

CHAPTER II

ELEMENTS OF PROBABILITY THEORY

EXERCISES II

PROBLEM 1

In a construction site the job C is composed of the jobs A and B succeeding each other. The job A can be completed in 4, 5 or 6 days with the probabilities 0,2, 0,5, and 0,3, respectively. The job B can be finished in 3, 4 or 5 days with the probabilities 0,3, 0,4, and 0,3, respectively.

- a) What are the simple events in the sample space of the job C?
- b) What are the probabilities of these events ?
- c) What is the probability that the job C can be completed in not more than 9 days ?
- d) What is the probability that the job C can be completed in at least 9 days ?
- e) What is the union of the two events defined in c. and d. ?
- f) What is the probability that the job B can be completed in 3 days on the condition that A is completed in 4 days ?
- g) What is the expected value of duration of the job C ?

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SOLUTIONS

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SOLUTION 1

$$A = \{ 4, 5, 6 \}$$

$$P(A=4) = 0,20$$

$$P(A=5) = 0,50$$

$$P(A=6) = 0,30$$

$$B = \{ 3, 4, 5 \}$$

$$P(B=3) = 0,30$$

$$P(B=4) = 0,40$$

$$P(B=5) = 0,30$$

a) $C = (4+3, 4+4, 4+5, 5+3, 5+4, 5+5, 6+3, 6+4, 6+5)$
 $C = \{ 7, 8, 9, 10, 11 \}$

b)

$$P[C = 7] = P[A = 4] \times P[B = 3] = 0,2 \times 0,3 = 0,06$$

$$P[C = 8] = P[A = 4] \times P[B = 4] + P[A = 5] \times P[B = 3] = 0,2 \times 0,4 + 0,5 \times 0,3 = 0,23$$

$$P[C = 9] = P[A = 4] \times P[B = 5] + P[A = 5] \times P[B = 4] + P[A = 6] \times P[B = 3] = 0,2 \times 0,3 + 0,5 \times 0,4 + 0,3 \times 0,3 = 0,35$$

$$P[C = 10] = P[A = 5] \times P[B = 5] + P[A = 6] \times P[B = 4] = 0,5 \times 0,3 + 0,3 \times 0,4 = 0,27$$

$$P[C = 11] = P[A = 6] \times P[B = 5] = 0,3 \times 0,3 = 0,09$$

c)

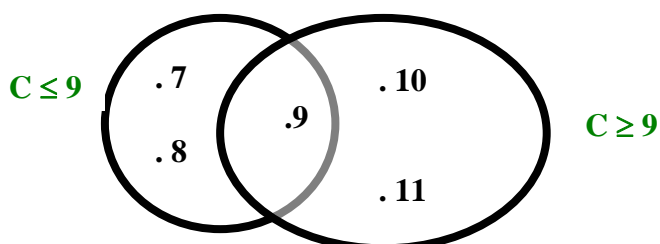
$$P(C \leq 9) = P[C = 7] + P[C = 8] + P[C = 9] = 0,06 + 0,23 + 0,35 = 0,64$$

d)

$$P(C \geq 9) = P[C = 9] + P[C = 10] + P[C = 11] = 0,35 + 0,27 + 0,09 = 0,71$$

e)

$$P[(C \leq 9) \cup (C \geq 9)] = P(C \leq 9) + P(C \geq 9) - P(C = 9) = 0,64 + 0,71 - 0,35 = 1,00$$



f)

$$P(B = 3 | A = 4) = \frac{P(A = 4 \cap B = 3)}{P(A = 4)} = \frac{0.2 \times 0.3}{0.2} = 0,3$$

g)

The expected value of duration of the job C

$$E [C] = 0,06 \times 7 + 0,23 \times 8 + 0,35 \times 9 + 0,27 \times 10 + 0,09 \times 11 = 9,1 \text{ Days.}$$