CHAPTER VIII

SAMPLING DISTRIBUTIONS, ASYMPTOTIC AND EXACT SAMPLING DISTRIBUTIONS

EXERCISES VIII **02.04.2002**

PROBLEM 1

The BOD (biological oxygen demand) in a stream has the mean **90 mg/l** and standard deviation **20 mg/l**. What is the probability that the annual mean BOD to be computed from **12** monthly samples will exceed **100 mg/l**?

a) Use the **asymptotic sampling distribution**.
b) Use the **exact sampling distribution**.

PROBLEM 2

The mean and the standard deviation of the breaking load in experiments performed on **25** concrete columns are found as **10 000 kg** and **500 kg**, respectively. Find the confidence interval of the mean at **Pc = %90**.

a) Use the **asymptotic sampling distribution**.
b) Use the **exact sampling distribution**.

CHAPTER VIII

SAMPLING DISTRIBUTIONS, ASYMPTOTIC AND EXACT SAMPLING DISTRIBUTIONS

EXERCISES VIII **SOLUTIONS** **02.04.2002**

SOLUTION 1

a)

$\sigma_{Xm} = \sigma_X / N^{1/2}$

N = 12 and $\sigma_X = 20$

$\sigma_{Xm} = 20 / 12^{1/2}$

$\sigma_{Xm} = 5{,}77$ *mg/l*

$Z = (X - X_m) / \sigma_{Xm}$

Z = (100 – 90) / 5,77

Z = 1,732

From the table the following probability is taken;

$P(X > 100) = F_1(Z)$

$P(X > 100) = F_1(1{,}732)$

$P(X > 100) = 0{,}042$

Thus the probability that the annual mean BOD to be calculated from samples recorded during 12 months is greater than 100 mg/l is ***0,042***

b)

Since the sample is a small one, it is preferred to use the t distribution instead of the normal distribution.

$t = (X - Xm) / (\sigma_X / N^{1/2})$

$t = (100 - 90) / (20 / 12^{1/2})$

t = 1,732

For n = N – 1 = 12 – 1 = 11 from the table;

$P(X > 100) = 0,057$

As it is seen; this value is greater than ***0,042*** calculated with the assumption of asymptotic normal distribution.

SOLUTION 2

a)

$\sigma_{Xm} = \sigma_X / N^{1/2}$

$N = 25$ and $\sigma_X = 500$

$\sigma_{Xm} = 500 / 25^{1/2}$

$\sigma_{Xm} = 100$ *kg*

(1-Pc) / 2

(1-Pc) / 2 = (1 – 0,9) / 2

(1-Pc) / 2 = 0,05

Can be read from the normal distribution table as;

$Z_{0,05} = 1,645$

The confidence interval will be an interval around X_m = 10 000 with a width of 1,645 * σ_{Xm} on both sides.

$b_1 = X_m - Z_{0,05} * \sigma_{Xm}$

$b_1 = 10000 - 1,645 * 100$

$b_1 = 9835$ *kg*

$b_2 = X_m + Z_{0,05} * \sigma_{Xm}$

$b_2 = 10000 + 1,645 * 100$

$b_2 = 10165$ *kg*

b)

Since the sample is a small one, it is more suitable to use the exact t distribution instead of the normal distribution. (For this it is necessary to assume that the breaking load is normally distributed)

$\sigma_{Xm} = \sigma_X / (N - 1)^{1/2}$

$N = 25$ and $\sigma_X = 500$

$\sigma_{Xm} = 500 / (25 - 1)^{1/2}$

$\sigma_{Xm} = 102$ kg

n = N - 1

n = 25 – 1

n = 24

(1-Pc) / 2

(1-Pc) / 2 = (1 – 0,9) / 2

(1-Pc) / 2 = 0,05

Can be read from the table as;

$t_{0,05} = 1,711$

Thus the limits of the confidence interval of the mean at %90 confidence level:

$b_1 = X_m - t_{0,05} * \sigma_{Xm}$

$b_1 = 10000 - 1,711 * 102$

$b_1 = 9825$ kg

$b_2 = X_m + t_{0,05} * \sigma_{Xm}$

$b_2 = 10000 + 1,711 * 102$

$b_2 = 10175$ kg