Combustion Dynamics: Diagnostics, Modeling and Control

S. Acharya, O. Tuncer, J. H. Uhm, and A. De Turbine Innovation and Energy Research (TIER) Center Louisiana State University, Baton Rouge, LA 70803, USA

Abstract

In a gas turbine combustor, the flow and pressure fields, the mixing of fuel and air and the associated combustion and heat release, are all essentially unsteady non-linear processes and are intrinsically interlinked with each other. Understanding this complex interaction and its implication on key performance metrics such as emissions, thrust or volumetric heat release, and pressure oscillations is a challenging task. In this paper, we present measurements that underscore the importance of the unsteady processes in a gas turbine combustor, and present two modeling approaches that can be used to qualitatively predict the unsteady behavior in the combustor.

Measurements of pressure, CH-chemiluminescence, and NOx are reported for both a nonpremixed system (liquid fuel representing an aero-engine application) and a premixed fuel system (natural gas and hydrogen-enriched natural gas representing a land-based power-generation application). It will be shown that both systems exhibit unsteady characteristics that have similar origins. For example, both systems are characterized by thermo-acoustic instabilities and realtime fluctuations in the NOx emissions.

A reduced order model is presented and is shown to qualitatively predict the dynamics of pressure, heat release and flame movement in the combustor. The reduced order model embodies the feedback coupling between the acoustic field (obtained from a one-dimensional acoustic wave equation) and the non-linear heat release (modeled as proportional to velocity fluctuations modified by a non-linear saturation effect) together with a level set approach for modeling the flame kinematics. The reduced order model is a cost-effective and fast approach for understanding and representing combustion dynamics. For a more comprehensive and detailed understanding of the unsteady combustion behavior a Large Eddy Simulation (LES) approach is recommended.

Also active control strategies for reducing the pressure oscillations and heat release dynamics, and the impact of this control on emissions are briefly presented and discussed. The overall goal of the present effort is to be able to simulate or model such control strategies.

Introduction

Combustion dynamics is a key issue in the design and operation of gas turbine combustors. While "dynamics" refers to unsteady flow and combustion behavior, it is usually used in the context of thermo-acoustic instabilities which can lead to performance degradation, unacceptable noise levels and structural damage [1-3]. Therefore, there is a large body of literature devoted to an understanding of thermo-acoustic combustion dynamics, and several strategies toward controlling these dynamics have been developed over the years. The literature on feedback control of combustion instabilities is extensive, and is documented in the reviews by Culick [4], Schadow and Gutmark [5], Candel [6] and McManus et al. [7]. Active control has traditionally been implemented by acoustic drivers (Lang et al. [8], Hoffmann et al. [9]), and by pulsed fuel injection, (Campos-Delgado et al. [10] and Uhm and Acharya [11]). Sivasegaram and Whitelaw [12] and Sivasegaram et al. [13] showed that the suppression of the dominant frequency in flows with competing frequency modes led to the excitation of a rival frequency. Yu et al. [14], Fleifi et al. [15], Wilson et al. [16] and Bhidayasiri et al. [17] found that control can be ineffective at high flow rates due to bimodal combustion instabilities and a shift in the position of the flame.

Open loop control for combustion instability has been also reported by a number of researchers. Recently Shcherbik et al. [18] modulated fuel injection rate with frequency of 300

Hz, separated from the resonant frequency of 375 Hz, and effectively damped or prevented the onset of large amplitude instabilities in a high pressure gas turbine combustor. Richards et al. [30] showed the possibility of stabilizing the oscillations of acoustic mode in natural gas turbine combustor by modulating fuel at a low frequency of 50 Hz. Richards et al. [19] also showed the ability of low-frequency open-loop control to suppress combustion instability. The open-loop studies above were all carried out with fuel modulation. McManus et al. [20] investigated open-loop control by exciting the flow periodically in the form of a sinusoidal cross-stream velocity perturbation applied just upstream of the flow separation in two-dimensional dump combustor. The periodic flow perturbation was shown to increase the volumetric energy release, reduce NOx emissions, extend the lean-flammability limit and decrease the pressure fluctuations. The flow perturbation was applied close to the region of positive Rayleigh index, but this was possible due to the simple configuration studied. More recently, Uhm and Acharya [11, 22, 23] have demonstrated that open-loop control with a high-momentum air-jet can be quite effective in reducing the dynamics.

In order to effectively model the combustion dynamics, it is critical to be able to resolve the large scale unsteadiness in the flow since these drive the instabilities. Traditional Reynolds-Averaged-Navier Stokes (RANS) are usually best in resolving steady-state behavior. In situations where there is strong flow unsteadiness, such as those encountered in the presence of combustion dynamics, it is unlikely that RANS can properly model the effect of the flow unsteadiness. Methods such as Direct Numerical Simulations (DNS) and Large Eddy Simulations (LES) are likely to be needed to properly capture the flow unsteadiness. These methods that attempt to capture the energy-carrying scales in the flow are inherently time-consuming, and require parallel computations. Therefore, for simulating the unsteadiness in real-time, simpler methods that focus on modeling the large-scale unsteadiness are needed. These methods, called Reduced Order Models (ROM), usually require the solution of ordinary differential equations, so that, their solutions can be obtained rapidly. The challenge lies in correctly capturing the dynamics of the heat release and acoustic fields that lead to the thermo-acoustic instabilities. In this paper, we discuss this approach, and present typical results.

Combustor Description

Measurements will be presented for two combustors: a non-premixed, liquid-fueled, swirlstabilized system used for aero-engine applications, and premixed natural gas system (with hydrogen enhancement) used for power-generation applications. Characteristics of the dynamics in these two different systems will be presented with the intent of emphasizing the similar nature of these dynamics.

The Non-Premixed Combustor: The experiments are carried out in a multi-fuel-feed combustor operating at nearly 150 kW heat release. The combustor configuration, shown in figure 1, consists of two concentric large area ratio nozzles. Swirl vanes with 45 degree angle are placed in the inner and outer air streams at the exit of each nozzle. The placement of eight liquid fuel atomizers (Arizona Mist with 0.3 mm orifices), equally spaced circumferentially, and located between the coaxial jet streams is provided. A high-momentum control air-jet, with straight-through holes of different sizes (1.5, 3, and 6 mm), is provided at the geometric center. For modulating the control air-stream, the air-delivery tube is connected to proportional-drive valve located outside the combustor (Fig. 1). The valve responds to the input signal proportionally with a flat response up to a maximum frequency of 300 Hz. Ethanol is used as the liquid fuel and is pressurized to a maximum of 18 bar in three fuel tanks by high-pressure nitrogen.

The combustor is equipped with a square shell with 19.5 cm sides and windows that are either stainless steel or quartz for optical access. A high sensitivity, water-cooled pressure transducer (Kistler 6061B) is mounted at an axial distance of 6 cm from the expansion plane to measure pressure oscillation and to provide the feedback signal for control. Light emission is recorded at the CH-radical wavelength using a photodiode with a 430 nm centered-optical filter, and these

measurements are assumed to be representative of the heat release fluctuations from the flame. Pressure and CH light signals are also used for identifying combustion instability, which is driven or damped through their phase relationship. Concentrations of NOx at the exit plane of the combustor were measured to investigate the influence of combustion instability and its control on emissions. The exhaust gas samples were collected with a water-cooled stainless-steel suction probe and transferred through an electrically heated PTFE sample line to a fast chemiluminescence detector (Cambustion DCS 400). The probe was installed 1 cm downstream of combustor exit. In order to explain NOx in detail, flame images were captured at the dump plane through the quartz window and at the combustor exit using an Olympus C5000 digital camera (460x480 pixels). The pressure and CH-signals are processed in real time using a digital signal processor (DS1103, DSPACE, 333 MHz Motorola power PC) to be used in active control. The spatial and temporal variations in the CH chemiluminesence (heat release) are also visualized using a Princeton Instruments PI-MAX 512x512 ICCD camera with a UV lens (Electrophysics) and a bandpass filter (DIOP) at 430nm. The images are triggered with respect to the pressure oscillations in the combustor to determine the phase-averaged distributions of the heat release at different instances of the pressure-oscillation cycle.



Figure 1: Coaxial fuel Spray Combustor.

The Premixed Combustor: A schematic of the premixed combustor is shown in Fig. 2. A 3.25" inner diameter quartz tube sits on a stainless steel flange, which defines the dump plane of the combustor. This enables optical access to the main re-circulation zone where combustion takes place facilitating photodiode, radical imaging, and PLIF measurements.

Combustion air is fed through an eight-blade 45° -swirl vane with inner and outer diameters of 19 and 36 mm respectively. Swirl not only provides stabilization at the dump plane but also facilitates the entrainment and mixing of the of fuel jets with the cross flow in the pre-mixing zone. Total height H of the combustor is 912 mm. The dump plane is located at x/H=0.6. The area expansion ratio at the dump plane is approximately 1:4. At the end of the combustor there is a conical section, which contracts the exhaust flow down to an exit diameter of 12.7 mm.

Fuel gases (CH₄ and H₂) are all individually supplied from compressed gas tanks and mixed inside a manifold before the combustor inlet. Their flow rates are monitored by separate mass flow meters. Flow rates of fuel gases are adjusted separately to achieve the desired blend. Air

necessary for combustion is supplied from an air compressor and its flow rate is adjusted with a needle valve.

<u>Measurements Exhibiting Representative</u> <u>Unsteady Behavior</u>

Nonpremixed Combustion: Figure 3a shows the pressure at two locations (in the combustor and 10 cm upstream of the dump plane). the CH chemiluminescence and NOx signals, for flow conditions (equivalence ratio of 0.89) leading to combustion instability. The spectra in figure 3b show a dominant frequency f_{a} of 230 Hz superimposed on a low frequency of 13 Hz. The low frequency modulation is due to the bulk mode of the air-delivery chamber that was described in detail in [22]. NOx signal and its spectrum also show a distinct peak at the bulk-mode oscillation frequency. NOx rms has a 10 ppm peak-topeak variation in figure 3a. The bulk-mode instability leads to flame movement at a low frequency, which causes a variation in the intensity of heat release and flame shape [22]. These two parameters are directly related to the formation of NOx in the flame region. This figure underscores the importance of low frequency dynamics in the pressure, heat release and NOx emissions.

Figure 4 shows transient pressure, CH light and NOx signals as the inner air flow rate (see Fig. 1) is varied by a modest amount: 0.46, 0.57 and 0.73 m^3/min (equivalence ratios of 0.92, 0.86, and 0.78 respectively). Strong combustion dynamics can be seen at an equivalence ratio of 0.86 (dominant frequency at 230 Hz) with an associated increase in pressure and CH-oscillations. In this range, the NOx values decrease significantly presumably because of large temperature (~CH) rms due to which the temperature field fluctuates between high and low values, and the residence time associated with the high temperature region is lower compared to the stable combustion regime. The high NOx at the lower



Figure 2: Schematic of the Combustor

flow conditions (equivalence ratio=0.92) is partly a consequence of the flame being long, extending out of the combustor, leading to long residence times in the flame regions. At the other two equivalence ratios, the flame is relatively compact. It is clear that the oscillations in the pressure field (and therefore the velocity field) play a key role in the heat release and NOx emissions, and this unsteadiness must be modeled correctly to predict the correct flame and emission behavior.



Figure 3: (a) Pressure signals in the combustor and delivery chamber, CH light emissions and NOx and (b) & (c) their spectra. Equivalence ratio: 0.89.



Figure 4: Transient combustion behavior with the variation of inner flow rate 0.45, 0.57 and 0.73 m^3/min

The key to controlling the oscillations is to alter the heat release dynamics in such a way that the driving coupling between the heat release and the combustor acoustics is removed. To enable this control over the heat release dynamics, a small fluidic actuator (3mm diameter nozzle), centrally located (Fig. 1), is used. It has been shown by Uhm and Acharya [21, 22] that if the control air-jet momentum is sufficiently large, even a small amount of control air (less than1% of the combustion air) can be used to effectively control the dynamics either in open-loop mode or in closed-loop mode. To illustrate this, the control air-jet is actuated at a low bandwidth of 1 Hz, in an open-loop manner, and the results are shown in Fig. 5. The spool position at the bottom of the curve indicates if the valve is open or closed. With the valve open (control air-jet on), pressure instability is suppressed, but the NOx levels are enhanced due to lower rms levels of the pressure, velocity and temperature fluctuations. With the valve closed, the instability reappears, and NOx levels decrease. Clearly, there is an optimal frequency as shown in Fig. 5(b)



Figure 5: (a) Effect of 1 Hz air jet modulation. (b) Effect of forcing frequency

Control with the air-jet actuator can also be achieved in the feedback (phase delay) mode where the pressure-signal, phase delayed, is used as the input signal to the control actuator. Results of this are shown in Fig. 6, and shows the effectiveness of this approach. An advantage of this approach is that the NOx penalty appears to be smaller compared to open-loop control. The drawback of this approach is the need for high bandwidth actuation.



Figure 6: Effect of phase delay control on combustion characteristics and NOx emission.

Premixed Combustion: Figure 7 shows the instability frequency in the premixed combustor, while Fig. 8 shows the pressure spectra. As in the non-premixed system, both the bulk mode and the longitudinal modes are excited. With increasing hydrogen percentage, flashback occurs, and the bulk mode (~30 Hz) is the predominant mode



Figure 7: Instability frequency as a function of equivalence ratio and % hydrogen



Figure 8 Pressure and Heat Release Spectra (Q_{air} = 340 lpm, Φ =0.7, 60% CH₄, 40% H₂ by volume)

Figure 9 shows the time histories of pressure, flashback and nitric oxide emission signals. All three of them exhibit oscillatory behavior. Nitric oxide emissions, a key performance metrics for the gas turbine combustor design, is directly affected by the unsteadiness of the combustion process. Any control strategy which is designed to alter the nature of unsteadiness (i.e. the suppression of thermo-acoustic instability), will therefore have a definite impact on pollutant emissions levels as well. Flashback signal which is measured through a fiber optic cable mounted upstream of the dump plane location also indicates flashback activity with strong temporal coherence with the pressure oscillations. Pressure signal attains its peak at 90-degree phase instant (see Fig. 9) and shortly afterwards, at around 135-degrees, the flashback photodiode records a peak value which signifies that the flame tip has reached the axial location of the fiber optic cable.



Figure 9 Time Histories of Pressure, Flashback and NO Signals ((Q_{air} = 374 lpm, Φ =0.6, 60% CH₄, 40% H₂ by volume)

Reduced Order Modeling

As presented in the measurements above, the combustion process is inherently unsteady in its very nature and key performance metrics exhibit unsteady characteristics. Large eddy simulation (LES) is the ultimate approach for addressing such unsteadiness, but is quite expensive computationally as a wide ranges of timescales need to be resolved. Therefore, reduced order modeling is useful in providing some key understanding. These simple models also have the advantage of being easily coupled with a control algorithm for real time control.

The unsteadiness in a combustion chamber is inherently dominated by its acoustics. An oscillator equation (see Eq. 1) is derived from the basic conservation principles that govern the motion of longitudinal and/or bulk acoustic modes. Here pressure is the product of a time dependent amplitude function and a spatially dependent mode shape $P = \overline{P}\eta(t)\Psi(x)$. Similarly, the acoustic velocity is also a function of those two quantities, i. e., $u = \dot{\eta}(t)\Psi'(x)k^{-2}$. The oscillator equation can then be expressed as:

$$\ddot{\eta} + \omega^2 \eta = \frac{(\gamma - 1)}{\overline{P}} E^{-1} \psi(x_f) \frac{dq'}{dt}$$
¹

The energy E of the mode shape ψ that appears in Eq. 1 is given by Eq. 2 as follows.

$$E = \int_{0}^{L} \psi^{2}(x) dx$$
²

For the sake of simplicity, and without loss of generality, only one longitudinal mode is assumed to be present inside the combustor. This corresponds to the dominant acoustic mode. In solving the mode shape closed and open boundary conditions are used at the inlet and outlet respectively. Also due to combustion across the flame front, a sudden jump occurs in temperature and thus in speed of sound. Therefore the jump conditions across the flame location are also taken into account in the calculation of the mode shape.

$$\psi(x) = \begin{cases} \cos\left(\frac{\alpha x}{c_u}\right) & x \le x_f \\ \frac{\cos(\alpha)}{\sin(\beta)} \sin\left(\frac{\varpi(L-x)}{c_d}\right) & x > x_f \end{cases}$$
3

In the above equation the constants α and β are defined as follows (Eqs.4-5).

$$\alpha = \varpi x_f / c_u \qquad 4$$

$$\beta = \overline{\omega} \left(L - x_f \right) / c_d \tag{5}$$

Lowest possible frequency of oscillation ω is the smallest root of the following equation. This corresponds to the dominant acoustic mode of the combustor. Other roots correspond to modes with higher frequencies. As only one mode is assumed to be present these need not be solved for.

$$\tan \alpha \tan \beta = (\rho_u c_u) / (\rho_d c_d)$$

In Eq. 1 the rate of unsteady heat release appears on the right hand side. Physically this term is the driving factor of the oscillations. In order to close the model one needs to relate acoustic fluctuations to the heat release fluctuations. Heat release fluctuates because of oscillations in the reactant mass flow due to velocity fluctuations. For small disturbances, the transfer function between heat release and mass flow perturbations can be expressed as a first order filter [24], however experimental evidence suggests that after certain amplitude heat release begins to saturate. The term in parenthesis on the right hand side of Eq. 7 is introduced to mimic that non-linear effect.

$$\frac{\dot{q}'(s)}{\dot{m}'(s)} = \frac{\beta}{s+\alpha} \left(1 - \left| \frac{\dot{m}'(s)}{\bar{m}} \right| \right)$$
⁷

Taking a further step, it is also possible to couple the combustor acoustics with flame hydrodynamics to gain further insight. For this purpose a level set based front tracking method is used to resolve flame front dynamics. Premixed flame stabilizes on the fuel injector tip, which acts like a center-body. Assuming an axisymmetric flow field and further assuming that combustion occurs on a surface whose axial position is given by a single-valued function z = f(r, t), the flame surface can be defined by a level-set of G-equation. After some manipulation, the level-set equation takes the following form.

$$-\frac{\partial f}{\partial t} + u = S_L \left(1 + \left(\frac{\partial f}{\partial r}\right)^2 \right)^{1/2}$$
8

Self-excited oscillations in a combustor can be so intense that flame can leave its attachment point and propagate upstream (flashback). For oncoming flow velocities less than flame speed flame cannot remain attached at the tip and starts to propagate upstream. Therefore the boundary condition of Eq. 8 at r = a should take this flashback phenomenon into consideration. In addition to de-attachment flame remains perpendicular to the center body to assure zero normal velocity on the wall of the flame-holder on both sides of the flame. Eq. 9, as per [25], prescribes the appropriate boundary conditions for the flame-front equation.

$$\frac{\partial f}{\partial r}\Big|_{r=a} = \begin{cases} \frac{\left(u^2 - S_L^2\right)^{1/2}}{S_L} & \text{if } u(t) \ge S_L \land f(a,t) = 0\\ 0 & \text{if } u \le S_L \lor f(a,t) < 0 \end{cases}$$

In Figure 10 we observe the development of a stable limit cycle oscillation starting from an arbitrary initial condition. Note that linear systems are either stable or unstable, that is, a disturbance either dies out (stable) or is amplified and the system response (pressure in this particular case) shoots to infinity (unstable). A limit cycle develops only when the system exhibits non-linear characteristics. Here the non-linearity between the velocity fluctuations and mass flow rate fluctuations, which is manifested in Eq. 7, is responsible for the existence of such a limit cycle behavior. Amplitude of these limit cycle oscillations obtained from simulations is dependent on the flow condition as in the actual case with experiments. Frequency of these

fluctuations at about 140 Hz. again correspond to dominant acoustic modes which is solved from Eq. 6 and is in close agreement with experiments.



Figure 10 Development of Limit Cycle Pressure Oscillations from an Arbitrary Initial Condition $(\phi=1.0, Q=540 \text{ lt/min at } x/L=0.85)$

As mentioned earlier flame hydrodynamics are also coupled to the thermo-acoustic model. This enables one to locate the flame front with respect to the limit cycle fluctuations and also to simulate flashback. 11 demonstrates the phase locked (with respect to pressure) position of the flame initiation front obtained from numerical simulations. Position of the flame front is shown at 90-degree phase intervals. For this case the ratio of laminar flame speed and mean axial flow u/S_L velocity is 0.6. In a typical pre-mixed gas turbine combustor flame speed Mach number is typically one order of magnitude less than the Mach number of the oncoming fluid flow, similar to the case investigated here. Position of the flame front is determined by solving Eq. 8 by a numerical strategy. Due to high frequency fluctuations, the flame front does not experience excessive curvature. It is seen from the figure that with flashback, the flame is inside the premixing section, moving up and down as in the case of experiments. Only around the 270-degree phase instance, the flame attaches to the tip of the center-body (injector) briefly and de-attaches again. During this limit cycle oscillation flame, reaches its maximum propagation distance around the 90-degree phase instance at which the heat release is at its peak point.



Figure 11 Flame Front Movement with Respect to Thermo-acoustic Instability Cycle

Figure 12 shows the effect of the airflow rate and equivalence ratio on the RMS amplitude of the limit cycle pressure oscillations. Note that the trends in experiments and simulations agree qualitatively. As the airflow rate is increased the pressure fluctuation amplitude increases substantially. This is due to the fact that there is more unsteady fuel mass to be burnt at the flame location which, in turn, causes higher amplitude fluctuations. RMS pressure fluctuation amplitude also increases slightly with increasing equivalence ratio as shown in 12 a & b. This increase is due to the change of the filter gain β , which appears in the heat release equation (see Eq. 7).

It can therefore be concluded that the developed mathematical framework of a reduced order model can be used to predict combustor behavior such as limit cycle amplitudes, dominant frequencies etc., and also helps to gain some useful insight into flame holding and flashback. Furthermore, this reduced order model can be coupled with a model-based control algorithm to manipulate performance metrics in a desired fashion (i.e. suppress thermo-acoustic instability, prevent flashback, reduce emissions etc.).



a. Numerical Simulation



b. Experimental

Figure 12 RMS Pressure Amplitudes As a Function of Flow Rate and Equivalence Ratio

Concluding Remarks

In this paper, we have shown the importance of unsteady pressure, heat release and emissions that characterize combustion dynamics. It is argued that to properly model such dynamics, LES is an appropriate tool, but that a simpler reduced order model can also be used to capture the large scale dynamical effects. For premixed natural gas flames, a simple reduced order model is introduced and shown to qualitatively produce the observed trends. It is shown through measurements that the combustion dynamics can be effectively controlled with a small high-momentum fluidic jet. Low bandwidth open loop control is shown to be effective in reducing the dynamics, but is associated with a NOx penalty.

Acknowledgements

This work was supported by funding from the Propulsion program of the Office Of naval Research, and through the Clean Power and Energy Research Consortium (CPERC) that receives funding from the state of Louisiana. This support is gratefully acknowledged.

References

- [1] J.A. Austin, J.R. Tilson, I.R.I. McKenzie, Proc. RTO AVT Symposium on Active Control Technology for Enhanced Performance Operation Capabilities of Military Aircraft, Land Vehicles and Sea Vehicles, Braunschweig 2000.
- [2] G.A. Richards, M.C. Janus, ASME: J. Engineering for Gas Turbines and Power 120 (1998) 294-302.
- [3] J.O. Keller, L. Vaneveld, D. Korschelt, G.L. Hubbard, A.F. Ghoniem, J.W. Daily, A.K. Oppenheim, AIAA J. 20 (1981) 254-262.
- [4] F.E.C. Culick, AGARD Conference on Combustion Instabilities in Liquid-Fuelled Propulsion System, Proc. No. 450 (1), Neuilly Sur Seine, 1988.
- [5] K.C. Schadow, E. Gutmark, Prog. Energy Combust. Sci. 18 (1992) 117-132.
- [6] S.M. Candel, Proc. Combust. Inst. 24 (1992) 1277-1296.
- [7] K.R. McManus, T. Poinsot, S.M. Candel, Prog.Energy Combust. Sci. 19 (1993) 1-29.
- [8] W. Lang, T. Poinsot, S. Candel, S., Combust. Flame 70 (1987) 281-289.
- [9] S. Hoffmann, G. Weber, H. Judith, J. Hermann, A. Orthmann, A., Proc. RTO AVT symposium on Gas Turbine Engine Combustion, Emissions and Alternative Fuels, Paper No. 40 (1998).
- [10] D.U. Dampos-Delgado, K. Zhou, D. Allgood, S. Acharya, Combust. Sci. Technol. 175 (1) (2003) 1-27.
- [11] J.H Uhm, S. Acharya, AIAA-2003-0339, Reno, Nevada, USA, Jan. 6-9, (2003).
- [12] S. Sivasegaram, J.H. Whitelaw, ASME: Active control of noise and vibration DSC-38 (1992) 69-74.
- [13] S. Sivasegaram, R-F. Tsai, R-F., J.H. Whitelaw, Combust. Sci. Technol. 105 (1995) 67-83.
- [14] K.H. Yu, Proc. RTO AVT Course on Active Control of Engine Dynamics, Brussels, Belgium, May (2001).
- [15] M. Fleifil, A.M. Annaswamy, J.P. Hathout, A.F. Ghoniem, A.F., Combust. Sci. Technol. 133 (1998) 227-260.
- [16] K.J. Wilson, E. Gutmark, K.C. Schadow, AIAA paper 91-0368, Jan. 7-10, Reno, Nevada (1991).
- [17] R. Bhidayasiri, S. Sivasegaram, J.H. Whitelaw, Thermofluids Section report TF/98/08, Imperial College of Science Technology and Medicine, London (1998).
- [18] D. Shcherbik, E. Lubarsky, Y. Neumeier, B.T. Zinn, K. McManus, T.F. Fric, and S. Srinivasan, GT2003-38103 paper, Proceedings of ASME Turbo Expo 2003 Power for

Land, Sea, and Air, June 16-19, Atlanta, Georgia, USA.

- [19] G.A. Richards, M.J. Yip, E. Robey, L. Cowell, and D. Rawlins, ASME: J. Engineering for Gas Turbines and Power 119(2) (1997) 340-343.
- [20] G.A. Richards, M. Janus, and E.H. Robey, J. Propulsion and Power 15(2) (1999) 232-240.
- [21] K.R. McManus, U. Vandsburger and C.T. Bowman, Combust. Flame 82 (1990) 75-92.
- [22] Uhm, J., and Acharya, S., Control of Combustion Instability Using a High-Momentum Air-Jet, *Combustion and Flame*, Vol. 139, 2004, pp. 106-125
- [23] Uhm, J., Acharya, S., Open-Loop Control of Combustion Instability with a High-Momentum Air Jet, *Combustion and Flame*, Vol. 142, No.4, 2005, pp. 348-363.
- [24] Park, S., Annaswamy, A., Ghoneim, A., 2002, "Heat Release Dynamics Modeling of Kinetically Controlled Burning", Combustion and Flame, Vol. 128, pp. 217-231
- [25] Dowling, A. P., 1999, "A Kinematic Model of a Ducted Flame", Journal of Fluid Mechanics, Vol. 394, pp. 51-72