Digital Design

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Grading

- 1st Midterm - % 18
  - 31th October 2013
- 2nd Midterm – % 18
  - 07 th December 2013
- 5 Homeworks - % 6
- Final Exam - % 34
- Lab - % 24

Goals and objectives

- The goal of this course is to:
  - provide a good understanding of the digital systems.
  - introduce the basic building blocks of digital design including combinational logic circuits, combinational logic design, arithmetic functions and circuits and sequential circuits.
  - Showing how these building blocks are employed in larger scale digital systems

- Having successfully completed this course, the student will:
  - acknowledge the importance of digital systems.
  - Design a digital circuit given a Boolean function.
  - Get familiar with typical combinatorial (adders, decoders, multiplexers, encoders) and sequential (D flip-flops, counters, registers, shift registers) components.
  - Understand how larger systems are organized.

References

- Text Books :
  - Digital Design, M. Morris Mano, Michael D. Ciletti,
  - Logic and Computer Design Fundamentals, 4/E, M. Morris Mano and Charles Kime , Prentice Hall, 2008.
- Slides and all anouncements : http://ee.yeditepe.edu.tr/labs/ee241/

Overview of Chapter 1

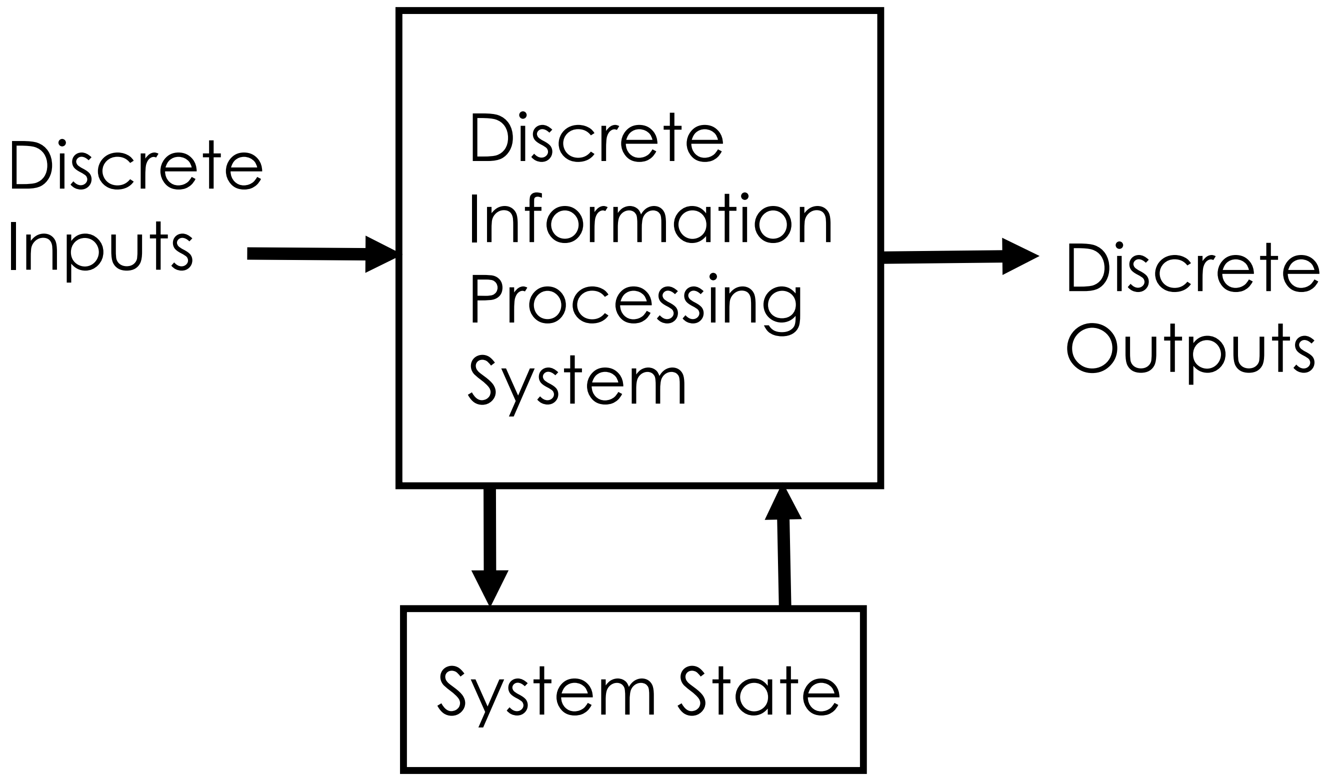
- **Digital Systems, Computers**
- **Information Representation**
- **Number Systems** [binary, octal and hexadecimal]
- **Arithmetic Operations**
- **Base Conversion**
- **Decimal Codes** [BCD (binary coded decimal)]

What is a Digital System?

- One characteristic:
  - Ability of manipulating discrete elements of information
- A *set* that has a finite number of elements contains discrete information
- Examples for discrete sets
  - Decimal digits {0, 1, …, 9}
  - Alphabet {A, B, …, Y, Z}
  - Binary digits {0, 1}
- One important problem
  - how to represent the elements of discrete sets in physical systems?

DIGITAL & COMPUTER SYSTEMS - Digital System

- **Takes a set of discrete information inputs and discrete internal information (system state) and generates a set of discrete information outputs.**



Types of Digital Systems

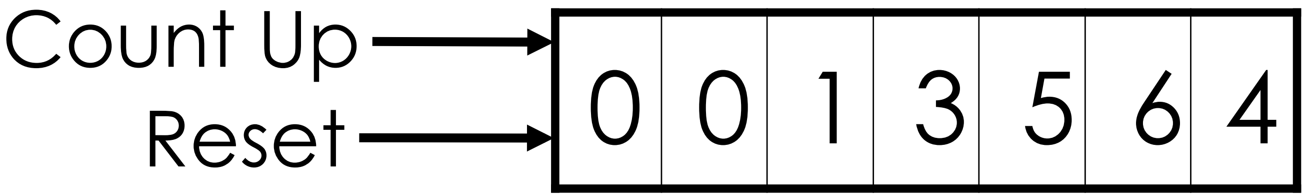
- No state present
  - Combinational Logic System
  - Output = Function(Input)
- State present
  - State updated at discrete times

    => Synchronous Sequential System
  - State updated at any time

    =>Asynchronous Sequential System
  - State = Function (State, Input)
  - Output = Function (State)
    or Function (State, Input)

Digital System Example:

A Digital Counter (e. g., odometer):

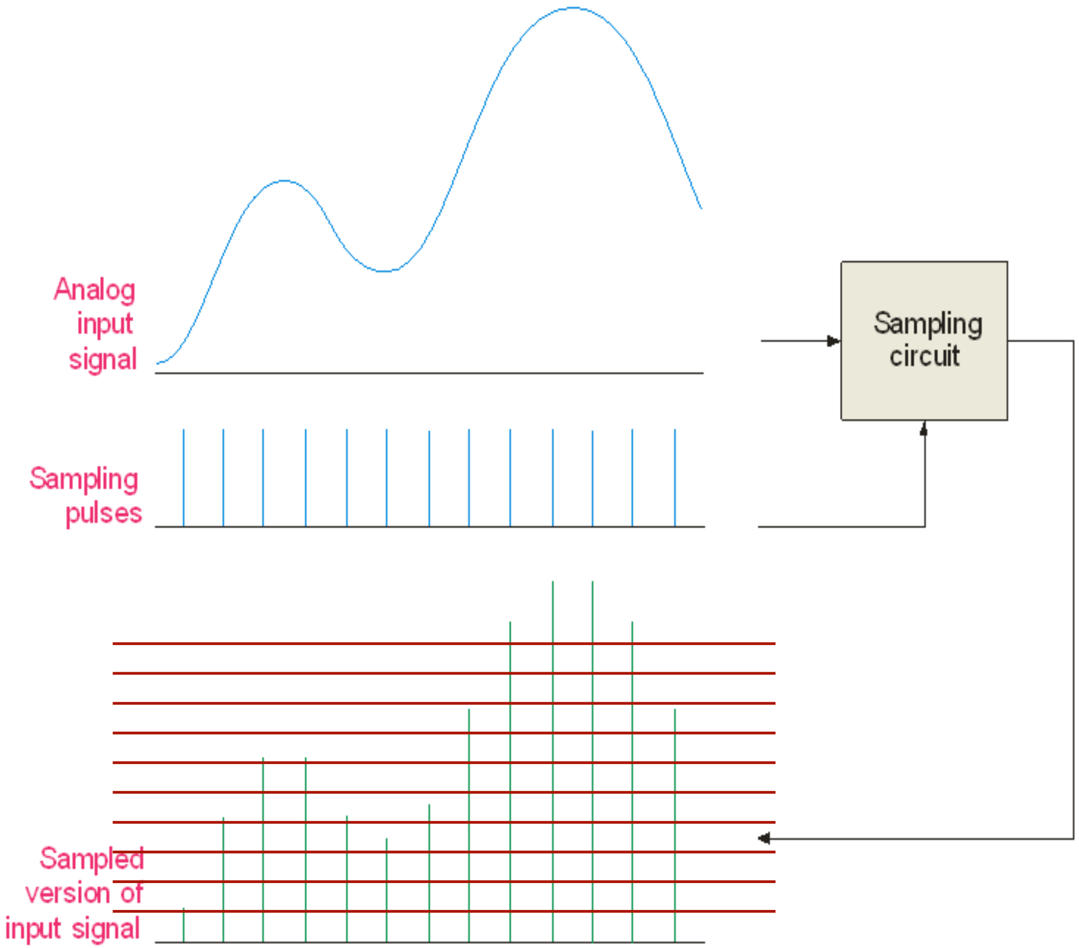


Inputs: **Count Up, Reset**
Outputs: **Visual Display**
State: **"Value" of stored digits**

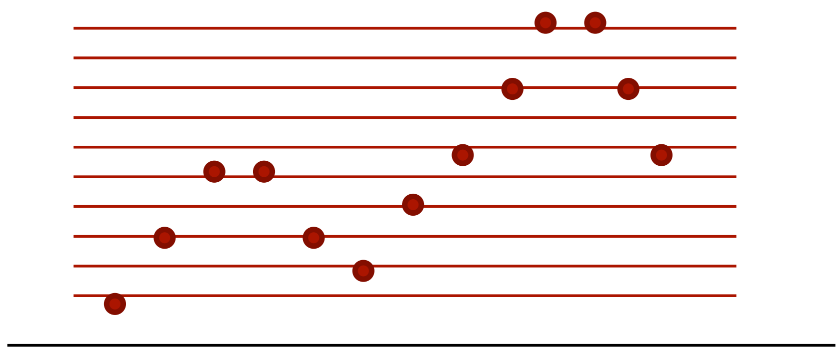
Analog – Digital Signals

- **The physical quantities in real world like current, voltage, temperature values change in a continuous range.**
- **The signals that can take any value between the boundaries are called analog signals.**
- **Information take discrete values in digital systems.**
- **Binary digital signals can take one of the two possible values: 0-1, high-low, open-closed.**

Conversion of Analog Signals to Digital Signals



▪Quantized Signal



How to Represent?

- In electronics circuits, we have electrical signals
  - voltage
  - current
- Different strengths of a physical signal can be used to represent elements of the discrete set.
- Which discrete set?
- Binary set is the easiest
  - two elements {0, 1}
  - Just two signal levels: 0 V and 4 V
- This is why we use binary system to represent the information in our digital system.

Binary System

- Binary set {0, 1}
  - The elements of binary set, 0 and 1 are called "binary digits"
  - or shortly "bits".
- How to represent the elements of other discrete sets
  - Decimal digits {0, 1, …, 9}
  - Alphabet {A, B, …, Y, Z}
- Elements of any discrete sets can be represented using groups of bits.
  - 9 → 1001
  - A → 1000001

How Many Bits?

- What is the formulae for number of bits to represent a discrete set of *n* elements
- {0, 1, 2, 3}
  - 00 → 0, 01 → 1, 10 → 2, ands 11 → 3.
- {0, 1, 2, 3, 4, 5, 6, 7}
  - 000 → 0, 001 → 1, 010 → 2, ands 011 → 3
  - 100 → 4, 101 → 5, 110 → 6, ands 111 → 7.
- The formulae, then,
  - ?
  - If n = 9, then 4 bits are needed

NUMBER SYSTEMS – Representation

- Positive radix, positional number systems
- A number with *radix* ***r*** is represented by a string of digits:

  $$A_{n-1}A_{n-2} \ldots A_1A_0 \,.\, A_{-1}\, A_{-2} \ldots A_{-m+1}\, A_{-m}$$

  in which $\mathbf{0 \leq A_i < r}$ and **.** is the *radix point*.
- The string of digits represents the power series:

$$(\text{Number})_r = \left(\sum_{i=0}^{i=n-1} A_i \cdot r^i \right) + \left(\sum_{j=-m}^{j=-1} A_j \cdot r^j \right)$$

(Integer Portion) + (Fraction Portion)

Representation of positive numbers

- Örnek: $215_{10}=(1101\ 0111)_2 = 1\cdot2^7+1\cdot2^6+0\cdot2^5+1\cdot2^4+0\cdot2^3+1\cdot2^2+1\cdot2^1+1\cdot2^0$

- Most Signaficant Bit (MSB)
- Least Signaficant Bit (LSB)

- The largest postive number that can be represented by 8 bits is:
  - $(1111\ 1111)_2=255_{10}$
- The smallest postive number that can be represented by 8 bits is:
  - $(0000\ 0000)_2=0_{10}$

Some Bases

Name	Base	Digits
Binary	2	0,1
Octal	8	0,1,2,3,4,5,6,7
Decimal	10	0,1,2,3,4,5,6,7,8,9
Hexadecimal	16	0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F

Some Numbers in Different Bases

Decimal (Base 10)	Binary (Base 2)	Octal (Base 8)	Hexa decimal (Base 16)
00	00000	00	00
01	00001	01	01
02	00010	02	02
03	00011	03	03
04	00100	04	04
05	00101	05	05
06	00110	06	06
07	00111	07	07
08	01000	10	08
09	01001	11	09
10	01010	12	0A
11	0101 1	13	0B
12	01100	14	0C
13	01101	15	0D
14	01110	16	0E
15	01111	17	0F
16	10000	20	10

Base Conversions

- From base-r to decimal is easy
  - expand the number in power series and add all the terms
- Reverse operation is somewhat more difficult
- Simple idea:
  - divide the decimal number successively by r
  - accumulate the remainders.
- If there is a fraction, then integer part and fraction part are handled separately.

Example: 46.6875_{10} to base 2

- Convert 46 to binary
  - 46/2=23 remainder 0
  - 23/2=11 remainder 1
  - 11/2=5 remainder 1
  - 5/2=2 remainder 1
  - 2/2=1 remainder 0
  - 1/2=0 remainder 1
- Convert 0.6875 to binary
  - 0.6875*2=1.375
  - 0.375*2=0.75
  - 0.75*2=1.5
  - 0.5*2=1
  - 0*2=0
- Put two results together with a radix point
  - 101110.10110_2

Example: 46.6831_{10} to hexadecimal (base 16)

- Convert 46 to base 16
  - 46/16=2 remainder 14
  - 2/16=0 remainder 2
- Convert 0.6831 to base 16
  - 0.6831*16=10.9296
  - 0.9296*16=14.8736
  - 0.8736*16=13.9776
  - 0.9776*16=15.6416
  - …
- Put two results together with a fraction dot
  - $2E.AEDF_{16}$

Conversion from base r to base decimal

- Convert 101110.10110_2 to base 10

$$101110_2 = 1 \cdot 32 + 0 \cdot 16 + 1 \cdot 8 + 1 \cdot 4 + 1 \cdot 2 + 0 \cdot 1$$

$$= 32 + 8 + 4 + 2$$

$$= 46$$

$$0.1011_2 = 1/2 + 1/8 + 1/16$$

$$= 0.5000 + 0.1250 + 0.0625$$

$$= 0.6875$$

Conversions between Binary, Octal and Hexadecimal

- Octal to Binary
  - $743.056_8 = 111\ 100\ 011.000\ 101\ 110_2$
- Hexadecimal to Binary
  - $A49.0C6_{16} = 1010\ 0100\ 1001.0000\ 1100\ 0110_2$
- Binary to Octal
  - $1 | 011 | 100 | 011.000 | 101 | 110 | 1_2 = 1343.0564_8$
- Binary to Hexadecimal
  - $1 | 1010 | 0100 | 1001.0010 | 1100 | 0110 | 1_2 = 1A49.2C68_{16}$

- Octal and hexadecimal representations are more compact.
- Therefore, we use them in order to communicate with computers directly using their internal representation

Representation of Negative Numbers

- In order to differ between positive and negative numbers the MSB is used.
  - If “0” positive
  - If “1” negative
- The positive numbers that can be shown by 8 bits are between 0000 0000 and 0111 1111, hence between 0 and + 127.
- 2’s complement method is used for representation of negative numbers.
  - 2’s complement of a positive number shows the negative of it.
- In order to find the 2’s complement of a number
  - 1’s complement is found: 0s are changed to 1s, 1s are changed to 0s.
  - 1 is added to 1’s complement of the number.

Examples for Negative Numbers

	0 0 0 0 0 1 0 1	+ 5
1's complement	1 1 1 1 1 0 1 0	
Addition of 1	+ 1	
2's complement	1 1 1 1 1 0 1 1	- 5

Negative number

Examples for Negative Numbers

	1 1 1 1 1 0 1 1	- 5
1's complement	0 0 0 0 0 1 0 0	
Addition of 1	+ 1	
2's complement	0 0 0 0 0 1 0 1	+ 5

Pozitive number

ARITHMETIC OPERATIONS - Binary Arithmetic

- **Single Bit Addition with Carry**
- **Multiple Bit Addition**
- **Multiple Bit Subtraction**
- **Multiplication**

Single Bit Binary Addition with Carry

Given two binary digits (X,Y), a carry in (Z) we get the following sum (S) and carry (C):

Carry in (Z) of 0:

Z	0	0	0	0
X	0	0	1	1
+ Y	+ 0	+ 1	+ 0	+ 1
C S	0 0	0 1	0 1	1 0

Carry in (Z) of 1:

Z	1	1	1	1
X	0	0	1	1
+ Y	+ 0	+ 1	+ 0	+ 1
C S	0 1	1 0	1 0	1 1

Addition of Positive Numbers

Carry	00000		01100	
X	01100	12	10110	22
Y	+10001	+17	+10111	+23
Sum	11101	29	101101	45

Note:

1. The carry input to the LSB is always '0'.
2. The sum of two n-bit numbers has n+1-bits.

Addition of Positive and Negative Numbers Represented By 2's Complement

Carry	0010		0100	
X	1101	-3	0011	3
Y	+0001	+1	+0010	+2
Sum	1110	-2	0101	5
	Negative		Positive	

Addition of Positive and Negative Numbers Represented By 2's Complement

Carry	11110		11100	
X	1101	-3	0011	3
Y	+1111	-1	+1110	-2
Sum	11100	-4	10001	1

negative

ignored

positive

ignored

Addition of Positive and Negative Numbers Represented By 2's Complement

Carry	1000		0000	
X	0100	4	1010	-6
Y	+0101	+5	+1101	-3
Sum	1001	9	10111	-9

Is the sum negative?

ignored

Is the sum positive?

- Overflow occured. The largest positive number that can be represented by 4-bits is +7. Larger numbers can not be reprsented by 4-bits.
- The smallest negative number that can be represented by 4-bits is -8. Smaller numbers can not be reprsented by 4-bits.
- The number of bits to be used in the representation of the numbers should be decided according to the boundaries of the inputs and the outputs of the operations.

Subtraction of Positive Numbers

Borrow	00000		00110	
X	10110	22	10110	22
Y	- 10010	-18	- 10011	-19
Difference	00100	4	00011	3

Note: The borrow input to the LSB is always '0'. If Y>X, then X and Y are exchanged and – sign is put in front of the result.

Subtraction of Numbers With Sign Bit and 2's complement

X	3	0011		0011	3
Y	-1	-0001	2's complement	+1111	+(-1)
Difference	2			10010	2

ignored positive

X	3	0011		0011	3
Y	-4	-0100	2's complement	+1100	+(-4)
Difference	-1			1111	-1

negative

X	3	0011		0011	3
Y	-(-1)	-1111	2's complement	+0001	+1
Difference	4			0100	4

positive

Subtraction of Numbers With Sign Bit and 2’s complement

X	1	0001		0001	1
Y	-(-7)	-1001	2’s complement	+0111	+7
Difference	8		Is the result negative?	1000	8
X	-5	1011		1011	-5
Y	-4	-0100	2’s complement	+1100	+(-4)
Difference	-9		ignored	10111	-9

Is the result positive?

- Overflow occured. The largest positive number that can be represented by 4-bits is +7. Larger numbers can not be represented by 4-bits.
- The smallest negative number that can be represented by 4-bits is -8. Smaller numbers can not be reprsented by 4-bits.
- The number of bits to be used in the representation of the numbers should be decided according to the boundaries of the inputs and the outputs of the operations.

Binary Multiplication

The binary multiplication table is simple:

0 * 0 = 0 | 1 * 0 = 0 | 0 * 1 = 0 | 1 * 1 = 1

Extending multiplication to multiple digits:

Multiplicand	**1011**
Multiplier	**x 101**
Partial Products	**1011**
	0000 -
	1011 - -
Product	**110111**

Binary Numbers and Binary Coding

- Flexibility of representation
  - Within constraints below, can assign any binary combination (called a code word) to any data as long as data is uniquely encoded.
- Information Types
  - Numeric
    - Must represent range of data needed
    - Very desirable to represent data such that simple, straightforward computation for common arithmetic operations permitted
    - Tight relation to binary numbers
  - Non-numeric
    - Greater flexibility since arithmetic operations not applied.
    - Not tied to binary numbers

Non-numeric Binary Codes

- Given *n* binary digits (called <u>bits</u>), a <u>binary code</u> is a mapping from a set of <u>represented elements</u> to a subset of the 2^n binary numbers.
- Example: A binary code for the seven colors of the rainbow
- Code 100 is not used

Color	Binary Number
Red	000
Orange	001
Yellow	010
Green	011
Blue	101
Indigo	110
Violet	111

Number of Elements Represented

- **Given *n* digits in radix *r*, there are r^n distinct elements that can be represented.**
- **But, you can represent m elements, m < r^n**
- **Examples:**
  - **You can represent 4 elements in radix *r* = 2 with *n* = 2 digits: (00, 01, 10, 11).**
  - **You can represent 4 elements in radix *r* = 2 with *n* = 4 digits: (0001, 0010, 0100, 1000).**
  - **This second code is called a "<u>one hot</u>" code.**