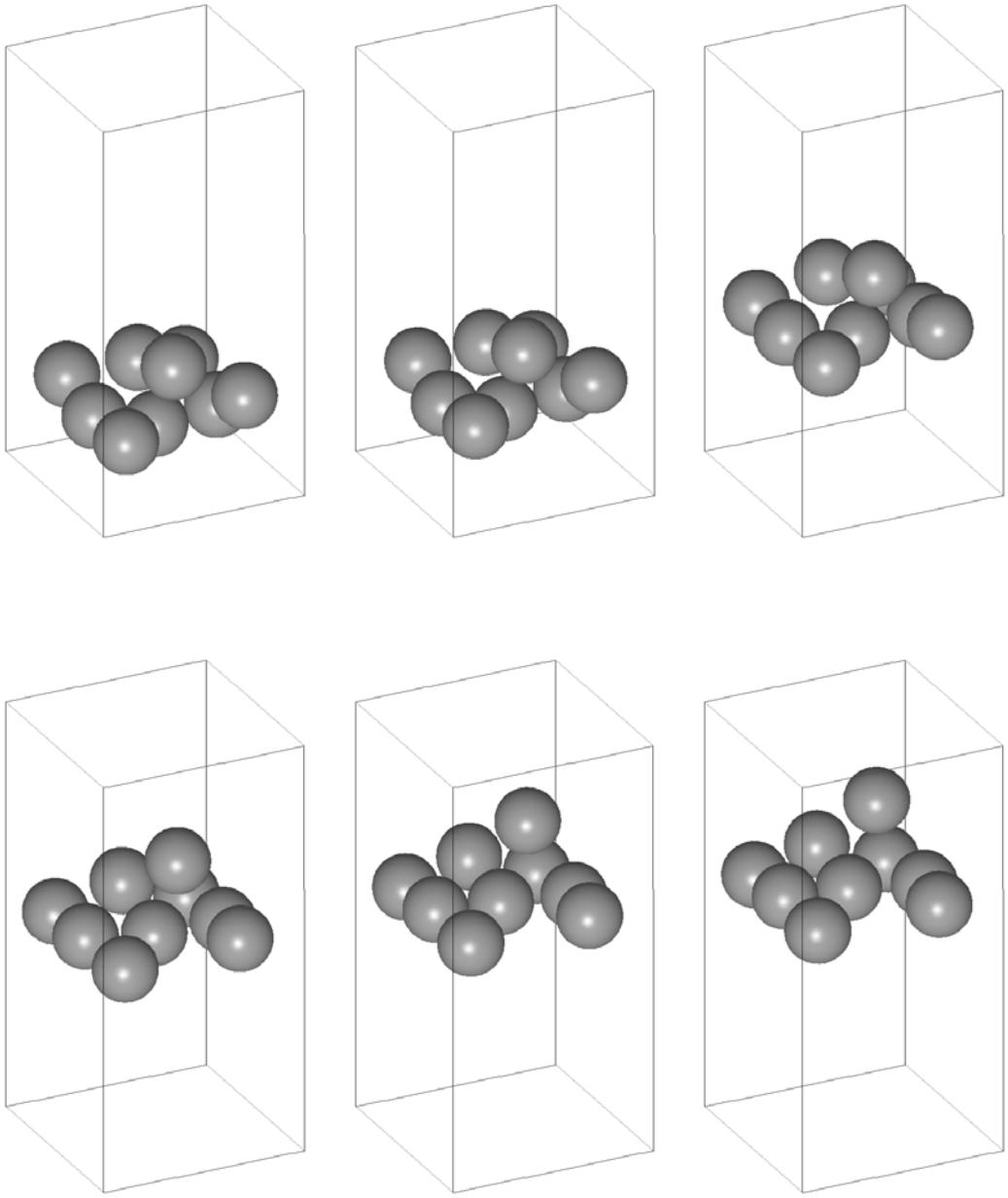


Figure 4.10: A fully three-dimensional simulation of the interaction of nine equal sized bubbles. Time progresses from left to right, top to bottom. The nondimensional time, $t^*$, is equal to 0.66, 6.66, 40, 66.66, 80, 86.66. The nondimensional time is scaled by $a/U_r$ and velocity is scaled by reference velocity, $U_r$. Computational domain size is $x/a = 6.66$, $y/a = 6.66$ and $z/a = 13.33$.

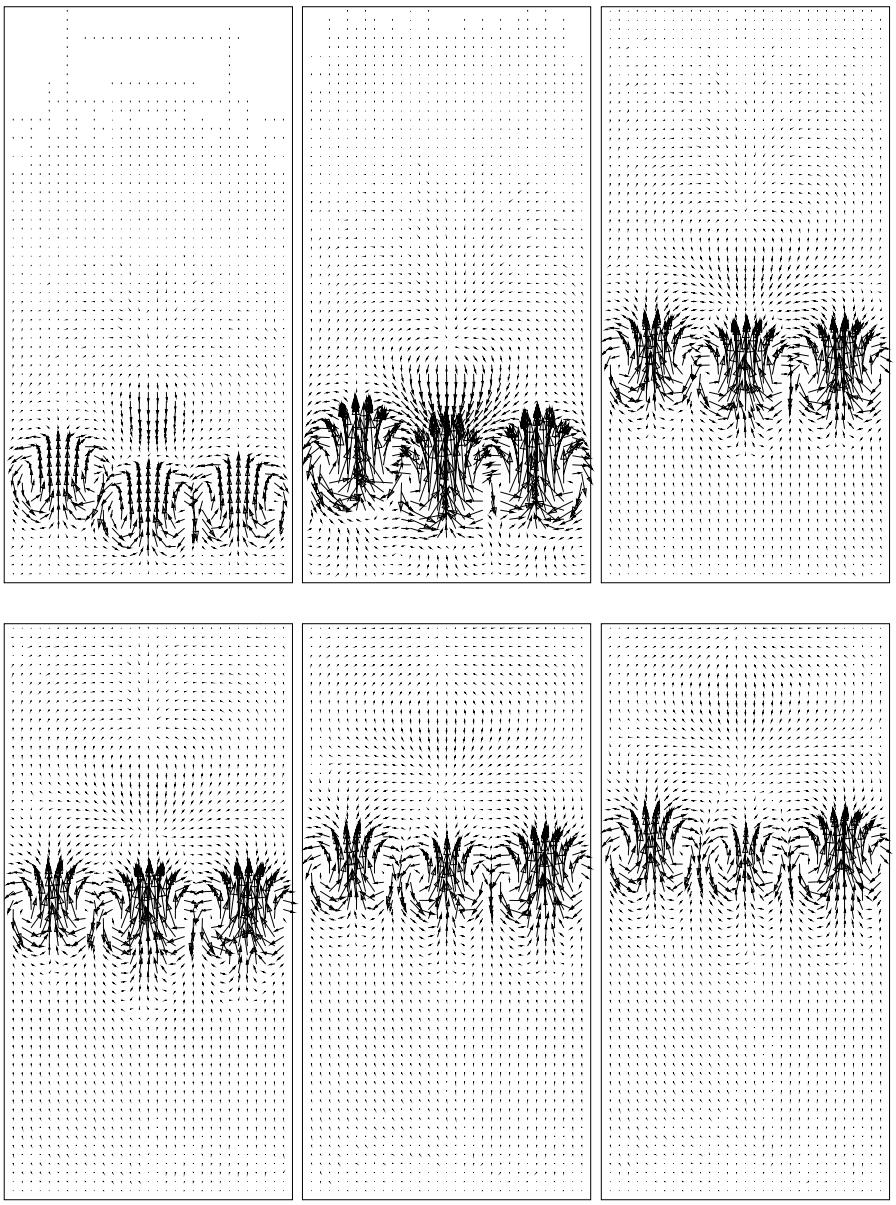


Figure 4.11: Velocity field for selected frames from the fully three dimensional simulation of nine-bubble interaction. The velocity field is shown at every other grid point in the middle plane of the computational box in $y$ direction. Time progresses from left to right. The nondimensional time, $t^*$, is equal to 0.66, 6.66, 40, 66.66, 80, 86.66. The nondimensional time is scaled by $a/U_r$ and velocity is scaled by reference velocity, $U_r$. Computational domain size is $x/a = 6.66$, $y/a = 6.66$ and $z/a = 13.33$.

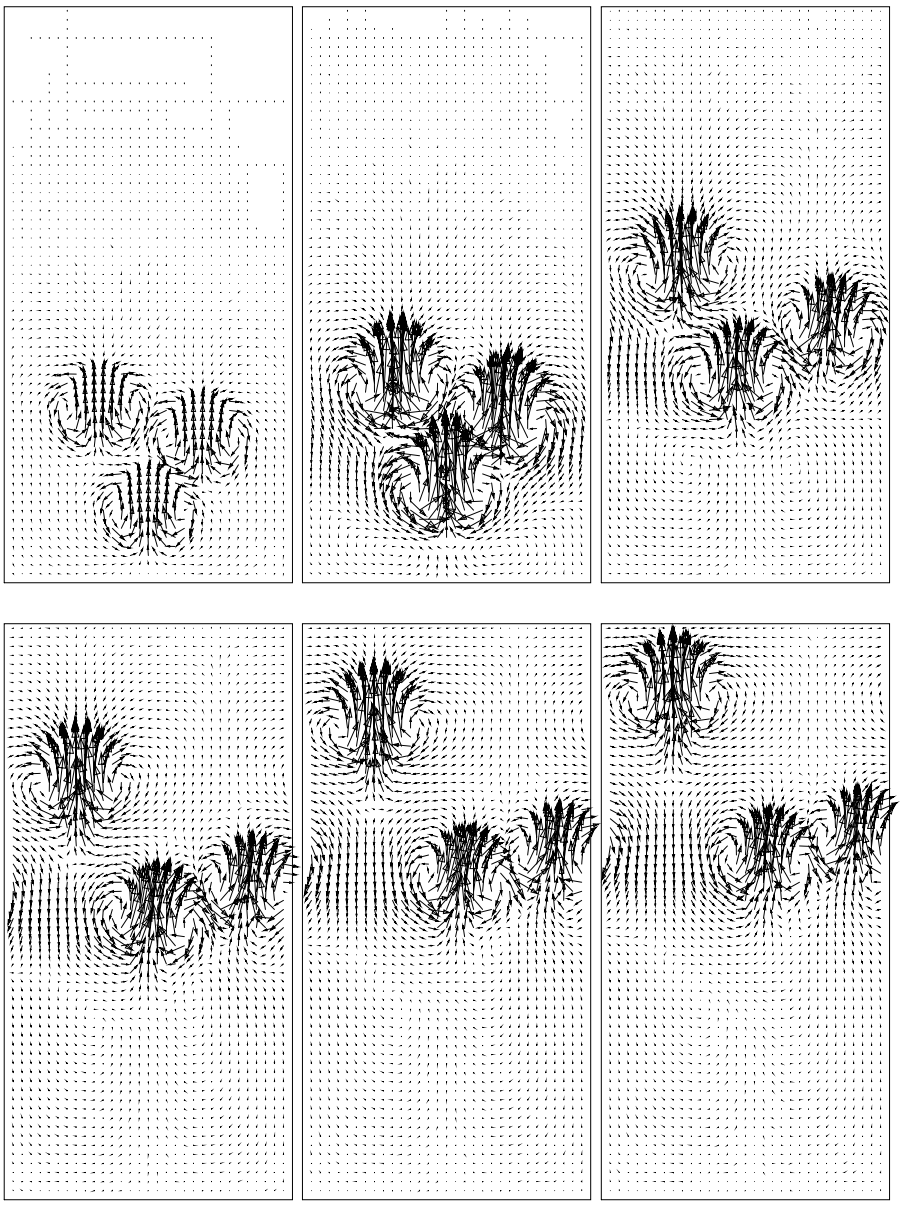


Figure 4.12: Velocity field for selected frames from the fully three dimensional simulation of nine-bubble interaction. The velocity field is shown at every other grid point in the middle plane of the computational box in $x$ direction. Time progresses from left to right. The nondimensional time, $t^*$, is equal to 0.66, 6.66, 40, 66.66, 80, 86.66. The nondimensional time is scaled by $a/U_r$ and velocity is scaled by reference velocity, $U_r$. Computational domain size is $x/a = 6.66$, $y/a = 6.66$ and $z/a = 13.33$.

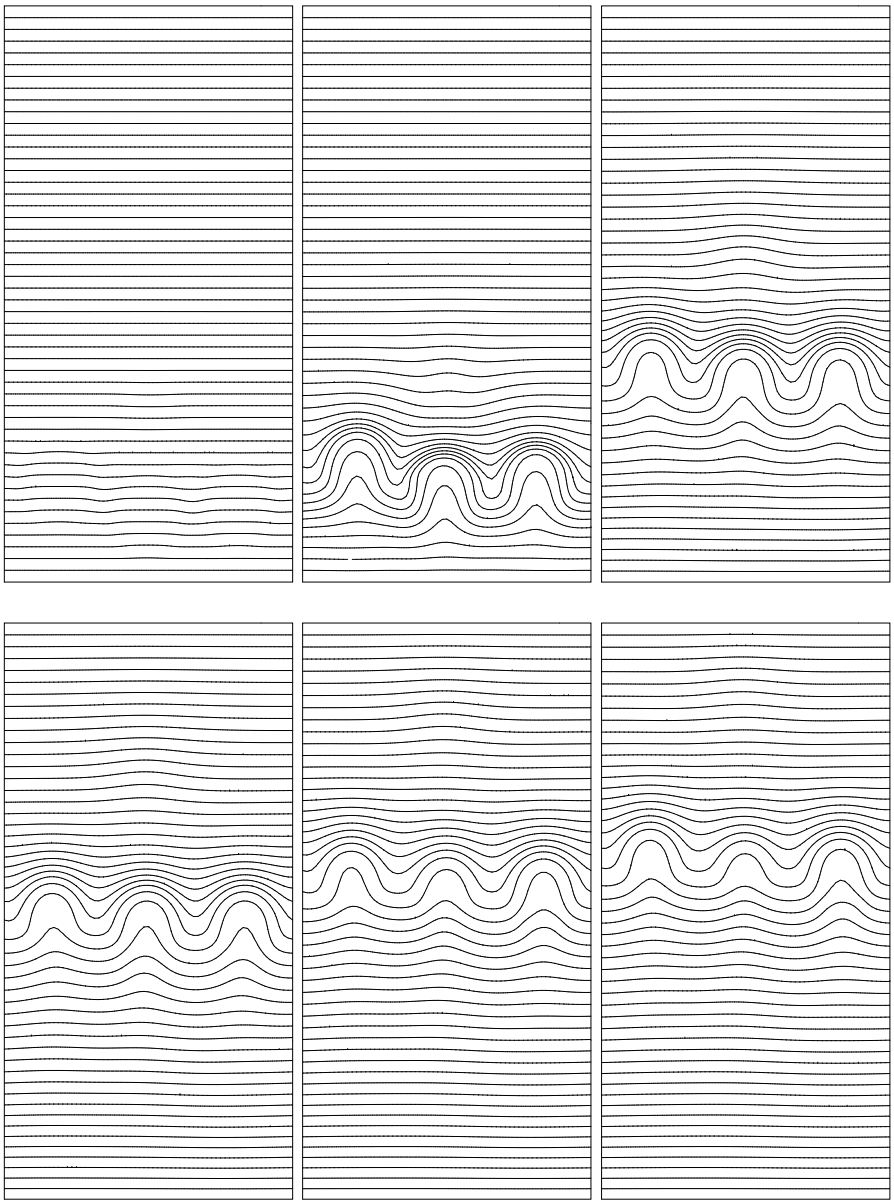


Figure 4.13: Temperature contours for selected frames from the fully three dimensional simulation of nine-bubble interaction. 50 equally spaced contours are shown in the middle plane of the computational box in $y$ direction. Time progresses from left to right. The nondimensional time, $t^*$, is equal to 0.66, 6.66, 40, 66.66, 80, 86.66. The nondimensional time is scaled by $a/U_r$ and temperature is scaled, after subtracting a reference temperature, by $a\nabla T_\infty$. Computational domain size is $x/a = 6.665$ $y/a = 6.66$ and $z/a = 13.33$.

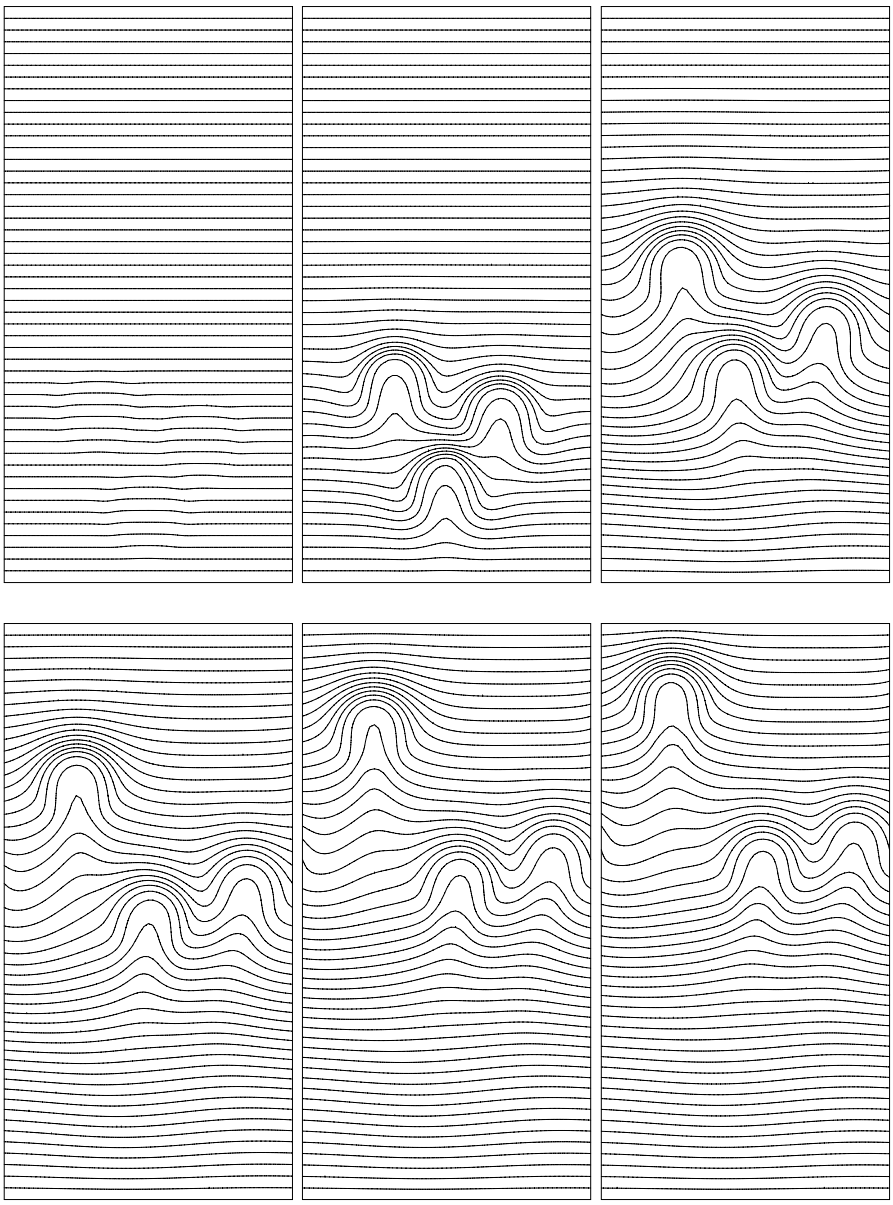
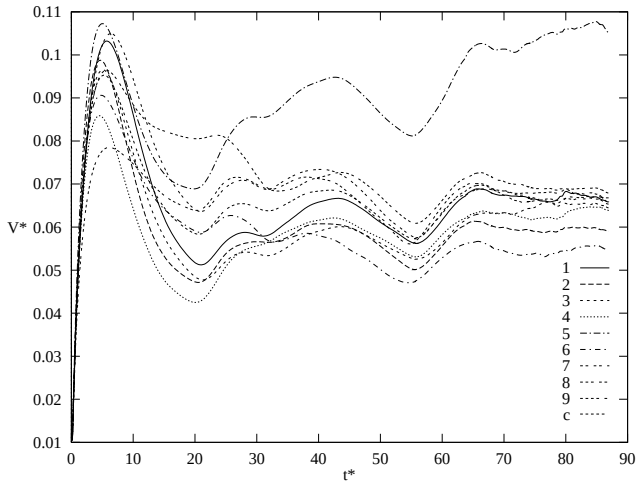
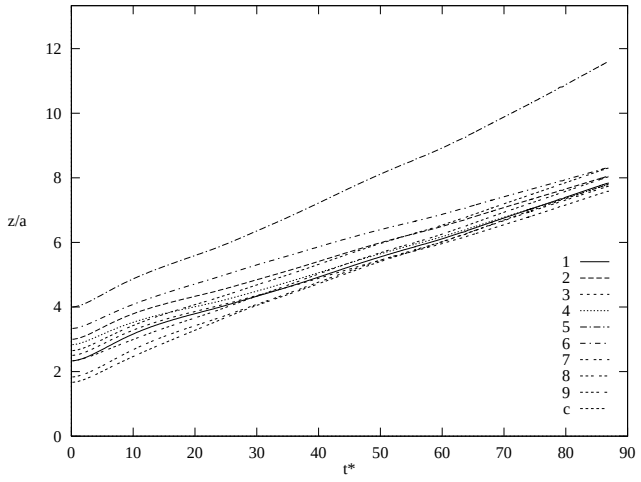


Figure 4.14: Temperature contours for selected frames from the fully three dimensional simulation of nine-bubble interaction. 50 equally spaced contours are shown in the middle plane of the computational box in $x$ direction. Time progresses from left to right. The nondimensional time, $t^*$, is equal to 0.66, 6.66, 40, 66.66, 80, 86.66. The nondimensional time is scaled by $a/U_r$ and temperature is scaled, after subtracting a reference temperature, by $a\nabla T_\infty$. Computational domain size is $x/a = 6.66$, $y/a = 6.66$ and $z/a = 13.33$.




(a) Migration velocity




(b) Vertical position

Figure 4.15: $(a)$ Migration velocity versus time $(b)$ $z$ component of the centroid of bubbles versus time, for nine-bubble simulation in Figure 4.10. Velocity is scaled by $U_r$, time by $a/U_r$ and $z$ axis by $a$.

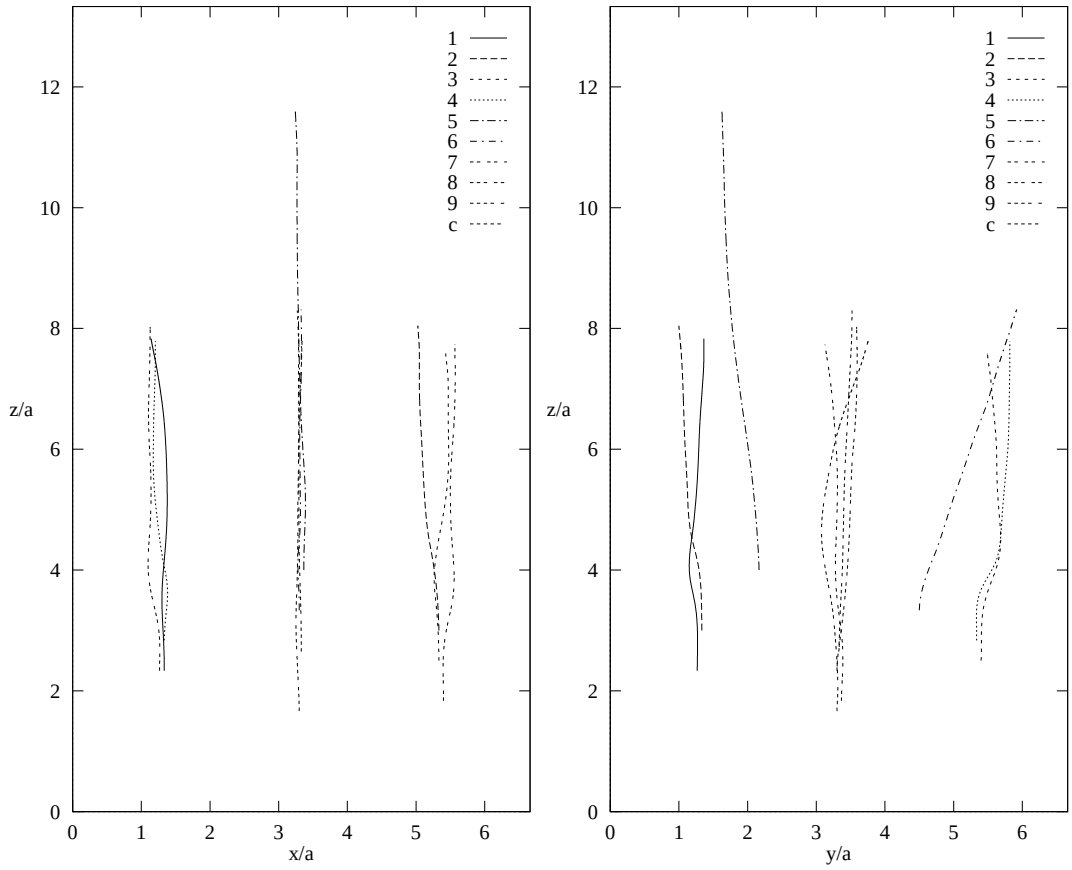



Figure 4.16: ($a$) $x-z$ trajectories of the center of mass of the bubbles (left). ($b$) $y-z$ trajectories of the center of mass of the bubbles (right). Both axis are scaled by the bubble radius $a$.

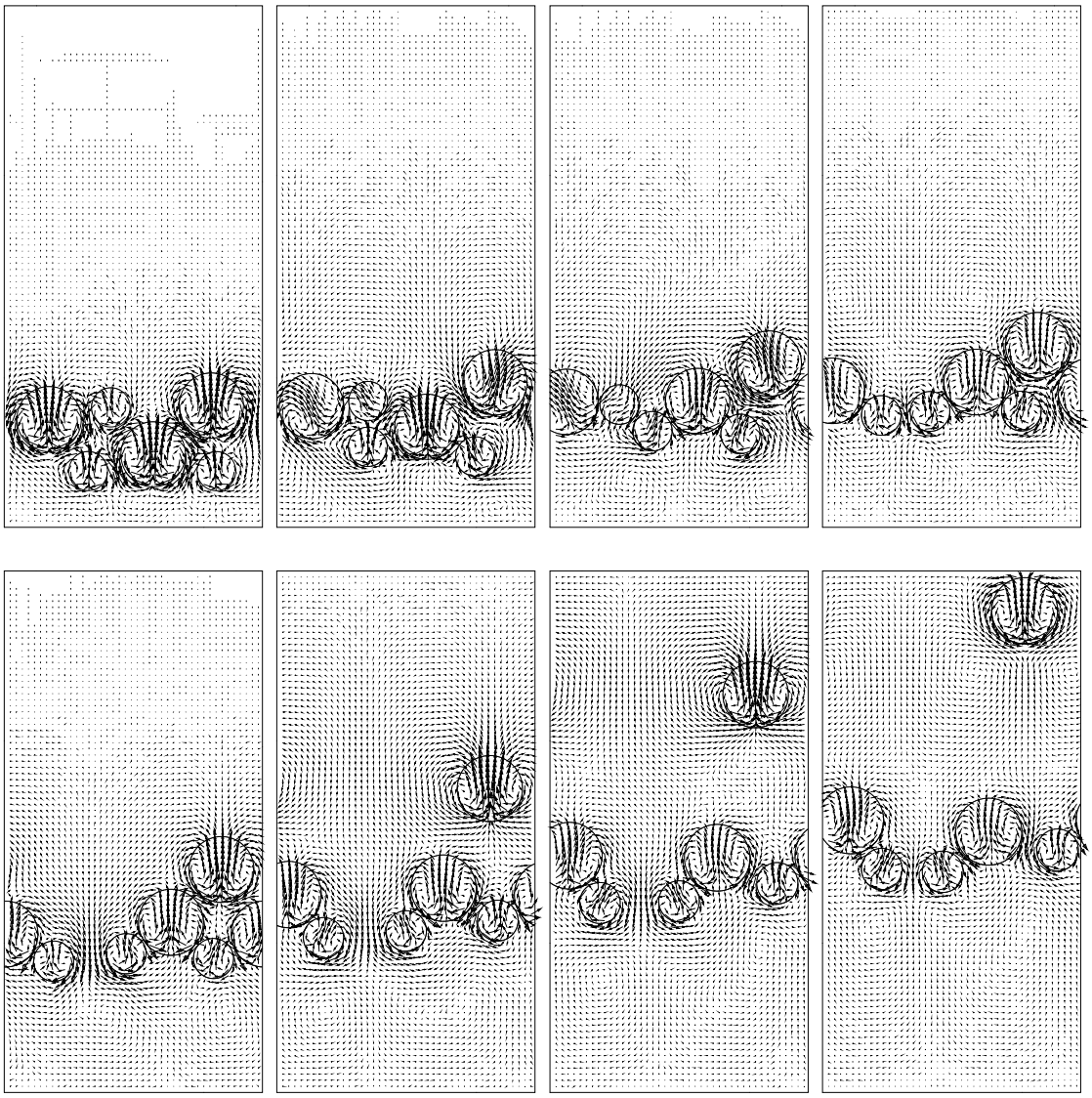


Figure 4.17: Velocity field for selected frames from the computation of six unequal-sized-bubble interaction. Third of the velocity field is shown at every third grid point. Time progresses from left to right. The nondimensional time, $t^*$, is equal to $2.5, 12.5, 25, 40, 62.5, 87.5, 112.5, 137.5$. The nondimensional time is scaled by $a_v/U_{rv}$ and velocity is scaled by reference velocity, $U_{rv}$. Computational domain size is $x/a_v = 10$ and $z/a_v = 20$.

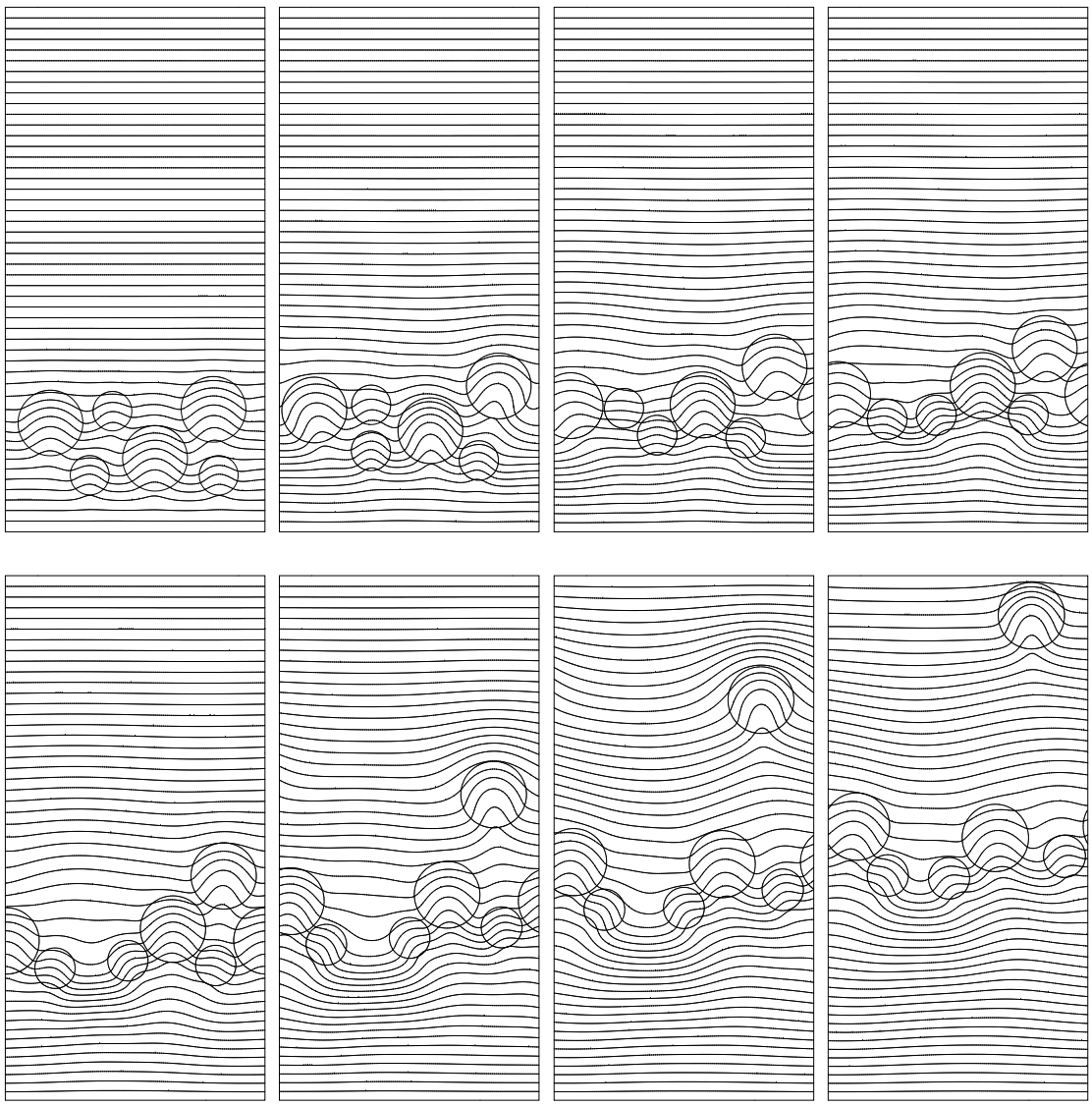
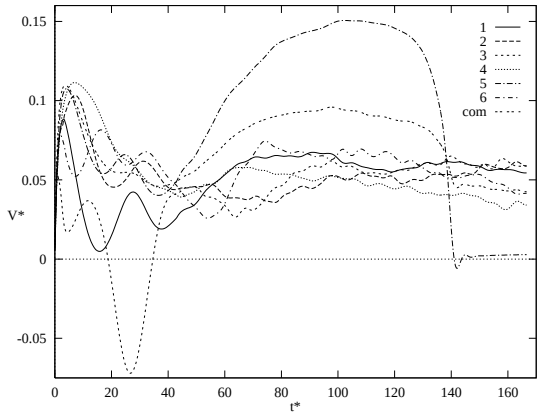
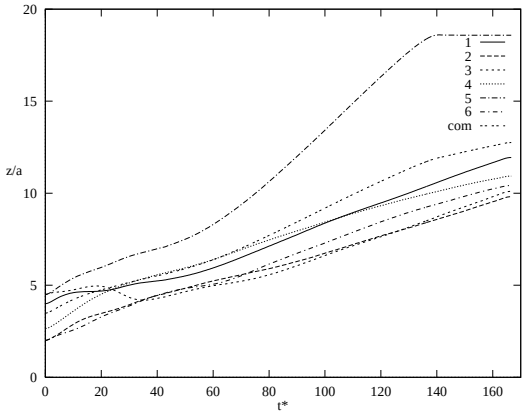


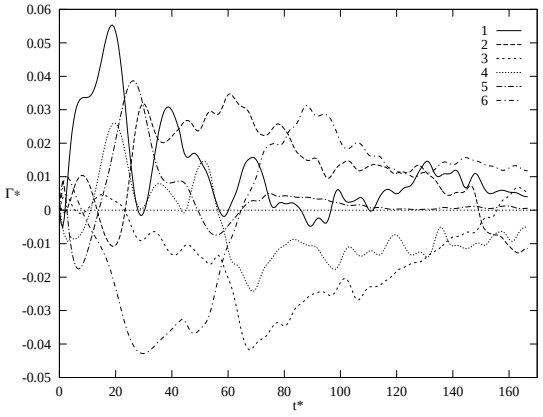
Figure 4.18: Temperature contours for selected frames from the computation of six unequal-sized-bubble interaction. 50 equally spaced contours are shown. Time progresses from left to right. The nondimensional time, $t^*$, is equal to $2.5, 12.5, 25, 40, 62.5, 87.5, 112.5, 137.5$. The nondimensional time is scaled by $a_v/U_{rv}$ and temperature is scaled, after subtracting a reference temperature, by $a_v \nabla T_\infty$. Computational domain size is $x/a_v = 10$ and $z/a_v = 20$.



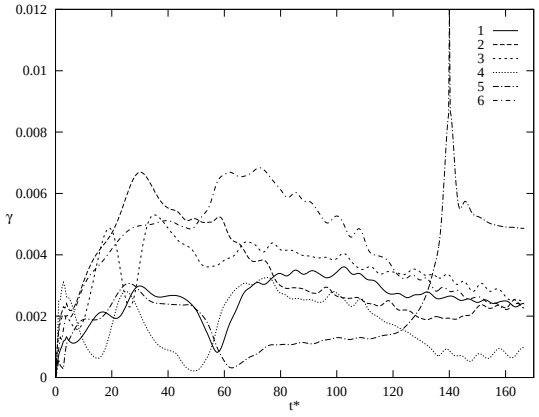

(a) Migration velocity




(b) $z$ component of the centroid of bubbles




(c) Internal circulation




(d) Deformation

Figure 4.19: Quantitative description of the six unequal-sized-bubble interaction in Figures 4.17 and 4.18. ($a$) Migration velocity versus time. ($b$) $z$ component of the centroid of bubbles versus time. ($c$) Internal circulation versus time. ($d$) Deformation versus time. Velocity is scaled by $U_{rv}$, $z$ axis by the averaged bubble radius, $a_v$, time by $a_v/U_{rv}$, and the internal circulation by $2\pi a_v U_{rv}$. Bubbles marked 1 to 6 from left to right in the domain. Com is the velocity of the center of mass.

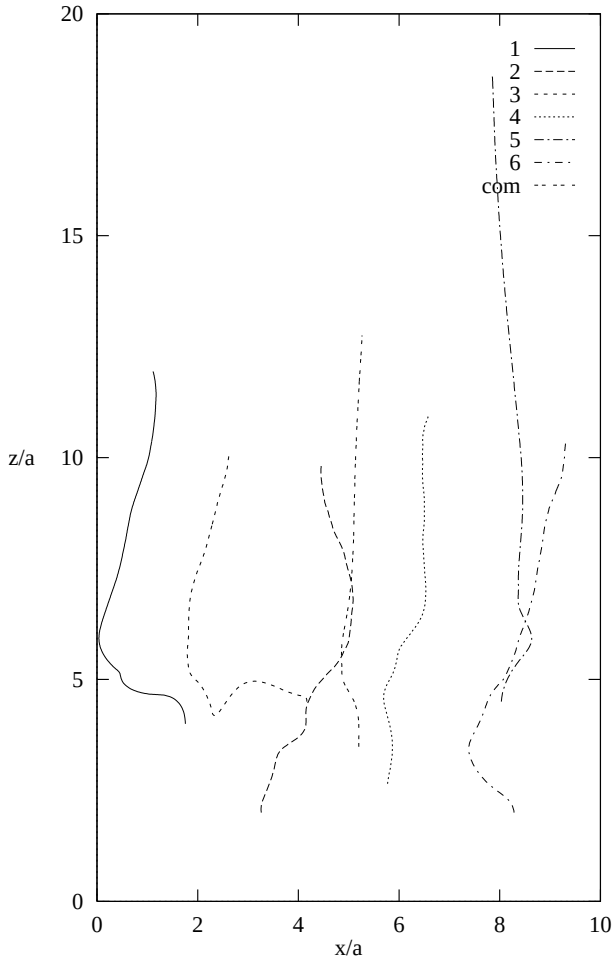



Figure 4.20: Trajectories of the center of mass of the bubbles. Both axis are scaled by the average bubble radius $a_v$.

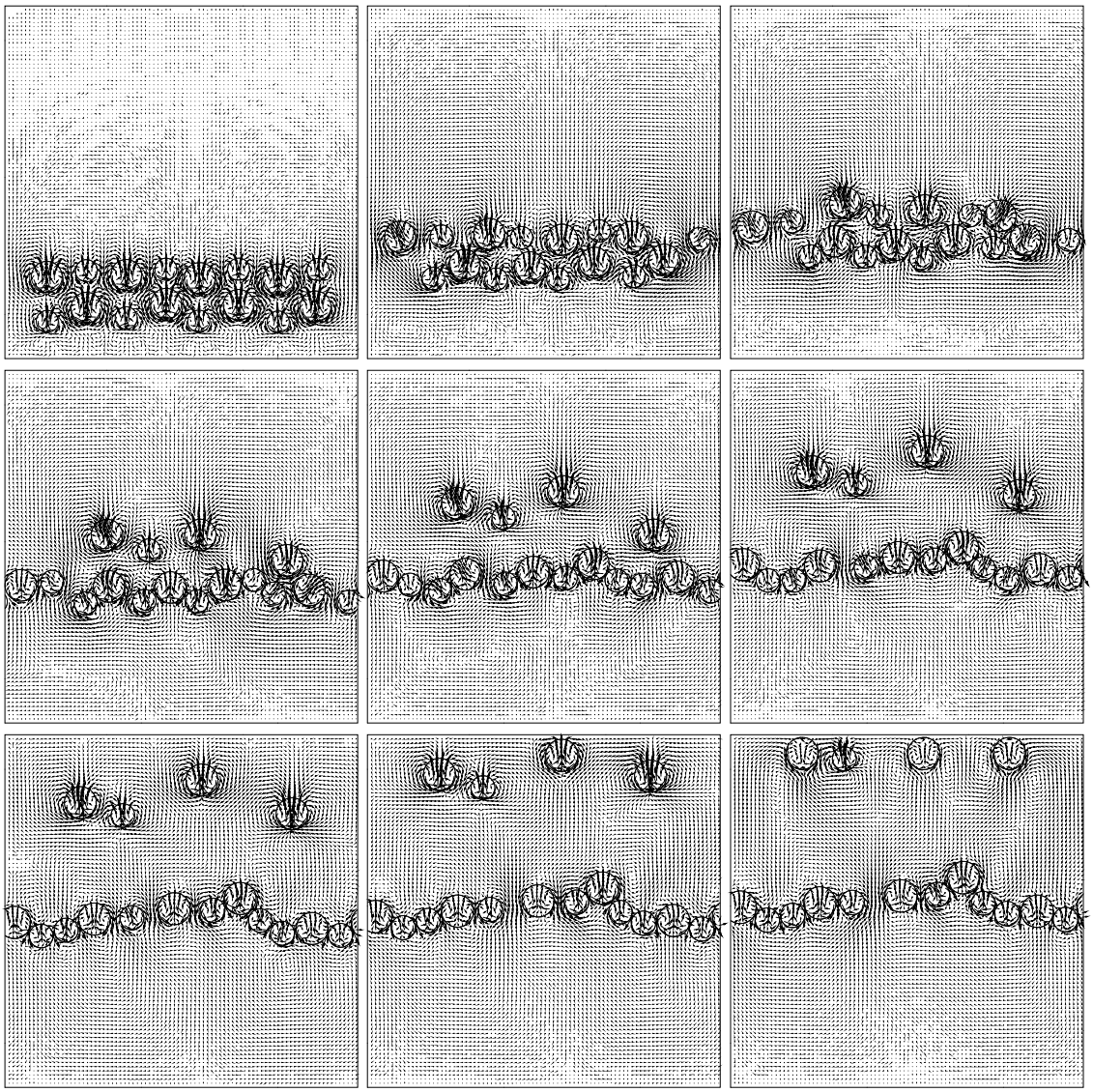
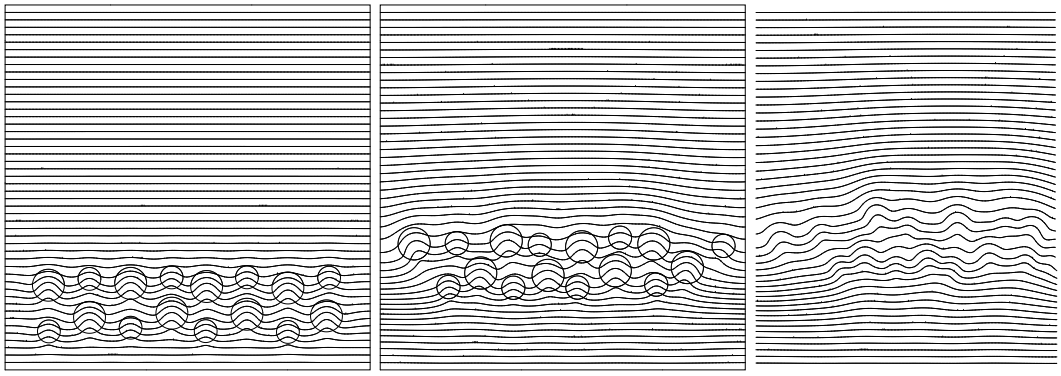
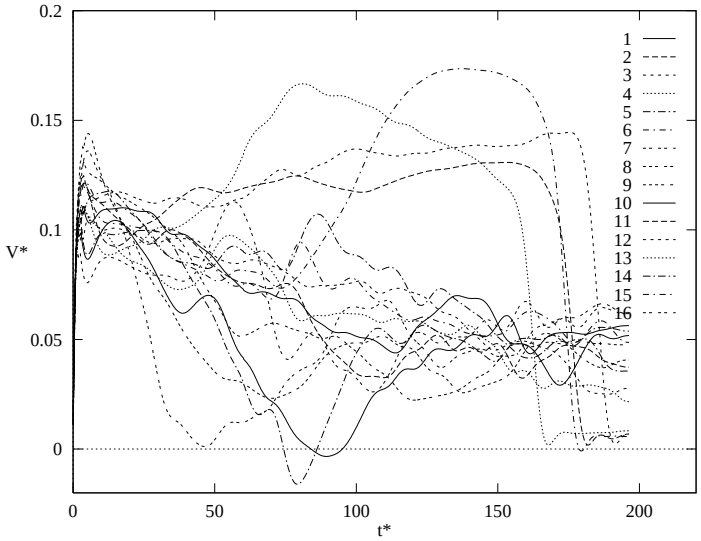
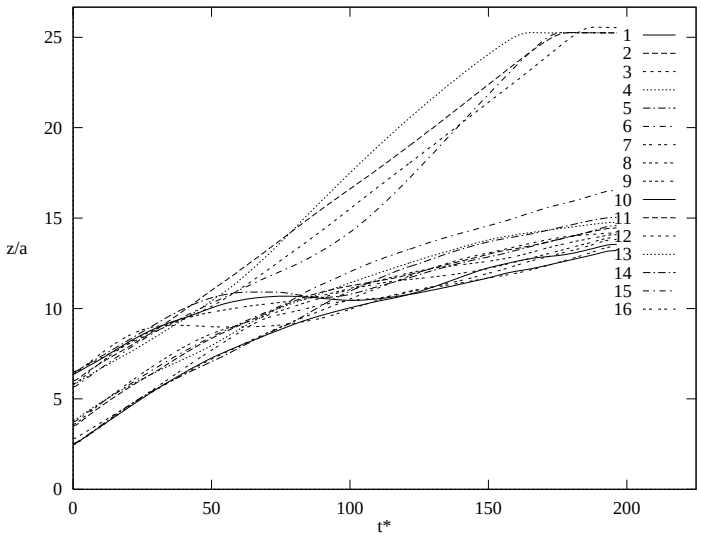


Figure 4.21: Velocity field for selected frames from the simulation of 16 unequal-sized bubble interaction. The velocity field is shown at every third grid point. Time progresses from left to right, top to bottom. The nondimensional time, $t^*$, is equal to 4, 40.5, 64.7, 89, 113.3, 137.6, 161.9, 182, 202.4. The nondimensional time is scaled by $a_v/U_{rv}$ and velocity by $U_{rv}$. Computational domain size is $x/a_v = 13.33$ and $z/a_v = 13.33$.






(a) Migration velocity




(b) $z$ component of the centroid of bubbles

Figure 4.23: $(a)$ Migration velocity versus time $(b)$ $z$ component of the centroid of bubbles versus time for 16 unequal-sized bubble interaction in Figures 4.21 and 4.22. Velocity is scaled by $U_{rv}$, time by $a_v/U_{rv}$ and $z$ axis by $a_v$.

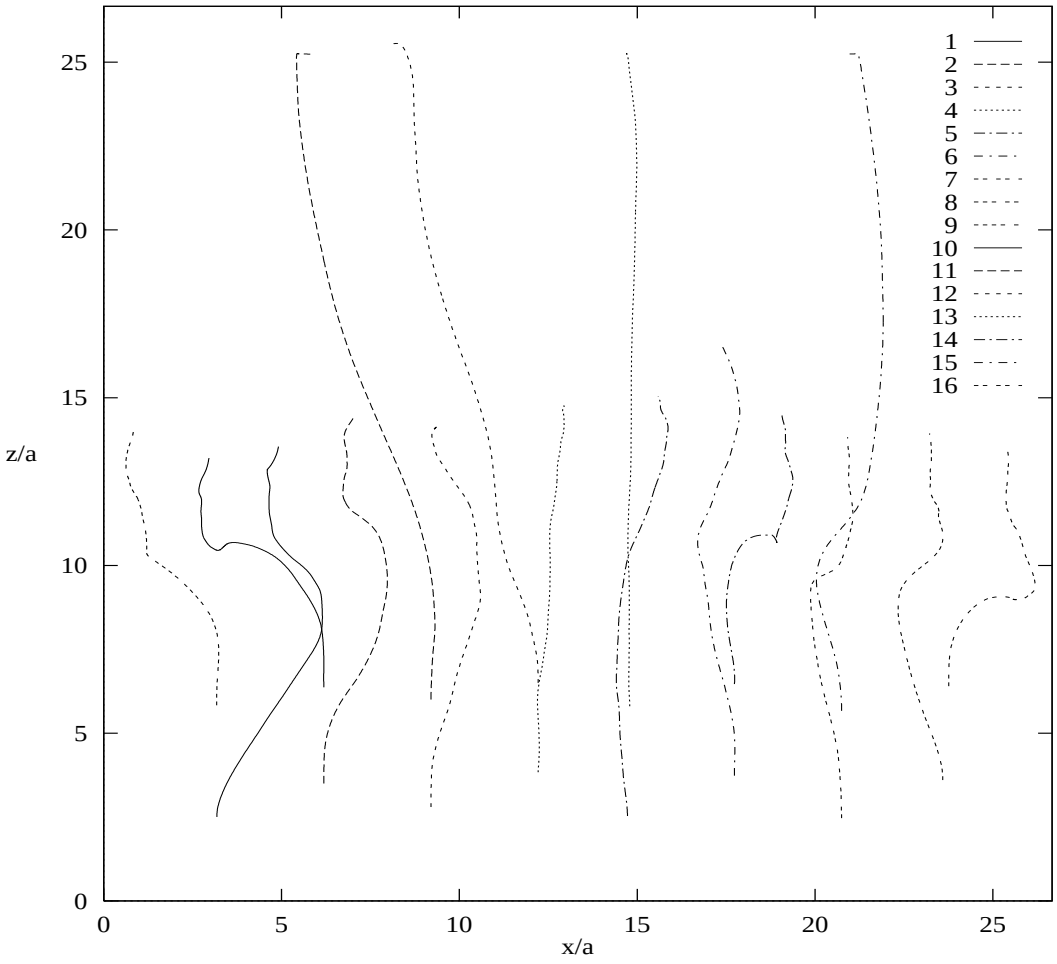



Figure 4.24: Trajectories of the center of mass of the bubbles. Both axis are scaled by the average bubble radius $a_v$.

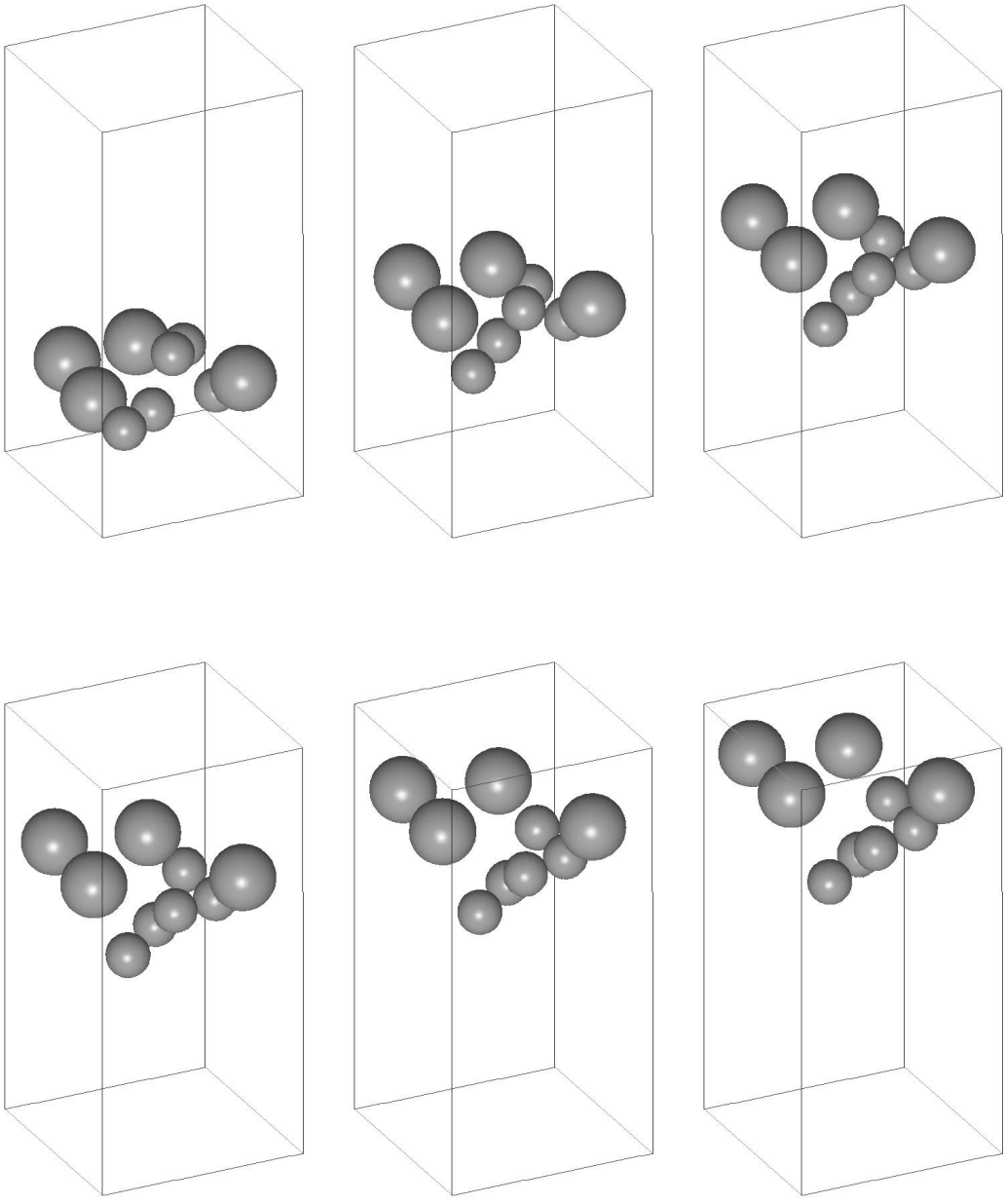


Figure 4.25: A fully three-dimensional simulation of the interaction of nine unequal-sized bubbles. Time progresses from left to right, top to bottom. The nondimensional time, $t^*$, is equal to 6.66, 40, 66.66, 80, 100, 114.2. The nondimensional time is scaled by $a_v/U_{rv}$ and velocity is scaled by average reference velocity, $U_{rv}$. Computational domain size is $x/a_v = 8$, $y/a_v = 8$ and $z/a_v = 16$.

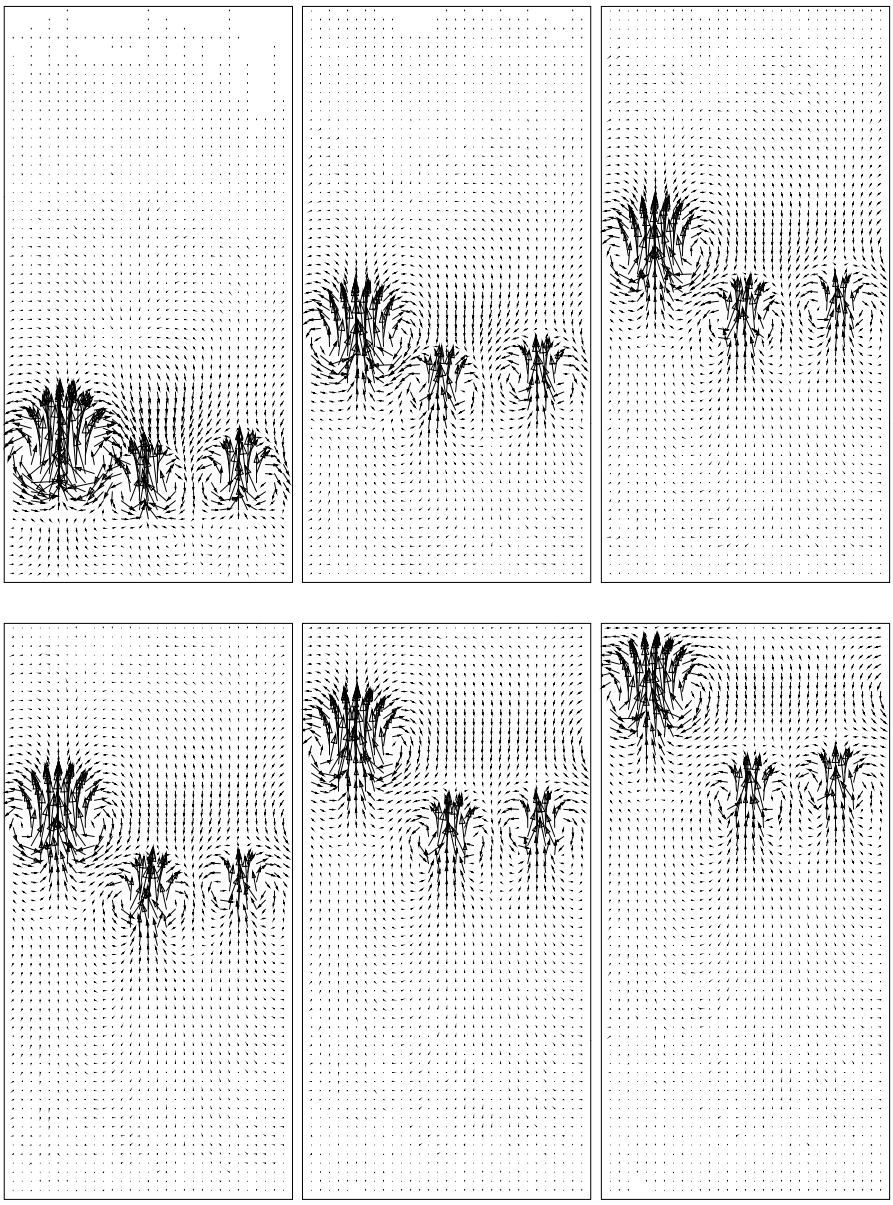


Figure 4.26: Velocity field for selected frames from the fully three dimensional simulation of nine unequal-sized bubble interaction. The velocity field is shown at every other grid point in the middle plane of the computational box in $y$ direction. Time progresses from left to right. The nondimensional time, $t^*$, is equal to 6.66, 40, 66.66, 80, 100, 114.2. The nondimensional time is scaled by $a_v/U_{rv}$ and velocity is scaled by average reference velocity, $U_{rv}$. Computational domain size is $x/a_v = 8$, $y/a_v = 8$ and $z/a_v = 16$.

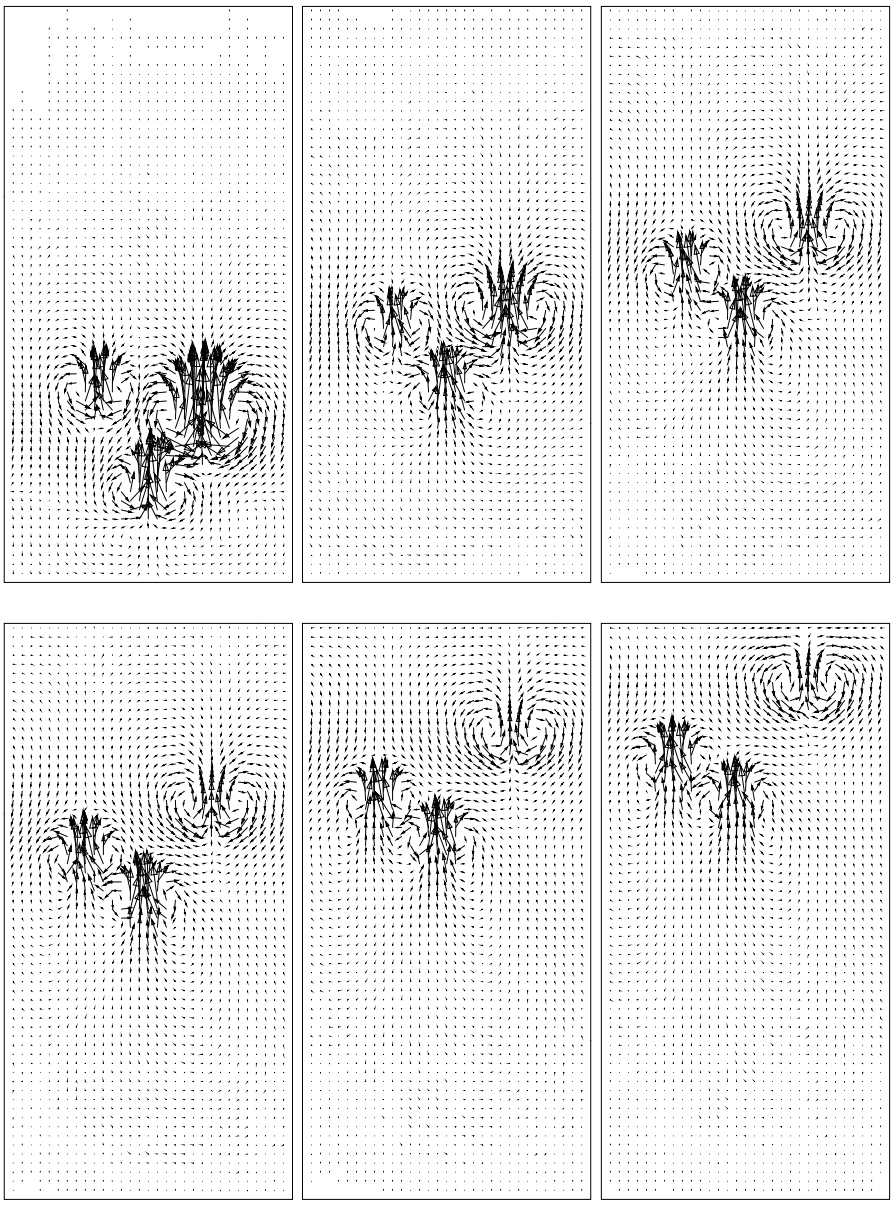


Figure 4.27: Velocity field for selected frames from the fully three dimensional simulation of nine unequal-sized bubble interaction. The velocity field is shown at every other grid point in the middle plane of the computational box in $x$ direction. Time progresses from left to right. The nondimensional time, $t^*$, is equal to 6.66, 40, 66.66, 80, 100, 114.2. The nondimensional time is scaled by $a_v/U_{rv}$ and velocity is scaled by average reference velocity, $U_{rv}$. Computational domain size is $x/a_v = 8$, $y/a_v = 8$ and $z/a_v = 16$.

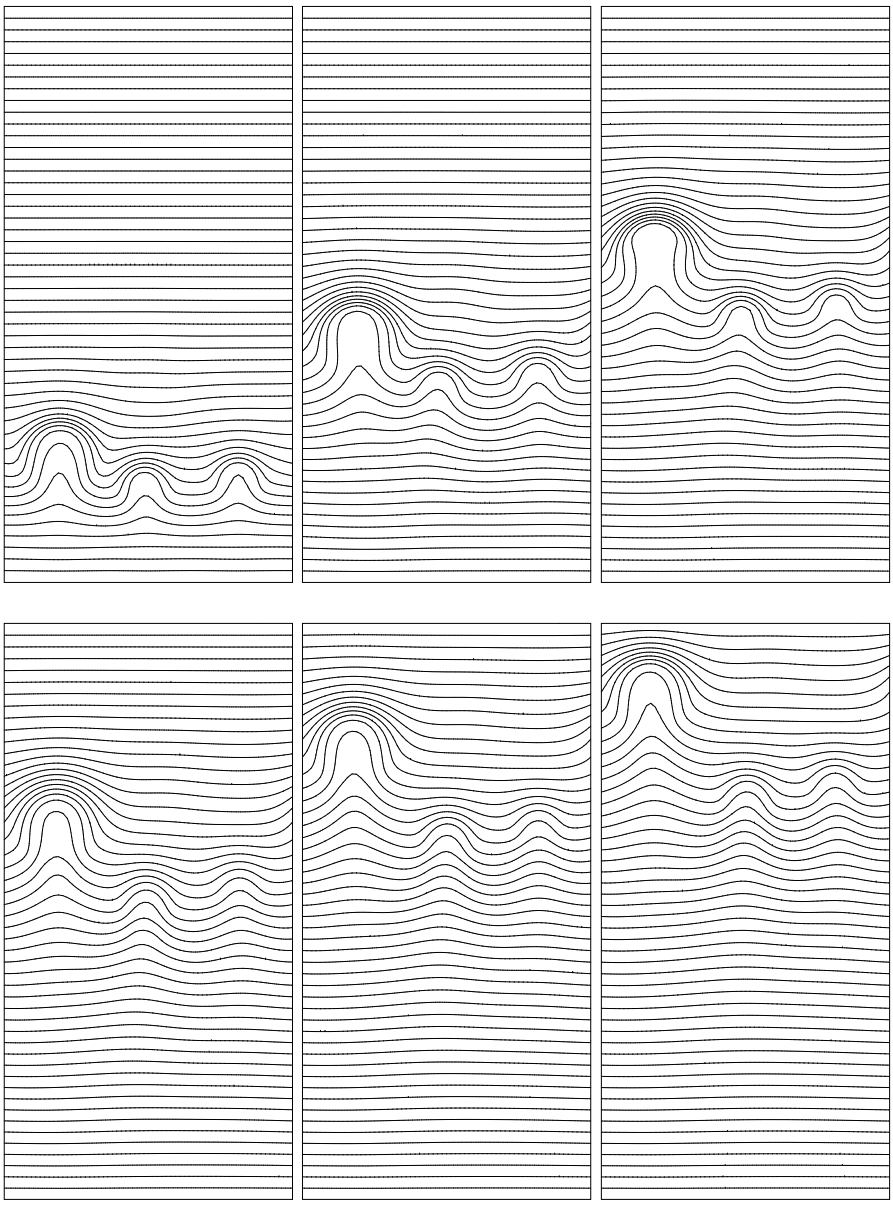


Figure 4.28: Temperature contours for selected frames from the fully three dimensional simulation of nine unequal-sized bubble interaction. 50 equally spaced contours are shown in the middle plane of the computational box in $y$ direction. Time progresses from left to right. The nondimensional time, $t^*$, is equal to 6.66, 40, 66.66, 80, 100, 114.2. The nondimensional time is scaled by $a_v/U_{rv}$ and temperature is scaled, after subtracting a reference temperature, by $a_v \nabla T_\infty$. Computational domain size is $x/a_v = 8$, $y/a_v = 8$ and $z/a_v = 16$.

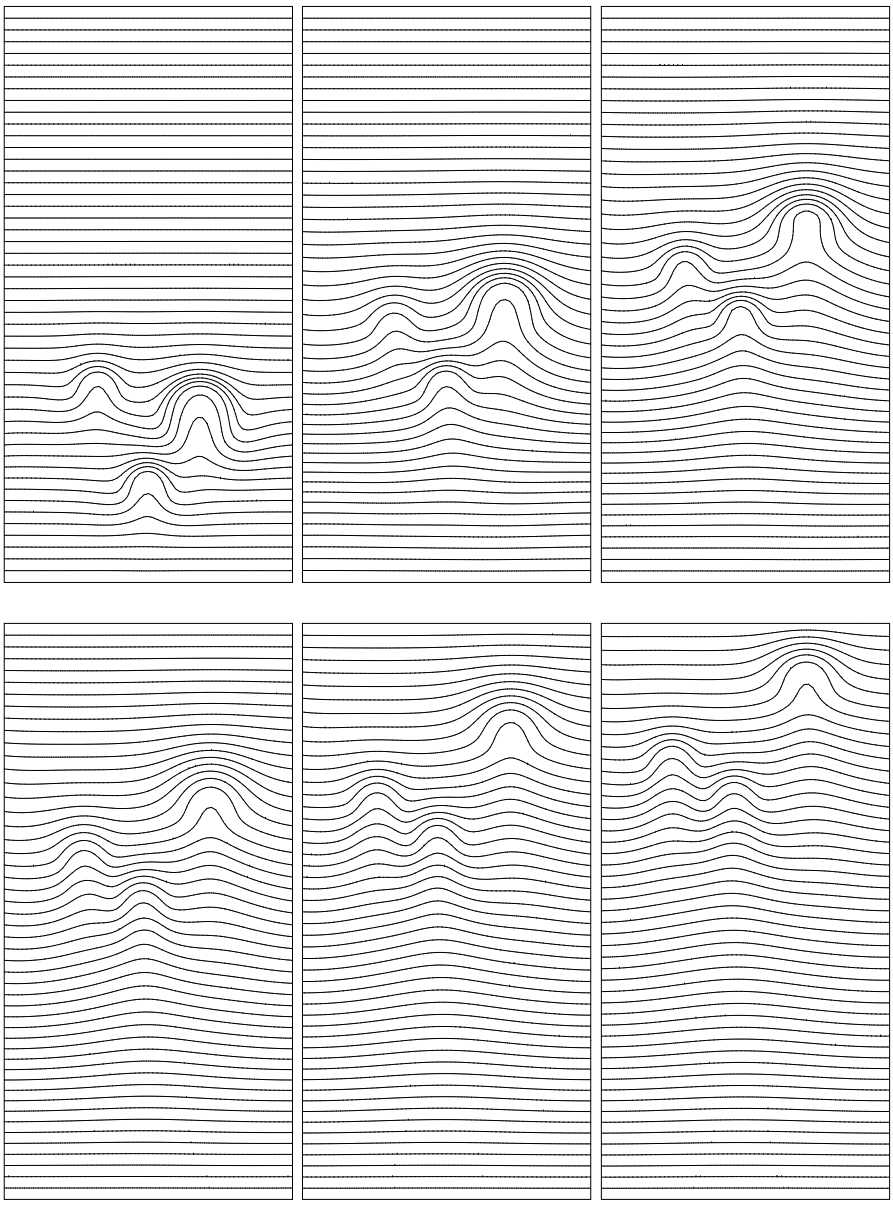
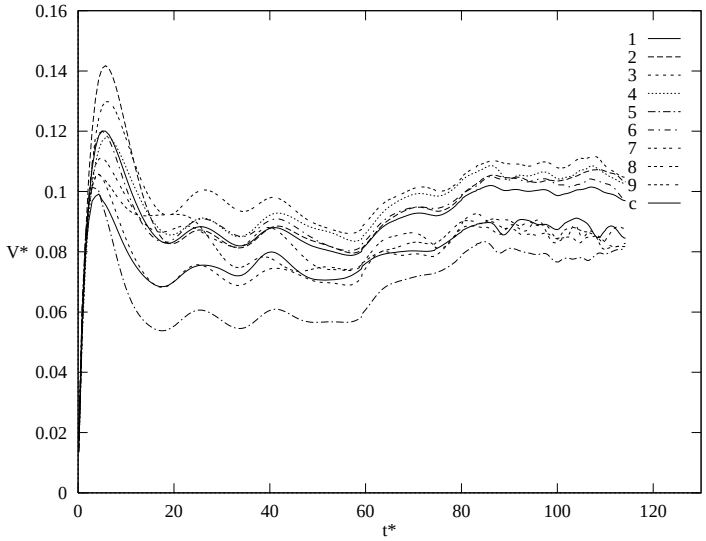
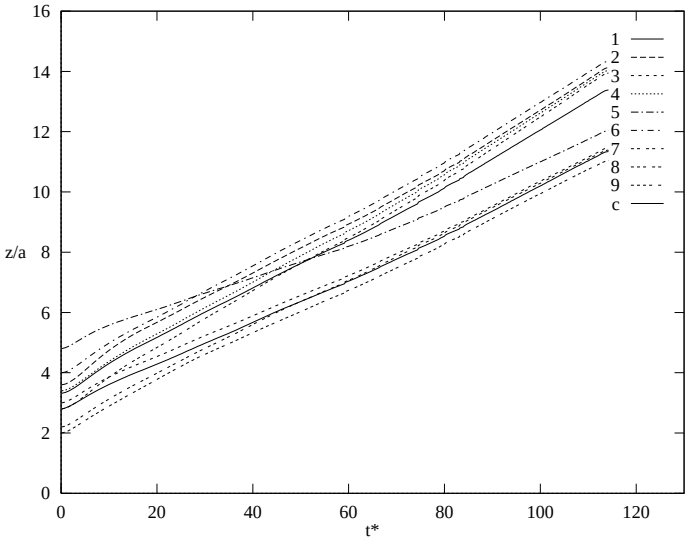


Figure 4.29: Temperature contours for selected frames from the fully three dimensional simulation of nine unequal-sized bubble interaction. 50 equally spaced contours are shown in the middle plane of the computational box in $x$ direction. Time progresses from left to right. The nondimensional time, $t^*$, is equal to 6.66, 40, 66.66, 80, 100, 114.2. The nondimensional time is scaled by $a_v/U_{rv}$ and temperature is scaled, after subtracting a reference temperature, by $a_v \nabla T_\infty$. Computational domain size is $x/a_v = 8$, $y/a_v = 8$ and $z/a_v = 16$.




(a) Migration velocity




(b) Vertical position

Figure 4.30: ($a$) Migration velocity versus time ($b$) $z$ component of the centroid of bubbles versus time, for 9 unequal-sized-bubble simulation. Velocity is scaled by $U_{rv}$, time by $a_v/U_{rv}$ and $z$ axis by average bubble radius, $a_v$. Even numbered bubbles are the larger ones.

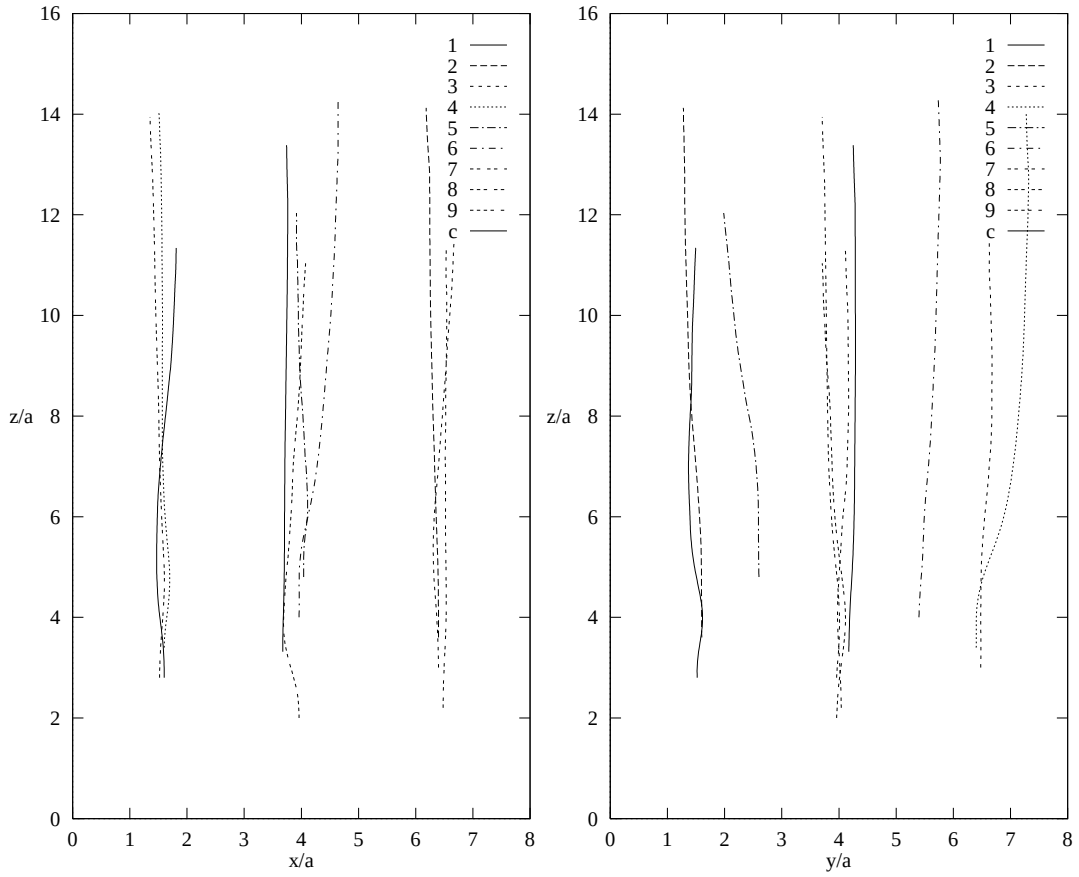



Figure 4.31: $(a)$ $x-z$ trajectories of the center of mass of the bubbles (left). $(b)$ $y-z$ trajectories of the center of mass of the bubbles (right). Both axis are scaled by the average bubble radius, $a_v$.