CHAPTER IX

STATISTICAL HYPOTHESES, HYPOTHESES TESTS (PARAMETERS)

EXERCISES IX

PROBLEM 1

It is required that the mean asphalt ratio in a construction material equals 5%. It is assumed that this ratio is normally distributed with a population standard deviation of 0,75%. In a small sample of three elements measured ratios are 4,2%, 4,7%, and 3,7%.

a) Can we assumed that the mean ratio equals 5%. ($\alpha = 0,1$)
b) Whether the ratio is significantly smaller than 5% or not?
c) Calculate the standard deviation of the measured values and solve the question (a) using t distribution.
d) Check if the measured standard deviation $\hat{S}_X = 0,005$ is significantly different from the population parameter $\sigma_X = 0,0075$. The variance has the X^2 distribution.

CHAPTER IX

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EXERCISES IX **SOLUTIONS** **09.04.2002**

SOLUTION 1

a)

The mean asphalt ratio is distributed normally with the standard deviation,

$\sigma_X / N^{1/2}$

$N = 3$, $\sigma_X = 0{,}0075$, and $\alpha = 0{,}1$

$H_0 : \mu_X = 5\%$ and $H_1 : \mu_X \neq 5\%$

From the table $Z_{0,05} = 1{,}65$

The acceptance region:

$b_1 = \mu_X - Z_{0,05} * \sigma_X / N^{1/2}$

$b_1 = 0{,}05 - 1{,}65 * 0{,}0075 / 3^{1/2}$

$b_1 = 0{,}0429$

$b_2 = \mu_X + Z_{0,05} * \sigma_X / N^{1/2}$

$b_2 = 0{,}05 + 1{,}65 * 0{,}0075 / 3^{1/2}$

$b_2 = 0{,}0571$

The mean of the three measurements is:

$x_m = (0{,}042 + 0{,}047 + 0{,}037) / 3$

$x_m = 0{,}042$

$x_m = 0{,}042$, which lies outside this region. Therefore the hypothesis $H_0 : \mu_X = 5\%$ is rejected at 10% significance level.

b)

From the table $Z_{0,1} = 1{,}28$

The lower boundary of acceptance region is

$\boldsymbol{b_1 = \mu_X - Z_{0,05} * \sigma_X / N^{1/2}}$

$b_1 = 0{,}05 – 1{,}28 * 0{,}0075 / 3^{1/2}$

$\boldsymbol{b_1 = 0{,}04445}$

Where $Z_{0,1} = 1{,}28$ is the standard normal variable with a probability of exceedance $\alpha = 0{,}1$. The measured mean $x_m = 0{,}042$ is smaller than 0,04445, and the hypothesis $H_0 : \mu_X =5\%$ is again rejected (but now the hypothesis that is accepted is $H_1 : \mu_X <5\%$).

c)

In the preceding tests it was assumed that the population standard deviation was known as $\sigma_X = 0{,}0075$

If this information was not given, we would have to work with the standard deviation of the measured values:

$\hat{S}_X = [(0{,}042 – 0{,}042)^2 + (0{,}047 – 0{,}042)^2 + (0{,}037 – 0{,}042)^2]^{0,5} / (3 – 1)^{0,5}$

$\boldsymbol{\hat{S}_X = 0{,}005}$

In this case the mean follows the t distribution. For *d.f.* = N-1 = 3 – 1 = 2 and $\alpha / 2 = 0{,}05$, the critical t value is read from the table as:

$t_{0,05} = 2{,}92$.

The boundaries of the region of acceptance:

$\boldsymbol{b_1 = \mu_X - t_{0,05} * \hat{S}_X / N^{1/2}}$

$b_1 = 0{,}05 – 2{,}92 * 0{,}005 / 3^{1/2}$

$\boldsymbol{b_1 = 0{,}0416}$

$\boldsymbol{b_2 = \mu_X + t_{0,05} * \hat{S}_X / N^{1/2}}$

$b_2 = 0{,}05 + 2{,}92 * 0{,}005 / 3^{1/2}$

$\boldsymbol{b_2 = 0{,}0584}$

Now the mean of the measurements $X_m = 0{,}042$ is inside the acceptance region and the hypothesis $H_0 : \mu_X =5\%$ is accepted. In small samples the acceptance region is wider when t distribution is used ($t_{0,05} = 2{,}92$ is much larger than $Z_{0,05} = 1{,}65$).

d)

$\hat{S}_X = 0{,}005$, $\sigma_X = 0{,}0075$.

For small samples the variance has the X^2 distribution with *d.f.* = N – 1 = 3 – 1 = 2 and for α = 0,1, it can be read the critical values of X^2 from the table as $X^2_{0,95} = 0{,}104$ and $X^2_{0,05} = 5{,}991$. value is read from the table as:

The boundaries of the region of acceptance:

$$b_1 = X^2_{0,05} * \sigma^2_X / (N - 1)$$

$$b_1 = 0{,}104 * 0{,}0075^2 / 2$$

$$b_1 = 2{,}9 * 10^{-6}$$

$$b_2 = X^2_{0,05} * \sigma^2_X / (N - 1)$$

$$b_2 = 5{,}991 * 0{,}0075^2 / 2$$

$$b_2 = 168{,}5 * 10^{-6}$$

The measured variance is $\hat{S}^2_X = 0{,}005^2 = 25 * 10^{-6}$, which is inside the region of acceptance. The hypothesis H_0: $\sigma^2_X = 0{,}0075^2$ is accepted.