\cdot\frac{1}{4}\cdot 4$ ✓

$$\Rightarrow R_Y(\tau) = \begin{cases} \frac{1}{2}\left(1 - \frac{|\tau|}{4}\right), & |\tau| \le 4 \\ 0, & |\tau| > 4. \end{cases}$$

(7.34) $R_Y(\tau) = E[Y(t)\,Y(t+\tau)] = E_W\left[E[Y(t)Y(t+\tau)\,|\,W]\right]$

Let $W=w$ & evaluate $E[Y(t)\,Y(t+\tau)\,|\,W=w]$. Since not a function of t, can take t at start of pulse (by WSS on Y)



Now for $\tau < w$ $\quad Y_1 = Y_2$

so $\quad Y_1 Y_2 = Y_1^2$

for $\tau > w$ $\quad Y_1 Y_2$ with $Y_1 \perp Y_2$

$$E[Y(t)Y(t+\tau)|W=w] = E[Y_1 Y_2] = E[Y_1]E[Y_2] = E^2[Y], \quad \tau > w$$
$$= E[Y_1^2] = E[Y^2], \quad \tau < w$$

or $\quad E[Y(t)Y(t+\tau)|W=w] = \begin{cases} E^2[Y] & \tau > w \\ E[Y^2] & \tau < w \end{cases}$

so

$$E[E[Y(t)Y(t+\tau)|W]] = E^2[Y]\, P[W<\tau] + E[Y^2]\, P[W>\tau]$$

& $E[X]=0$, $P[W>\tau] = \int_\tau^\infty \lambda e^{-\lambda w} dw = e^{-\lambda\tau}$

$$E[Y^2] = \int_{-\infty}^{+\infty} y^2 f_Y(y)\,dy = \sigma^2$$ ($E[Y]=0$ since the process is zero mean

$$\therefore R_Y(\tau) = \sigma^2 e^{-\lambda\tau}, \quad \tau > 0$$

Since $R_Y(\tau)$ must be even in τ, we have

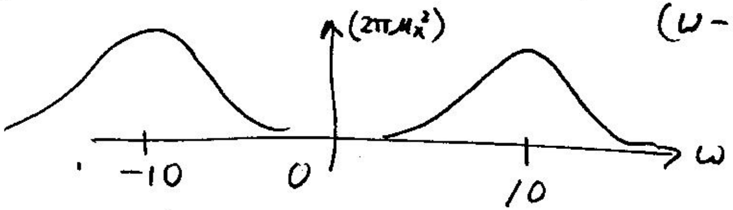
$$R_Y(\tau) = \sigma^2 \exp -\lambda|\tau| \quad \text{for } -\infty < \tau < +\infty.$$

$$S_Y(\omega) = \frac{2\lambda\sigma^2}{\omega^2+\lambda^2} \quad \text{for } -\infty < \omega < +\infty$$

$\tau = 0, \quad R_Y(0) = \sigma^2$ = mean square value of X, as it should be.

$\tau = \infty, \quad R_Y(\infty) = 0$ = mean value of the process squared. Seperated elements become uncorrelated.

(7.35) a) $S_X(\omega) = 2\pi \mu_X^2 \delta(\omega) + \dfrac{10}{(\omega-10)^2+4} + \dfrac{10}{(\omega+10)^2+4}$



$\left(e^{-\alpha|\tau|} \Leftrightarrow \dfrac{2\alpha}{\omega^2+\alpha^2} \right)$

b) $\dfrac{1}{2\pi}\displaystyle\int_{\omega_1}^{\omega_2} (\cdot)\, d\omega + \dfrac{1}{2\pi}\int_{-\omega_2}^{-\omega_1} (\cdot)\, d\omega$

c) $S_Y(\omega) = S_X(\omega) \cdot |H|^2(\omega)$

with $|H|^2 = \dfrac{j\omega+4}{(j\omega+6)(j\omega+5)} \cdot \dfrac{-j\omega+4}{(-j\omega+6)(-j\omega+5)} = \dfrac{\omega^2+16}{(\omega^2+36)(\omega^2+25)}$

(7.7) Poisson pmf: $P_N(n;t) = \frac{\left(\int_0^t \lambda(v)\,dv\right)^n}{n!} e^{-\int_0^t \lambda(v)\,dv} u(n)$

a) $\mu_N(t) = E[N(t)] = \int_0^t \lambda(v)\,dv \quad , \; t \geq 0$

$$= \int_0^t (1+2v)\,dv$$

$$= v + v^2 \Big|_0^t$$

$$= t + t^2 \,, \quad t \geq 0.$$

b)

Let $t_2 > t_1 \geq 0$, then

$$N(t_1)N(t_2) = N(t_1)\left[N(t_1) + (N(t_2) - N(t_1))\right]$$

so $R_N(t_1,t_2) = E[N^2(t_1)] + \mu_N(t_1)\left(\mu_N(t_2) - \mu_N(t_1)\right)$

$$E[N^2(t)] = \int_0^t \lambda(v)\,dv + \left(\int_0^t \lambda(v)\,dv\right)^2$$

$$= (t+t^2) + (t+t^2)^2$$

so for $t_2 > t_1$

$$R_N(t_1,t_2) = (t_1+t_1^2) + (t_1+t_1^2)^2 + (t+t_1^2)\left(t_2+t_2^2 - (t_1+t_1^2)\right)$$

$$= (t_1+t_1^2) + (t_1+t_1^2)(t_2+t_2^2).$$

in general then $R_N(t_1,t_2) = \left[\min(t_1,t_2) + \min^2(t_1,t_2)\right]\left[1 + \max(t_1,t_2) + \max^2(t_1,t_2)\right]$

d) Use CLT with $\mu_N = t+t^2$, $\sigma_N^2 = t+t^2$ to yield

$$P \simeq \frac{1}{2} + \operatorname{erf}\left(\frac{t^2}{\sqrt{t+t^2}}\right) \sim \frac{1}{2} + \operatorname{erf}(t).$$