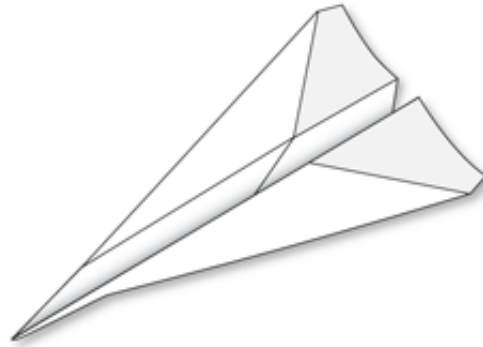


MAT-202E NUMERICAL METHODS HOMEWORK 4

Due date for source code submission via e-mail: 28.12.2008 @ 12 pm
Due date for the submission of the report: 29.12.2008

In the figure you see a classic dart shaped paper airplane. Your task is to analyze the flight trajectories of this craft under a number of initial conditions. You are to make some simplifying assumptions, such as treating the aircraft as a point mass.

This sophisticated flight vehicle has a wingspan of 12 cm. and a corresponding length of 28 cm, resulting in an aspect ratio AR and a wing reference area S of 0.86 and 0.017 m², respectively.



Under the point mass assumption longitudinal motions of the paper airplane are governed by the following set of ordinary differential equations.

$$\begin{aligned}\dot{V} &= -C_D \left(\frac{1}{2} \rho V^2 \right) S / m - g \sin \gamma \\ \dot{\gamma} &= \left[C_L \left(\frac{1}{2} \rho V^2 \right) S / m - g \cos \gamma \right] / V \\ \dot{h} &= V \sin \gamma \\ \dot{r} &= V \cos \gamma\end{aligned}$$

where, V, γ , h, and r are the airspeed, flight path angle, height and range respectively. Environmental variables are air density ρ (1.225 kg/m³ at sea level), and gravitational acceleration $g=9.807$ m/s². Parameters of the airplane are contained in the following variables; C_D the drag coefficient, C_L the lift coefficient, reference area S and finally mass m. The lift coefficient can be modelled as a linear function of the angle of attack α ,

$$C_L = C_{L_\alpha} \alpha$$

Derivative of the lift slope can be estimated as,

$$C_{L_\alpha} = \frac{\pi AR}{1 + \sqrt{1 + (AR/2)^2}} \quad (\text{per radians})$$

Accordingly the drag coefficient is modeled as,

$$C_D = C_{D_0} + \epsilon C_L^2$$

with $C_{D_0} = 0.02$ and the induced drag factor, where $\epsilon = 1/(\pi AR)$; ϵ is referred to as the so-called Oswald efficiency factor and is estimated to be 0.9 in this case.

In this problem attitude dynamics are neglected, such that the angle of attack α is taken as a control variable. Its trimmed value is a function of the center-of-mass location and the amount of deflection of the wing's trailing edge.

If one defines a state vector $x = [V \quad \gamma \quad h \quad r]^T$, and a control variable $u = \alpha$, which equals to the angle of attack. Then the model can be expressed in the general form of, $\dot{x} = f(x, u)$

where,

$$f(x, u) = \begin{bmatrix} -C_D \left(\frac{1}{2} \rho x_1^2 \right) S / m - g \sin x_2 \\ \left[C_L \left(\frac{1}{2} \rho x_1^2 \right) S / m - g \cos x_2 \right] / x_1 \\ x_1 \sin x_2 \\ x_1 \cos x_2 \end{bmatrix}$$

Now $f(x, u)$ is a non-linear function of its arguments for a number of reasons. First of all $C_D(u)$ is a quadratic function of u . Moreover, several non-linear operations involve elements of the state, including the square of x_1 (which is the airspeed V), division by x_1 , sines and cosines of x_2 , together with the product of x_1 with functions depending upon x_2 .

Three longitudinal paths are typical of a paper airplane; ***constant angle descent, vertical oscillation, and loop***. Numerical integration of these “system of ordinary differential equations”, should produce these paths depending on the initial conditions.

Before proceeding with numerical integration let us determine the angle of attack that produces the highest lift/drag ratio. If one plots C_L/C_D versus the angle of attack, he would see that the maximum occurs at an angle of attack of $\alpha=9.3^\circ$. This yields a best lift to drag-ratio of 5.2 for this configuration. Hence take the angle of attack fixed at 9.3° for all your simulations.

This angle of attack also produces the shallowest equilibrium glide scope with $\gamma=-11^\circ$ and a constant speed of $V=3.7$ m/s.

- i. Write a function f that takes the state vector x , and the control variable u as arguments and returns the rate of change of the state vector dx/dt as the output.
- ii. Write another function that integrates x in time. You are free to choose the numerical method. This function within itself might contain a number of subroutines for your convenience. The initial time of simulation should be $t_i=0$ seconds and the final time should be $t_f=6$ seconds.
- iii. Calculate flight trajectories with the following initial conditions (Take initial throwing height $H=2.0$ m). Don't forget to convert γ into radians!
 - a. $X_0=[3.70 \quad -11^\circ \quad H \quad 0]$ (Hint: These initial conditions should yield a constant angle descent path as discussed above, since these initial conditions correspond to equilibrium values. Time derivatives of the airspeed V , and the glide angle γ are both zero in this case.)
 - b. $X_0=[3.70 \quad 0 \quad H \quad 0]$ (Hint: You will be getting an oscillating glide due to zero initial path angle γ)
 - c. $X_0=[5.55 \quad 0 \quad H \quad 0]$ (Try the effect of further increasing initial velocity. What happened to the amplitude of vertical oscillations? What about their period?)
 - d. $X_0=[7.40 \quad 0 \quad H \quad 0]$ (Hint: Your paper plane should make a loop with these initial

- conditions)
- iv. Discuss and thoroughly analyze your results. You need to plot the following before you discuss; height vs range, velocity (airspeed V) versus time, flight path angle γ vs time, altitude h vs time and finally range r versus time.
 - v. Justify the choice of your time integration scheme (e.g. stability and accuracy of the method and the step size of your choosing).

Homework Rules

You should present your own work during the course. Any unethical behaviour shall be penalized seriously.

You will post your homework in a zip file named as 'HW#-ID#.zip' to the address tuncero@itu.edu.tr

The zip file should include the necessary source codes and executables (m-files, FORTRAN codes, etc.)

Homework Format

Homework should consist of the following parts. Problem Definition and Discussion and Conclusion parts should be typed up using a word processor software. Neatness counts.

1. Cover Page (The cover page should state the name and ID number of the student clearly, cover page also needs to include the course name and homework number)

In addition, cover page must also include the following statement at the bottom;

“I have neither given nor received any unauthorized help for this assignment”

Your Signature

2. Problem Definition
3. Results (In Numerical or Graphical Format)
4. Discussion
5. Appendix (Source Code, etc.)

Source code should include the necessary comments and results should be presented with at least 5 digits.