EXERCISES III-SUPPLEMENT

PROBLEM-1

The joint probability density function (pdf) of a bivariate r.v. (X, Y) is given by

$$f_{xy}(x,y)=\begin{cases} ke^{-(ax+by)} & x>0, y>0 \\ 0 & otherwise \end{cases}$$

where a and bare positive constants and k is a constant

a) Determine the value of k.
b) Are X and Y independent

SOLUTION

a)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{xy}(x,y)dxdy = k\int_{0}^{\infty}\int_{0}^{\infty} e^{-(ax+by)}$$

$$= k\int_{0}^{\infty} e^{-(ax)}dx\int_{0}^{\infty} e^{-(by)}dy = \frac{k}{ab} = 1$$

Thus k=ab.

b) The marginal pdf of X is

$$f_x(x) = abe^{-ax}\int_{0}^{\infty} e^{-by}dy = ae^{-ax} \qquad x>0$$

The marginal pdf of Y is

$$f_y(y) = abe^{-by}\int_{0}^{\infty} e^{-ax}dx = be^{-bx} \qquad y>0$$

Since $f_{xy}(x,y) = f_x(x)f_y(y)$, X and Y are independent.

PROBLEM-2

The joint probability density function (pdf) of a bivariate r.v. (X, Y) is given by

$$f_{xy}(x,y)=\begin{cases} \frac{1}{y}e^{-x/y}e^{-y} & x>0, y>0 \\ 0 & otherwise \end{cases}$$

Find P(X>1/ Y=y).

SOLUTION

$$f_y(y) = \int_{0}^{\infty} f_{xy}(x,y)dx = \frac{1}{y}e^{-y}\int_{0}^{\infty} e^{-x/y}dx$$

$$= \frac{1}{y}e^{-y}\left[-ye^{-x/y}\Big|_{x=0}^{x=\infty}\right] = e^{-y}$$

The conditional pdf of X is

$$f_{x/y}(x,y) = \frac{f_{xy}(x,y)}{f_y(y)} = \begin{cases} \frac{1}{y} e^{-x/y} & x > 0, y > 0 \\ 0 & otherwise \end{cases}$$

Then

$$P(X > 1 / Y = y) = \int_1^\infty \frac{1}{y} e^{-x/y}$$

$$= -e^{-x/y} \Big|_{x=}^{x=\infty} = e^{-1/y}$$