WORKSHEET 4

Course: Mat101E
Content: Applications of Derivatives

1. Find the absolute extreme values of the following functions on the given interval. Then graph the function. Identify points on the graph where absolute extreme occur and include their coordinates.

   (a) $f(x) = \sec x, \quad -\pi/2 < x < \pi/2$

   (b) $g(x) = x^2 - 2|x| + 2 \quad -1/2 \leq x \leq 3/2.$

2. Find the absolute and local extrema, and points of inflection, if any, of the following functions on the given intervals.

   (a) $f(x) = \sqrt{(1-x^2)(1+2x^2)}, \quad [-1, 1]$

   (b) $f(x) = 2\sin x + \sin 2x, \quad [0, \frac{3\pi}{2}]$

   (c) $f(x) = 2\cos^3 x + 3\cos x, \quad [0, \pi]$

3. Describe the concavity of the graph of $f(x) = 2\cos^2 x - x^2$ on the interval $[0, \pi]$.

4. Find the extreme values of the function where they occur.

   (a) $y = x^3 - 3x^2 + 3x - 2$

   (b) $y = \dfrac{x+1}{x^2+2x+2}$

   (c) $y = x^{2/3}(x^2 - 4)$

   (d) $f(x) = \begin{cases} 3 - x &, \quad x < 0 \\ 3 + 2x - x^2, & x \geq 0 \end{cases}$

5. Find the critical points and classify the extreme values.
   (a) $f(x) = (x+7)(11-3x)^{1/3}$ (b) $f(x) = 2\cos^3 x + 3\cos x, \quad [0, \pi]$

6. Determine whether the function $f(x) = \dfrac{\sqrt{1-x^2}}{x^2+3}, \quad [-1, 1]$ satisfies the hypotheses of the Rolle's Theorem on the given interval? If so, find the admissible values of c.

7. Do the following functions satisfy the conditions of the Mean Value theorem? If so, find the admissible values of c.

   (a) $f(x) = \sqrt{x - x^2}, \quad [0, 1].$

   (b) $f(x) = \begin{cases} \dfrac{\sin x}{x}, & -\pi \leq x < 0 \\ 0 &, \quad x = 0 \end{cases}$

   (c) $y = \tan \sqrt[3]{x}, \quad [-\frac{\pi^3}{4^3}, \frac{\pi^3}{4^3}]$

8. For what values of a, m, and b does the function

$$f(x) = \begin{cases} 3 & , \quad x = 0 \\ -x^2 + 3x + a, & 0 < x < 1 \\ mx + b & , \quad 1 \leq x \leq 2 \end{cases}$$

satisfy the hypotheses of the Mean Value Theorem on the interval $[0, 2]$?

9. Given that $|f'(x)| < 1$ for all real number x, show that

$$|f(x_1) - f(x_2)| < |x_1 - x_2|$$

for all real numbers x_1 and x_2.

10. Using the Mean Value Theorem show that

$$|\sin b - \sin a| < |b - a|$$

for any numbers numbers a and b.

11. Let $f(x) = (x - 2)^{2/3}$ $[0, 4]$ .Does the function $f(x)$ satisfy the hypotheses of the Rolle's Theorem on the given interval?

12. Let $f(x)$ be a function such that $f(0) = 0$, $f'(x) = \dfrac{x^2}{1 + x^2}$ for all x.

    Show that $0 < f(x) < x$ for $x > 0$.

13. Suppose that $f(x)$ is differentiable on $[0,\ 1]$ and that its derivative is never zero. Show that $f(0) \neq f(1)$.

14. Show that $f(x) = x^4 + 3x + 1$ has exactly one zero in the interval $[-2, -1]$.

15. Show that $f(x) = x^3 + 2x + 2$ has exactly one zero in the interval $[-2, 0]$.

16. Let $f(x) = x\sqrt{8 - x^2} + 1$.
    (a) Find the function's natural domain.
    (b) Find the intervals on which the function is increasing and decreasing.
    (c) Then identify the function's local extreme values, if any, saying where they are taken on.
    (d) Which, if any of the extreme values are absolute.
    (e) Graph the curve $y = f(x)$.

17. Graph the following functions in details.

    (a) $y = x - 3x^{2/3}$ (b) $y = \dfrac{x^2 - x + 1}{x - 1}$ (c) $y = \dfrac{(x - 1)^2}{x + 2}$ (d) $y = \dfrac{1}{4 - x^2}$

18. Find two numbers such that their sum is 10 and the product of the square of the one and cube of the other is as large as possible.

19. Find the estimated value of $\tan 61^\circ$.

20. Suppose that $f(x)$ is differentiable on $[0, 1]$ and that its derivative is never zero. Show that $f(0) \neq f(1)$

21. Let $f(x)$ be differentiable at every value of x and suppose that $f(1) = 1$, that $f' < 0$ on $(-\infty, 1)$ and that $f' > 0$ on $(1, \infty)$.

    (a) Show that $f(x) \geq 1$ for all x .

    (b) Must $f'(1) = 0$? Give reasons for your answer.