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A PROCEDURE FOR THE ONLINE MEASUREMENT OF A VEHICLE'S INERTIA TENSOR ON A STEWART PLATFORM

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ABSTRACT

Online inertia tensor measurement of a vehicle suspended on a six degree of freedom Stewart Platform is performed via a recursive least squares algorithm. Inertia tensor parameters are updated online while the platform simultaneously rotates about its yaw, pitch and roll axes. Position control of the Stewart Platform is achieved by controlling each of the leg lengths within a decoupled PID loop after solving the desired joint lengths through inverse kinematics. An accelerometer mounted on the movable platform provides real time acceleration data and load cells on each leg provide the force data. Thus the governing dynamic equations of the upper platform are cast in a least squares form and the six independent entries of the inertia tensor are estimated recursively in order to gain speed and save memory. In simulations fast convergence to actual values are obtained for all parameters of interest.

Keywords: Stewart platform, inertia tensor, online recursive parameter estimation

STEWART PLATFORMU ÜZERİNDE ÇEVRİMİÇİ ARAÇ EYLEMSİZLİK TENSÖRÜ ÖLÇÜMÜ İÇİN BİR YÖNTEM

ÖZET

Altı serbestlik dereceli bir Stewart Platformu üzerine sabitlenmis bir taşıtın ardışık en küçük kareler yöntemiyle eylemsizlik tensörünün ölçümü gerçekleştirilmiştir. Eylemsizlik tensörü parametreleri araç aynı anda üç dönme ekseni etrafında hareket ederken çevrimiçi olarak güncellenmektedir. Stewart platformunun konum kontrolü her bir bacağı ters kinematik probleminin çözümü sonucunda istenilen boya ayrı bir PID denetleyicisi kullanılmak suretiyle gerçekleştirilir. Hareketli platform üzerine monte edilen bir ivme ölçer gerçek ivmelenme verisini, her ayaktaki kuvvet sensörleri ise kuvvet verisini sağlar. Netice olarak üst platformun dinamik denklemleri en küçük kareler biçiminde yazılmış ve eylemsizlik tensörünün altı bağımsız değiskeni hız kazanmak ve hafıza tasarrufu sağlamak amacıyla ardışık olarak en küçük kareler yöntemiyle tahmin edilmilştir. Benzetimler sonucunda ilgili tüm değerlerin gerçek değerlerine hızla yakınsadığı görülmüştür.

Anahtar kelimeler: Stewart platformu, eylemsizlik tensörü, çevrimiçi ardışık parametre tahmini

1. INTRODUCTION

Experimental research on vehicle dynamics and vehicle control depends tightly on the knowledge of vehicle parameters. While, the main challenge resides in the determination of tire parameters, another aspect of the problem consists in having a good knowledge of the vehicle inertial parameters. A number of studies have appeared in the literature concerning the online determination of the latter. In [1], recursive least squares are used to determine mass and road grade during driving conditions. In [2], a methodology based on multiple models and switching for real-time estimation of center of gravity (CG) position of an automotive vehicle is presented. While in [3], an active steering controller is designed so as to be robust against changes in vehicle CG and yaw moment of inertia. The importance of determining inertial parameters during driving is obvious: vehicle dynamics controllers must quickly adapt themselves to new operating conditions and ensure vehicle stability at all times. However, it is also clear that on board parameter estimation algorithms need good initial guesses for inertial parameters. Various industrial apparatus exist such as the VIMF (vehicle inertia measurement facility) [4] produced by S.E.A., Incorporated (SEA), and the VIMM (vehicle inertial measurement machine) [5] produced by ika. The former consists of a set of various experimental setups for the separate computation of yaw, roll and pitch inertias. The latter is a three degree of freedom testing table where all inertial parameters are claimed to be rapidly determined.

The Stewart platform has been widely used as a simulation platform in both the automotive and aerospace industries. In this paper, we propose a new application area and demonstrate the relative technical simplicity of adding an inertial parameter estimation unit to the system. In Section 2, kinematics of the Stewart platform are presented. The PID position control law is described in Section 3. Section 4 deals with the equations of motion for the upper platform. The estimation of the inertia tensor is given in Section 5. Finally, conclusions are made in Section 6.

2. KINEMATICS OF THE STEWART PLATFORM

Parallel mechanisms are essentially robots in which the linkages are connected to a base and a moving platform. Prismatic joints change the link lengths at the joints. According to another definition parallel mechanisms are chains whose only few joints are actuated. Most celebrated of such mechanisms is the modified version of the mechanism originally developed by Stewart [6]. In literature this mechanism is called the Stewart Platform Mechanism (SPM) and it has six controllable degrees of freedom. A typical SPM has six length adjustable legs connecting two rigid platforms one of which is fixed to the earth. Legs can either be formed by pistons and cylinders or by a series of elements linked by joints. The lower connecting points can either be co-planar or they can lie on different planes altogether. For the latter case, extra sensors are needed. Therefore in literature, first case is commonly investigated. Upper platform has six degrees of freedom with respect to the base. If constant lengths are assigned to the legs mechanism becomes a structure. Geometrically speaking each leg is a plane tangent to the circle passing through three vertices of the platform [7].

It was through Gough's proposal that the mechanism gained its fully parallel form. Therefore it is sometimes referred to as the Gough-Stewart platform. Since Stewart has first proposed to use this mechanism as a flight simulator it is more common to use the name SPM. Hunt has proposed to use this mechanism as a robotic manipulator.

Forward kinematic problem of the Stewart involves finding the rotation and translation of the upper platform with respect to the base when the leg lengths are known. In other words one needs to find the rotation matrix that maps the vectors from the upper platforms coordinate frame to a coordinate frame fixed on earth. Forward kinematic problem is important because one needs to know all the possible orientations of the mechanism at a given set of leg lengths. On the other hand the inverse kinematic problem involves finding the desired leg lengths when the translation and rotation of the upper platform are known. Inverse relation between serial and parallel mechanisms manifests itself here too. In serial mechanisms forward kinematic problem is trivial and the inverse problem is hard. Conversely in parallel mechanisms the inverse problem is easy and the forward problem is hard to solve. In order to follow a prescribed orbit in real time, by looking at the online dynamics of inverse kinematics, one can conclude that the forward kinematics has more than one solution [8-10]. There exists more than one configuration for a single entry of leg lengths. On the other hand inverse kinematics has a unique solution. In other words the desired orientation of the upper platform can only be formed by a single entry of leg lengths.

A schematic view of a 6-6 Stewart platform is shown in Figure 1. One can start the kinematic analysis by fixing two coordinate frames; one $\{\mathcal{B}\}$ fixed to the center of immobile base, the other $\{\mathcal{P}\}$ fixed at the center of the circle defined by the vertices of the movable platform.

Polar angles of the vertices of upper and lower platforms can be similarly expressed as follows (1-2)

.

$$\lambda_i = \frac{(i-1)\pi}{6} \tag{1}$$

$$\Lambda_i = \frac{(i-1)\pi}{6} \tag{2}$$

Position of a reference point P with respect to coordinate frame $\{\mathcal{P}\}$ is given as follows (3).

$$\boldsymbol{P_i} = \begin{bmatrix} r_p \cos(\lambda_i) \\ r_p \sin(\lambda_i) \\ 0 \end{bmatrix} \equiv \begin{bmatrix} P_{ix} \\ P_{iy} \\ P_{iz} \end{bmatrix} \tag{3}$$

Similarly position of a reference point B with respect to coordinate frame {B} is given as follows (4). Here r denotes the radius of the base and upper platform depending on the subscript.

$$\boldsymbol{B_i} = \begin{bmatrix} r_B \cos(\Lambda_i) \\ r_B \sin(\Lambda_i) \\ 0 \end{bmatrix} \equiv \begin{bmatrix} B_{ix} \\ B_{iy} \\ B_{iz} \end{bmatrix} \tag{4}$$

We also let $\mathbf{O_p}$ denote the position of the platform center with respect to the base. From inverse kinematics [5] necessary leg lengths are obtained from the following mathematical relationship (5).

$$l_i = \frac{R_{\alpha\beta\gamma}\boldsymbol{P_i} + \boldsymbol{O_p} - \boldsymbol{B_i}}{\left|R_{\alpha\beta\gamma}\boldsymbol{P_i} + \boldsymbol{O_p} - \boldsymbol{B_i}\right|} \tag{5}$$

In the above equation $R_{\alpha\beta\gamma}$ is the rotation matrix which transforms from the stationary base coordinate system to the platform coordinate system.

$$R_\alpha = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_\beta = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & -\sin\beta \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \tag{6}$$

$$R_\gamma = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Figure 1. Vehicle Mounted on the Platform

Multiplication of individual components gives $R_{\alpha\beta\gamma}$(7).

$$R_{\alpha\beta\gamma} = R_\alpha R_\beta R_\gamma \tag{7}$$

3. PID POSITION CONTROL

Desired trajectory signal to the platform is defined to provide zero translation and to give simultaneous roll, pitch and yaw motions at a chosen amplitude and frequency for each of these rotational components (8). This approach avoids zero or small entries in the regression matrix.

$$x_d = 0$$
$$y_d = 0$$
$$z_d = 0$$

$$\begin{aligned} \alpha_d &= A_1 \sin\omega_1 t \\ \beta_d &= A_2 \sin\omega_2 t \\ \gamma_d &= A_3 \sin\omega_3 t \end{aligned} \tag{8}$$

Each of the leg lengths are controlled within a decoupled PID loop after solving the desired joint lengths through inverse kinematics discussed in the previous section. Leg lengths are measured through linear transducers mounted on the piston cylinder mechanisms. Proportional, integral and derivative gains are fine tuned in order to achieve the best time response, the control objective consisting in producing leg forces F_{xi} so that actual leg lengths $l_{act\ i}$ track desired leg lengths l_{di} (Figure 2).

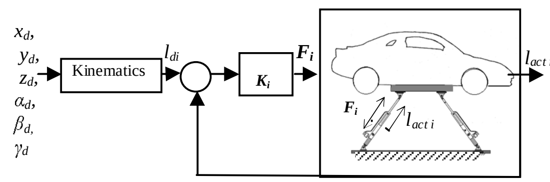

Figure 2. Vehicle Mounted on the Platform

4. EQUATIONS OF MOTION FOR PLATFORM

Inertia matrix of a solid body can be expressed as follows.

$$I = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \tag{9}$$

Inertia tensor is a 3x3 symmetric matrix (9). Therefore it only has six independent components. Since translational and rotational motion are decoupled from one another it suffices to write only the three governing ordinary differential equations (10-12) related to rotational motion in order to solve for the inertia matrix. By measuring the forces acting on the upper platform through each of the six legs one can solve for the three components of the torque.

$$I_{xx}\frac{dw_x}{dt} - I_{xz}\frac{dw_z}{dt} - I_{xy}\frac{dw_y}{dt} = \tau_x + I_{yz}\left({w_y}^2 - {w_z}^2\right) + I_{xz}w_x w_y - I_{xy}w_z w_x + \left(I_{yy} - I_{zz}\right)w_y w_z \tag{10}$$

$$I_{yy}\frac{dw_y}{dt} - I_{xy}\frac{dw_x}{dt} - I_{yz}\frac{dw_z}{dt} = \tau_y + I_{xz}({w_z}^2 - {w_x}^2) + I_{xy}w_z w_y - I_{yz}w_y w_x + (I_{zz} - I_{xx})w_x w_z \tag{11}$$

$$I_{zz}\frac{dw_z}{dt} - I_{yz}\frac{dw_y}{dt} - I_{xz}\frac{dw_x}{dt} = \tau_z + I_{xy}\left({w_x}^2 - {w_y}^2\right) + I_{yz}w_z w_x - I_{xz}w_y w_z + \left(I_{xx} - I_{yy}\right)w_x w_y \tag{12}$$

Platform orientation can be obtained by integrating the measured accelerations acting on the platform. Necessary acceleration data is obtained from the accelerometer mounted on the upper platform.

5. INERTIA TENSOR ESTIMATION

If one assigns the unknown entries of the inertia tensor to a vector (13) then the equations of motion of the upper platform can be cast into a least squares form with a parameter vector and a regression matrix (14). Note that the quantities appearing in the regression matrix are known through force and acceleration measurements.

$$\theta = [I_{xx} \quad I_{yy} \quad I_{zz} \; I_{xy} \quad I_{xz} \quad I_{yz}]^T \tag{13}$$

Then equations through (10-12) are essentially of the following form (14).

$$\left\{ \frac{d}{dt}\begin{bmatrix} w_x & 0 & 0 & -w_y & -w_z & 0 \\ 0 & w_y & 0 & -w_x & 0 & -w_z \\ 0 & 0 & w_z & 0 & -w_x & -w_y \end{bmatrix} - \begin{bmatrix} 0 & w_y w_z & -w_y w_z & -w_z w_x & w_x w_y & (w_y{}^2 - w_x{}^2) \\ -w_z w_x & 0 & w_x w_z & w_y w_z & (w_z{}^2 - w_x{}^2) & -w_x w_y \\ w_x w_y & -w_x w_y & 0 & (w_x{}^2 - w_y{}^2) & -w_y w_z & w_z w_x \end{bmatrix} \right\} \theta = \begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} \tag{14}$$

Consequently linear parameter estimation problem can be written as in the next equation (15). Over hat denotes estimated quantities and ε denotes the estimation error. Note that the output vector y is chosen as the torque vector. Regression matrix φ contains measured (through the accelerometer mounted on the platform) rotational acceleration and velocity data terms.

$$\varphi^T \hat{\theta} + \varepsilon = \hat{y} \tag{15}$$

Unknown parameter vector θ is updated at each iteration of the estimation loop. Parameter update law [11] is given by the following equation (16).

$$\hat{\theta}(t) = \hat{\theta}(t-1) + K(t)\left(y(t) - \varphi^T \hat{\theta}(t-1)\right) \tag{16}$$

K matrix appearing in (19) is given below (17)

.

$$K(t) = P(t-1)\varphi(t)\left(I + \varphi^T(t)P(t-1)\varphi(t)\right)^{-1} \tag{17}$$

P matrix appearing in (17) is given in (18). P is a diagonal matrix. Diagonal entries of P indicate the variance (thus accuracy) of the estimated quantity. The smaller the corresponding entry in P the better the corresponding component of θ.

$$P(t) = \left(I - K(t)\varphi^T(t)\right)P(t-1) \tag{18}$$

Equations (16-18) are updated at each successive iteration of the parameter estimation loop in order to obtain the current estimate of θ which is denoted by $\hat{\theta}(t)$. Results of these estimations are shown in Figures 3-8 for each of the six inertia elements. It can be observed that for all unknown inertia terms estimated values converge rapidly to their exact values. Number of iterations for a good convergence is typically less than 100. In simulations so far sensor noise effects have not been included. With noise effects included one can expect that the average number of iterations to increase by one or two orders of magnitude depending on the signal to noise ratio (SNR) of the measurement processes.

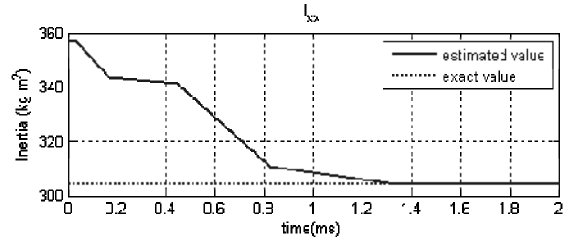

Figure 3. Estimation of I_{xx}

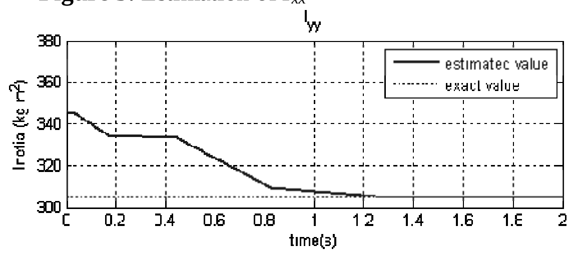

Figure 4. Estimation of I_{yy}

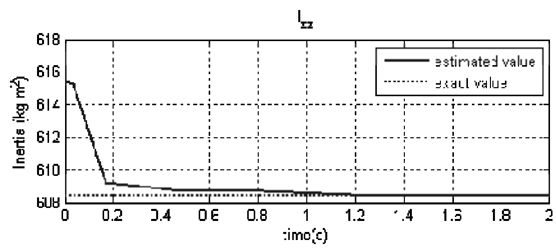

Figure 5. Estimation of I_{zz}

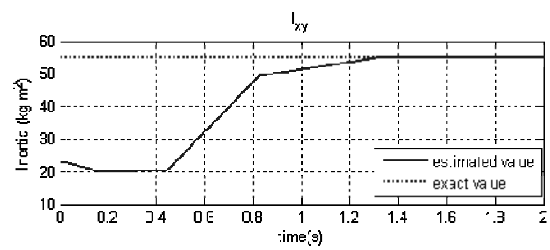

Figure 6. Estimation of I_{xy}

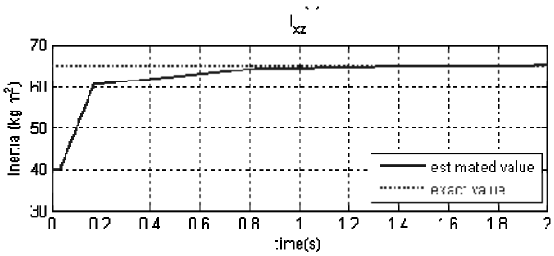

Figure 7. Estimation of I_{xz}

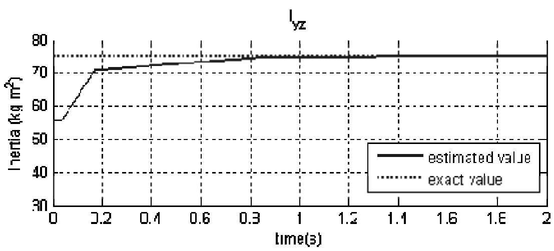

Figure 8. Estimation of I_{yz}

6. CONCLUSION

A novel technique for vehicle inertia measurement is demonstrated. This technique has the advantage of simultaneously measuring all components of the inertia tensor with a single test setup unlike the ones used in industry today. Typical test time is in the order of seconds once the vehicle is fixed on the measurement platform. Online recursive method is found to be a very efficient solution method for the problem of interest. All in all the proposed test setup can be claimed to be an improvement over the existing industrial ones albeit some additional complexity due to the added degrees of freedom which nonetheless can be viewed as an advantage at the same time.

Prospective research shall focus on testing sensor noise effects extensively as well as the detailed design of hydraulics.

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