CHAPTER IV

PARAMETERS, BERNOULLY TRIALS

EXERCISES IV

PROBLEM 1

In a construction “windows” and “doors” are two consecutive jobs each of which can be done at **3** different speeds. According to the speed chosen, the probabilities of both the cost and the time of completion is change. If the two jobs are not completed in **10** months, the contractor has to pay a fine of **500** Million monthly. The contractor has prepared the following table depending on his earlier experience.

JOB	SPEED	MONTHLY COST (MILLION)	THE PROBABILITY OF COMPLETION IN 4 MONHS	THE PROBABILITY OF COMPLETION IN 5 MONHS	THE PROBABILITY OF COMPLETION IN 6 MONHS
WINDOWS	A	400	0,2	0,5	0,3
	B	500	0,3	0,6	0,1
	C	600	0,6	0,4	0
DOOR	D	200	0,1	0,4	0,5
	E	300	0,3	0,4	0,3
	F	400	0,6	0,3	0,1

a) Calculate the expected (mean) value of the cost for different speeds.

b) Calculate the expected value of total cost. (First, calculate the probability of completion in **11** months, second, the probability of completion in **12** months, third, the expected value of construction cost to be paid, fourth, the expected value of fine to be paid, and finally the expected value of total cost.)

PROBLEM 2

The probability mass function $p(x_i)$ of the annual precipitation (X, cm) at a gauging station is given below:

x_i	0	15	20	25	30	35	40	45	50	55
$p(x_i)$	3/30	1/30	2/30	4/30	6,5/30	6/30	3,5/30	2/30	1/30	1/30

a) Calculate the mean of the annual precipitation,
b) Calculate the variance of the annual precipitation,
c) Calculate the standard deviation of the annual precipitation,
d) Calculate the coefficient of variation of the annual precipitation.

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EXERCISES IV **SOLUTIONS** **26.02.2002**

SOLUTION 1

a) If "windows" is performed with speed A monthly cost will be 400 Million and the work will be completed in 4 months with a probability of 0,2, in 5 months with a probability of 0,5, and in 4 months with a probability of 0,3 (since 3 simple events exist, the sum of their probabilities 0,2+0,5+0,3=1)

Now let us calculate the expected (mean) value of the cost for different speeds.

For the "windows" is performed with speed A, B, and C the expected (mean) value of cost is;

0,2*4*400+0,5*5*400+0,3*6*400 = 2040 (Million)
0,3*4*500+0,6*5*500+0,1*6*500 = 2400 (Million)
0,6*4*600+0,4*5*600+0,0*6*600 = 2640 (Million)

For the "windows" is performed with speed D, E, and F the expected (mean) value of cost is;

0,1*4*200+0,4*5*200+0,5*6*200 = 1080 (Million)
0,3*4*300+0,6*5*300+0,3*6*300 = 1500 (Million)
0,6*4*400+0,3*5*400+0,1*6*400 = 1800 (Million)

b) There are 18 alternatives with different speeds to the two jobs are not completed in 10 months.
p(11 months) = $p\{[(I=5) \cap (II=6)] \cup [(I=6) \cap (II=5)]\}$

$p[(I=5) \cap (II=6)] = p(I=5) * p(II=6) = 0,5 * 0,5 = 0,25$
$p[(I=6) \cap (II=5)] = p(I=6) * p(II=5) = 0,3 * 0,4 = 0,12$

p(11 months with speed AD) = $p\{[(I=5) \cap (II=6)] \cup [(I=6) \cap (II=5)]\} = 0,25 + 0,12 = 0,37$

Similarly we can calculate other probabilities:

p(11 months with speed AD) = $p\{[(I=5) \cap (II=6)] \cup [(I=6) \cap (II=5)]\} = 0,37$
p(11 months with speed AE) = $p\{[(I=5) \cap (II=6)] \cup [(I=6) \cap (II=5)]\} = 0,27$
p(11 months with speed AF) = $p\{[(I=5) \cap (II=6)] \cup [(I=6) \cap (II=5)]\} = 0,14$
p(11 months with speed BD) = $p\{[(I=5) \cap (II=6)] \cup [(I=6) \cap (II=5)]\} = 0,34$
p(11 months with speed BE) = $p\{[(I=5) \cap (II=6)] \cup [(I=6) \cap (II=5)]\} = 0,22$
p(11 months with speed BF) = $p\{[(I=5) \cap (II=6)] \cup [(I=6) \cap (II=5)]\} = 0,09$
p(11 months with speed CD) = $p\{[(I=5) \cap (II=6)] \cup [(I=6) \cap (II=5)]\} = 0,20$
p(11 months with speed CE) = $p\{[(I=5) \cap (II=6)] \cup (I=6) \cap (II=5)]\} = 0,12$
p(11 months with speed CF) = $p\{[(I=5) \cap (II=6)] \cup [(I=6) \cap (II=5)]\} = 0,04$

Similarly for 12 months:

p(12 months) = p[(I=6) ∩ (II=6)]

p[(I=6) ∩ (II=6)] = p(I=6) * p(II=6) = 0,3 * 0,5 = 0,15

p(12 months with speed AD) = p[(I=6) ∩ (II=6)]= 0,15

Similarly we can calculate other probabilities:

p(12 months with speed AD) = p[(I=6) ∩ (II=6)] = 0,15
p(12 months with speed AE) = p[(I=6) ∩ (II=6)] = 0,09
p(12 months with speed AF) = p[(I=6) ∩ (II=6)] = 0,03
p(12 months with speed BD) = p[(I=5) ∩ (II=6)] = 0,05
p(12 months with speed BE) = p[(I=6) ∩ (II=6)] = 0,03
p(12 months with speed BF) = p[(I=6) ∩ (II=6)] = 0,01
p(12 months with speed CD) = p[(I=6) ∩ (II=6)] = 0,0
p(12 months with speed CE) = p[(I=6) ∩ (II=6)] = 0,0
p(12 months with speed CF) = p[(I=6) ∩ (II=6] = 0,0

Expected value of fine can be calculated as:

For speeds AD; 0,37 * 1 * 500 + 0,15 * 2 * 500 = 335 similarly;
For speeds AE; = 225
For speeds AF; = 100
For speeds BD; = 220
For speeds BE; =140
For speeds BF; = 55
For speeds CD; = 100
For speeds CE; = 60
For speeds CF; = 20

Now expected value of total cost can be calculated as:

For speeds AD; 2040 + 1080 + 335 = 3455 similarly;
For speeds AE; = 3765
For speeds AF; = 3940
For speeds BD; = 3700
For speeds BE; = 4040
For speeds BF; = 4255
For speeds CD; = 3820
For speeds CE; = 4200
For speeds CF; = **4460**

The greatest value will be the expected value of total cost (**4460**). (It is better to show the results on a table given below)

JOB	SPEED	MONTHLY COST (MILLION)	THE PROBABILITY OF COMPLETION IN 4 MONHS	THE PROBABILITY OF COMPLETION IN 5 MONHS	THE PROBABILITY OF COMPLETION IN 6 MONHS	EXPECTED VALUE OF COST (MILLION)	SPEED	THE PROBABILITY OF COMPLETION IN 11 MONTHS	THE PROBABILITY OF COMPLETION IN 12 MONTHS	EXPECTED VALUE OF FINE (MILLION)	EXPECTED VALUE OF TOTAL COST (MILLION)
WINDOWS (I)	A	400	0,2	0,5	0,3	**2040**	AD	0,37	0,15	335	3455
	B	500	0,3	0,6	0,1	2400	AE	0,27	0,09	225	3765
	C	600	0,6	0,4	0	2640	AF	0,14	0,03	100	3940
DOOR (II)	D	200	0,1	0,4	0,5	**1080**	BD	0,34	0,05	220	3700
	E	300	0,3	0,4	0,3	1500	BE	0,22	0,03	140	4040
	F	400	0,6	0,3	0,1	1800	BF	0,09	0,01	55	4255
EXPECTED VALUE OF THE MAX TOTAL COST (MILLION)						**4460**	CD	0,2	0	100	3820
							CE	0,12	0	60	4200
							CF	0,04	0	20	**4460**

SOLUTION 2

a) $\mu_X = \Sigma\, x_i * p(x_i)$

= 0 * 3/30 + 15 * 1/30 + 20 * 2/30 + 25 * 4/30 +30 * 6,5/30 +35 * 6/30 + 40 * 3,5/30 + 45 * 2/30 +50 * 1/30 + 55 * 1/30

= 29,83

b) $Var(X) = \Sigma(x_i - \mu_X)^2 * p(x_i)$

= 148,58

c) $\sigma_X = (Var(X))^{1/2}$

= 12,19

d) $C_{vX} = \sigma_X / \mu_X$

= 0,41