

We write equilibrium equation of a fluid particle;

$$P_1(\pi r^2) - P_2(\pi r^2) - \tau(2\pi rL) = 0 \quad \text{Or} \quad \tau = \frac{(p_1 - p_2)r}{2L}$$

Since $(p_1 - p_2)/\gamma$ equals to loss of energy or h_L , we multiply both sides by $\frac{\gamma}{\gamma}$

$$\tau = \frac{\gamma r}{2L} \left(\frac{p_1 - p_2}{\gamma} \right) \quad \text{Or} \quad \tau = \frac{\gamma h_L}{2L} r$$

We extract h_L from the equation;

$$h_L = \frac{2\tau L}{\gamma r} = \frac{4\tau L}{\gamma d}$$

And from Darcy – Weisbach formula;

$$h_L = f \left(\frac{L}{d} \right) \left(\frac{V^2}{2g} \right)$$

We equalize the both h_L s, and extract τ ;

$$\frac{4\tau L}{\gamma d} = f \left(\frac{L}{d} \right) \left(\frac{V^2}{2g} \right)$$

$$\tau = f \left(\frac{\gamma}{g} \right) \left(\frac{V^2}{8} \right)$$

Since $\frac{\gamma}{g} = \rho$;

$$\tau = \frac{f\rho V^2}{8}$$