PET467E
A Note on Rock Compressibility

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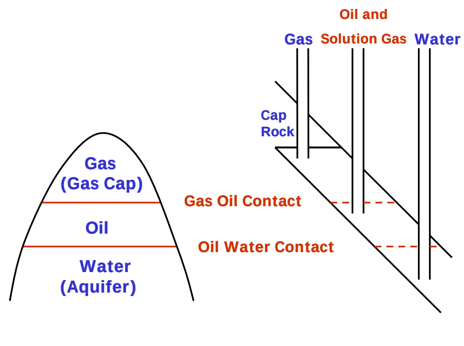
Rock Compressibility

- A reservoir consists of an impervious cover or cap rock overlying a porous and permeable rock.
- The density differences between the oil, gas and water phases can result in boundary regions between them known as fluid contacts (oil-water and gas-oil contacts).



Cross-section of a combination drive type anticline reservoir

Reservoir Pressures

- In a liquid column representing vertical pore fluid continuity the pressure at any point is approximated by the relationship

$$p_{f,D}=D.G_l+C$$

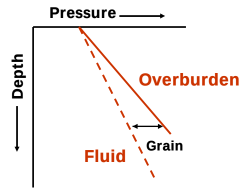
where D is the depth and G_l is the pressure exerted by unit height of liquid (pressure gradient). For fresh water, G_l is 0.433 psi/ft (9.79 kpa/m, 10 m/atm). Here, C is a constant.

- Note: psi = pound per square inch
  psia = absolute pressure, psig = gauge pressure

Rock Compressibility

- As sediment builds up, overburden pressure increases as rock is compacted.
- For a hydrocarbon reservoir, the reservoir rock grains are subjected to external pressure (stress) due to overburden and to internal pressure represented by the fluid pressure.

Reservoir Pressures



- There is a balance in a reservoir system between the pressure gradients representing rock overburden pressure (p_{ob}), pore fluids pressure (p_f) and sediment grain pressure (p_g):

  $p_{ob} = p_f + p_g$

  - p_f is also called reservoir pressure and a typical value of overden pressure (p_{ob}) is 1 psi/ft

Rock Compressibility

- As reservoir fluids are produced, the rock grains may expand causing a reduction in porosity which causes fluid to be expelled. (expansion of rock grains)
- At the same time, the overburden may cause a decrease in the bulk volume causing fluid to be expelled (rock compaction). In the extreme case, we get subsidence.

Rock Compressibility

- Of principal interest to the reservoir engineer is the change in the pore volume of the rock or also reffered to as effective rock compressibility or formation compressibility.

Rock Compressibility

•Effective rock compressibility is

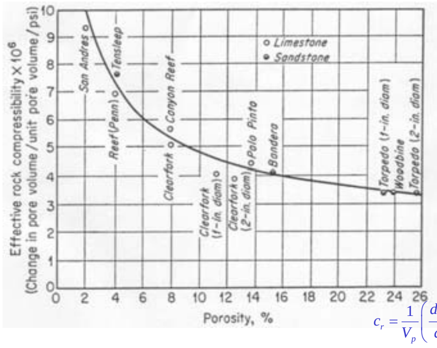
$$c_r = \frac{1}{V_p}\frac{dV_p}{dp} = \frac{1}{\phi V_b}\frac{d(\phi V_b)}{dp} =$$

$$\frac{1}{\phi V_b}\left(\phi\frac{d(V_b)}{dp} + V_b\frac{d(\phi)}{dp}\right) = \frac{1}{V_b}\frac{d(V_b)}{dp} + \frac{1}{\phi}\frac{d(\phi)}{dp}$$

where, p is the pore pressure, and assumes that overburden pressure (p^*) is held constant.

•Typical values are 1 to 5 times 10^{-6}

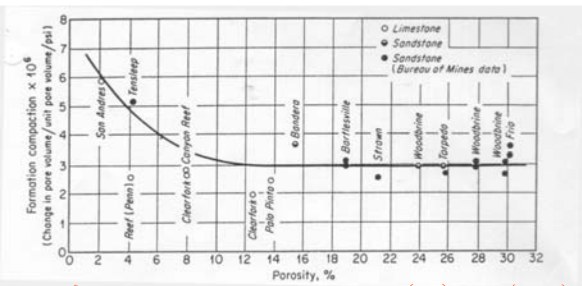
Effective Reservoir Rock Compressibilities (Hall)



p^* external pressure
p internal pressure

$$c_r = \frac{1}{V_p}\left(\frac{dV_p}{dp}\right)_{p^*} = \frac{1}{\phi V_b}\left(\frac{d(\phi V_b)}{dp}\right)_{p^*}$$

Formation Compaction Component of Total Rock Compressibility (Hall)



p^* external pressure
p internal pressure

$$c_c = -\frac{1}{V_p}\left(\frac{dV_p}{dp^*}\right)_p = -\frac{1}{\phi V_b}\left(\frac{d(\phi V_b)}{dp^*}\right)_p$$

Rock Compressibility

- Three kinds of compressibility can be distinguished in rocks: (a) rock matrix compressibility (c_s), (b) rock bulk volume compressibility (c_b), (c) pore compressibility (c_r)

$$c_b = (1-\phi)c_s + \phi c_r$$

Rock Compressibility

- Rock matrix compressibility (c_s) is the fractional change in volume of the solid rock material with a unit change in internal pressure
- Rock bulk compressibility (c_b) is the fractional change of bulk volume of the rock with a unit change in internal pressure.
- Pore compressibility (c_r) is the fractional change of pore volume of rock with a unit change in internal pressure.

A Derivation For Rock/Pore/Bulk Compressibilities

- For a fixed external pressure, and varying internal fluid pressure (as is done in the experiments conducted by Hall, we may derive the following expression:

$$c_b = \phi c_r + (1-\phi) c_s$$

$$c_r = \frac{1}{\phi}\left(\frac{1}{V_b}\frac{\partial V_b}{\partial p}\right) - \frac{(1-\phi)}{\phi}\left(\frac{1}{V_s}\frac{\partial V_s}{\partial p}\right) = \frac{1}{V_p}\frac{\partial V_p}{\partial p}$$

$$c_r = \frac{c_b}{\phi} - \frac{(1-\phi)}{\phi} c_s$$

A Derivation For Rock/Pore/Bulk Compressibilities

- According to Geerstma (1957), we should treat variations of bulk volume and pore volue (or porosity) with respect to both external pressure, p^* and internal pressure, p.

$$dV_p(p^*, p) = \left(\frac{\partial V_p}{\partial p^*}\right)_p dp^* + \left(\frac{\partial V_p}{\partial p}\right)_{p^*} dp$$

$$dV_b(p^*, p) = \left(\frac{\partial V_b}{\partial p^*}\right)_p dp^* + \left(\frac{\partial V_b}{\partial p}\right)_{p^*} dp$$

A Derivation For Rock/Pore/Bulk Compressibilities

- Geerstma derives:

$$\frac{1}{V_b}\left(\frac{\partial V_b}{\partial p}\right)_{p^*} = \frac{1}{V_b}\left(\frac{\partial V_b}{\partial p^*}\right)_p - c_m$$

$$\frac{1}{V_p}\left(\frac{\partial V_p}{\partial p^*}\right)_p = \frac{1}{\phi}\left[\frac{1}{V_b}\left(\frac{\partial V_b}{\partial p^*}\right)_p - c_m\right]$$

$$\frac{1}{\phi}\left(\frac{\partial \phi}{\partial p^*}\right)_p = -\frac{1}{\phi}\left(\frac{\partial \phi}{\partial p}\right)_{p^*} = \frac{(1-\phi)}{V_b \phi}\left(\frac{\partial V_b}{\partial p^*}\right)_p - \frac{c_m}{\phi}$$

A Derivation For Rock/Pore/Bulk Compressibilities

- Geerstma definitions:

$$\hat{c}_b = \frac{1}{V_b}\left(\frac{\partial V_b}{\partial p^*}\right)_p \; ; \; \hat{c}_r = \frac{1}{V_p}\left(\frac{\partial V_p}{\partial p^*}\right)_p$$

$$c_m = \frac{1}{V_m}\frac{dV_m}{dp} = \frac{1}{V_m}\frac{dV_m}{dp^*} = c_s$$

$$\hat{c}_p = \frac{1}{\phi}\left(\frac{\partial \phi}{\partial p^*}\right)_p = -\frac{1}{\phi}\left(\frac{\partial \phi}{\partial p}\right)_{p^*} = \frac{(1-\phi)}{\phi}\hat{c}_b - \frac{c_m}{\phi}$$

$$\hat{c}_r = \frac{1}{\phi}\left[\hat{c}_b - c_m\right]$$

Notes

- We can predict change of porosity with pressure from:

$$\phi(p_2) = \phi(p_1)\left[1 + c_r\left(p_2 - p_1\right)\right]$$

or simply

$$\phi_2 = \phi_1\left[1 + c_r\left(p_2 - p_1\right)\right]$$

- As fluid pressure decreases to a value p_2 from an initial pressure p_1, porosity decreases slightly.

Note on Reservoir Temperatures

- Reservoir temperature may be expected to confirm to the regional or local geothermal gradient. In many basins, this is around 0.034 K/m (2 ^{o}F/100 ft).
- A figure of 4 ^{o}F/100 ft is a high gradient, whereas 0.5 ^{o}F/100 ft is low.

Note on Reservoir Temperatures

- A 5000-ft well in a region with an average ambient (surface) temperature of 65^{o}F may be expected to have a bottom-hole temperature of roughly

$$65 + 5000 \times (2/100) = 165^oF$$

- It is a reasonable assumption that reservoir condition processes tend to be isothermal (temperature is constant throughout the reservoir).

Correlations for c_r

- It is usually the one of the difficult parameters to obtain and effective rock compressibilities are rarely measured.
- There are several correlations for c_r depending on the rock type (see Horne's book "Modern Well Test Analysis).

$$c_r = \exp\left(4.026 - 23.07\phi + 44.28\phi^2\right)x10^{-6}$$ Consolidated limestones

$$c_r = \exp\left(5.118 - 36.26\phi + 63.98\phi^2\right)x10^{-6}$$ Consolidated sandstones