CHAPTER III
DISTRIBUTION OF RANDOM VARIABLES
MULTIVARIABLE DISTRIBUTIONS
EXERCISES-3

1) The soil of the foundation of a column has a strenght in the range of 6 and 15 kg/cm^2 with the probability density function:

$$f(x)=C(1-x/15)$$

- **a)** Determine the value of C.
- **b)** What is the probability of failure when the column load is 7,5 kg/cm^2 ?
- **c)** What should the design load be if the probability of failure is required not to exceed %1?

2) The dailiy flows X and Y of two streams flowing into the reservoir of a dam have the joint joint probability density function:

$$f(x,y)=C(4000-x)/4000 \qquad 0 \le x \le 4000 \quad 0 \le y \le 2000$$

- **a)** Determine the value of **C**.
- **b)** What is the probability **P(X>2Y)** ?
- **c)** Obtain the marginal probability density function of **X** and **Y**
- **d)** Find the conditional probability density function of **f(x|y)**
- **e)** What is the probability that $X \ge 2000$ on a day When **Y=1000** ?
- **f)** Are the variables **X** and **Y** independent ? why?

3) The joint probability density function of the variables **X** and **Y** is:

$$f(x,y)=4xy \qquad 0 \le x \le 1 \quad 0 \le y \le 1$$

Find the marginal probability density function of **X** and **Y**.

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1)

a) $\int_6^{15} C(1-x/15)dx = \int_6^{15} Cdx - (1/15)C.x.dx = [C.x - (1/30).C.x^2]\Big|_6^{15} = 1$

$= [15.C.(1-15.(1/30)] - [6.C.(1-6(1/30)] = 1 \Rightarrow C = 1/2,7 \Rightarrow C = 0,37$

b) $F(x) = \int_6^{x} f(x).dx$

$\int_6^{x} 0,37.(1-x/15).dx = \int_6^{x} 0,37.dx - (1/15).0,37.x.dx = [0,37.x - (1/2)(1/15)0,37.x^2]\Big|_6^{x}$

$\Rightarrow F(7,5) = 0,30$

c)

$\int_6^{x} 0,37.(1-x/15).dx = 0,01 = [0,37.x - (1/2).(1/15).0,37.x^2]\Big|_6^{x}$

$\Rightarrow x^2 - 30x + 144,81 = 0 \Rightarrow x \cong 6,05 kg/cm^2$

2)

a) $\int_0^{4000}\int_0^{2000} C\frac{4000-x}{4000}.dx.dy = 1 \Rightarrow C = 2,5.10^{-7}$

b)

$P[X \geq 2Y] = \int_0^{4000}\int_0^{0,5x} \frac{C}{4000}(4000-x).dx.dy$

by carrying out the integrations; $P[X \geq 2Y] = \frac{1}{3}\frac{(2000).(4000)}{2}C = \frac{1}{3}$

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c) The marginal distributions of X and Y are formally

$$f_x(x) = \int_0^{2000} C\frac{4000 - x}{4000}dy = 2000C\frac{4000 - x}{4000} = \frac{1}{2000}\frac{4000 - x}{4000} \qquad 0 \le x \le 4000$$

$$f_y(y) = \int_0^{4000} C\frac{4000 - x}{4000}dx = 2000C \qquad 0 \le y \le 2000$$

d) $$f_{X|Y}(x, y) = \frac{f(x, y)}{f_y(y)} = \frac{C(4000 - x)/4000}{2000.C} = \frac{1}{2000}\frac{4000 - x}{4000} \qquad 0 \le x \le 4000$$

(Notice that this is unchanged from the marginal distribution $f_x(x)$)

e) $$\int_{2000}^{4000} f_x(x)\,dx = \int_{2000}^{4000} \left(\frac{1}{2000} - \frac{x}{8.10^{-6}}\right)dx = \left(\frac{x}{2000} - \frac{x^2}{16.10^{-6}}\right)\Big|_{2000}^{4000} = 0{,}25$$

f)Yes,the variables are independent.Because the knowledge of Y has not altered the distribution of X and hence has not provided us any new information in our quest to predict or describe the behaviour of X. **(** $f_x(x) = f_{X|Y}(x, y)$**)**

3) The marginal PDF of X: $\int_0^1 4xy.dy = 2xy^2 \,|_0^1 = 2x$

The marginal PDF of Y: $\int_0^1 4xy.dx = 2yx^2 \,|_0^1 = 2y$