CHAPTER I

INTRODUCTION

1.1 Motivation

Bubbles and drops in an ambient fluid with a temperature gradient will move toward the hot region due to thermocapillary forces. Surface tension decreases with increasing temperature and the nonuniform surface tension of the bubble surface causes shear stresses that are transmitted to the outer fluid by viscous forces, thus inducing a motion of the bubble in the direction of the thermal gradient. In space, where buoyancy forces are negligible, thermocapillary forces can be dominant. For material processing in microgravity, thermal migration can be used, for example, to remove gas bubbles or liquid drops in melts before solidification. It has been suggested that it is possible to produce high quality glass in space because of the ability to process it without a container (Uhlmann (1982)). However, bubbles due to chemical reactions have to be removed to achieve this. Thermocapillary migration could provide a way to do this in the absence of gravity. Vapor bubbles may also form due to evaporation in liquid-rockets which are used to power space vehicles and space probes (Ostrach (1982)). Similarly, the cooling system of the Space Shuttle uses liquids from which gas bubbles might form. The bubbles in these systems should be managed in order to have a properly working systems. Thermocapillary migration can also be important in the design of two-phase heat exchangers for space applications. Accumulations of bubbles on heated surfaces can act like an insulator and prevent heat transfer to the surface. To understand the interaction of many bubbles during thermocapillary induced motion and to investigate the effect of the various governing parameters, here we solve the full Navier-Stokes equations and the energy equation for both fluids, computationally.

1.2 Historical Background

In this section, we review some of the previous experimental, theoretical, and numerical investigations that have been carried out on thermocapillary migration of bubbles and drops.

Thermal migration of gas bubbles was first examined by Young *et al.* (1959), both theoretically and experimentally. Young *et al.* were able to hold a buoyant gas bubble stationary by applying a downward temperature gradient, found a first order approximation to the terminal velocity in the limit of negligible convective transport of momentum and energy, under the assumption that the bubble maintains a spherical shape. They also verified that the temperature gradient required to hold a bubble stationary was proportional to the bubble radius and that this gradient is independent of viscosity as predicted by their theoretical model.

Hardy (1979) performed experiments similar to Young *et al.* He reduced some of the experimental inaccuracies in the experiments of Young *et al.* and also obtained the temperature gradient needed to counter the buoyancy effect. His results were in good agreement with the theoretical predictions of Young *et al.*

Thompson (1979) and Thompson *et al.* (1980) used a NASA zero gravity drop tower to do experiments in 5.2 seconds of free fall. They used nitrogen bubbles in different host fluids. The results showed that the thermocapillary motion of bubbles occured in each test fluid except distilled water. Thompson suggested that the phenomena did not occur in distilled water due to surfactant contamination.

Meritt and Subramanian (1988) performed experiments on bubbles in silicon oil in a downward temperature gradient, overcoming the buoyant rise of the bubble. Although the bubbles in those experiments increased in size by as much as 100% during the experiment, due to the mass transfer from the liquid, their results showed good agreement with the work of Young *et al.* The surface tension, which was extracted from the experiments, was also in good agreement with earlier measurements of Hardy (1979).

Experimental investigations of the thermocapillary migration of liquid drops is a more recent field of research than work using gas bubbles. Wozniak and Siekman (1989) carried out a low gravity thermocapillary migration experiment on liquid drops in the European sounding rocket program. They reported that for high Reynolds (Re) and Marangoni (Ma) numbers, the measured migration velocities were close to the prediction given by Subramanian (1983). For intermediate non-dimensional numbers, however, they observed larger deviations from the theoretical model of Subramanian (1983).

Barton and Subramanian (1989) have completed measurements of drop migration speeds in circumstances where convective transport effects were negligible. These results are generally in agreement with the predictions of Young *et al.*

Rashidna and Balasubramaniam (1991) carried out experiments on drops in silicon oil using density matched systems. Drops moving towards the hot region in a vertical temperature gradient were observed. After a long time, however, drops began moving towards the cooler side. They attributed this surprising behavior to mass transfer between the phases, causing the drops to become more dense than the host-fluid.

Bratukhin (1976) derived an analytical solution for the thermocapillary flow of bubbles and drops based on a power series expansion in terms of the Marangoni number, and found the particle velocity, the fluid velocity, the temperature field, and the pressure field by using an Oseen approximation. Thus, his results are valid for small Reynolds numbers, to $O(Re)$. In zero gravity, and in the limit of zero Reynolds number, his formula is the same as the one found by Young *et al.*

Later, Subramanian (1981) obtained the migration velocity of a gas bubble for small, but nonzero convective heat transfer by using asymptotic expansion technique. In his analysis, it is assumed that the bubble is nondeformable and that the Reynolds number is small enough to be taken as zero. The migration velocity of the bubble is given up to $O(Ma^2)$. As the results show, the effect of convective transport of energy is to reduce the migration velocity of bubbles. Subramanian (1983) later extended his work to liquid drops. In the proper limit, his results give the correct value of the migration velocity for gas bubbles. For certain physical properties, it was shown that the drop velocity can be higher with increasing Marangoni number; in contrast, for bubbles, the effect is always a reduction in the migration velocity.

Balasubramaniam and Chai (1987) have given an exact solution for the migration velocity of a single drop in the limit of negligible convective transport of energy. They also computed the shape of the droplet, when deformations from a spherical shape are small. Their results are in agreement with previous results, such that the bubbles tend to deform oblately, and that droplets tend to elongate in the flow direction while droplets of the same density as the ambient fluid do not deform.

Shankar and Subramanian (1988) reconsidered the thermocapillary migration of a gas bubble in the limit of a zero Reynolds number at values of the Marangoni number ranging from 0 to 200, solving the energy equation by a finite difference method. They confirmed that increasing Marangoni number decreases the migration velocity of a gas bubble and presented a very simple formula for the bubble migration velocity for $Ma > 25$, by fitting their numerical solution.

Siekman and Szymczyk (1988) numerically solved the thermocapillary motion for a gas bubble, accounting fully for the convective transport of energy and momentum while assuming a nondeformable bubble. Their results show that convective energy and momentum transport effects tend to reduce the bubble migration velocity.

Balasubramaniam and Lavery (1989), extended the work of Siekman and Szymczyk (1988) and, for a large range of non-dimensional numbers, numerically solved the problem for an isolated axisymmetric spherical bubble. They found that the scaled bubble velocity is more sensitive to the Marangoni number at a fixed Reynolds number than to the Reynolds number at a fixed Marangoni number.

Haj-hariri *et al.* (1990) have examined the inertial effects on the thermocapillary velocity of a drop. It was found that with the convective transfer of heat neglected, droplets with densities higher/lower than the outside liquid deform to prolate/oblate spheroidal shapes, at small values of the Capillary and Reynolds numbers. It was shown that the migration velocity could increase, decrease, or remain unchanged depending on the ratios of physical properties.

Chen and Lee (1992) investigated numerically the effect of surface deformation on the terminal velocity of a single bubble and concluded that surface deformation reduces the terminal velocity considerably.

Other investigators have examined the thermocapillary motion for two bubbles or drops, but only for small Marangoni and Reynolds numbers. Meyyapan *et al.* (1983)

investigated the motion of two bubbles moving along their line of centers. They found that each bubble moves with the same velocity that it has if it is isolated. Their analysis also assumed that convective transport of energy and momentum is sufficiently small so it can be neglected and that the bubbles do not deform. When the bubbles differed in size, the smaller bubble moved faster than if isolated while the larger bubble moved slower. However, these interaction effects generally were small.

Meyyapan and Subramanian (1984) extended the analysis of Meyyapan *et al.* (1983) to the motion of two bubbles oriented arbitrarily with respect to the temperature gradient, using an approximate method. They found that a small bubble does not affect the motion of a larger bubble to any significant degree. It was further shown that if two bubbles are close and oriented with their line of centers perpendicular to the temperature gradient, the small bubble sometimes moves opposite to the direction of the temperature gradient.

Feuillebois (1989) has given an exact solution for the problem considered by Meyyapan *et al.* (1983).

The motion of two liquid drops oriented arbitrarily with respect to a temperature gradient was examined theoretically by Anderson (1985) in zero Reynolds and Marangoni number limit. In his analysis, where the method of reflections was used to solve the governing equations, the convective transport of energy and momentum as well as gravitational effects were neglected. He also utilized his two-drop theory to find the effect of the volume fraction of the drops on the mean drop velocity in a bounded suspension. It is shown that the mean velocity of a suspension is lower than for a single drop.

Acrivos *et al.* (1990) have studied the thermocapillary motion induced in a cloud of bubbles by a uniform temperature gradient under the assumptions that the bubbles are all the same size, that the surface tension is high enough to keep the bubbles spherical, and that the bubbles are non-conducting. It was shown that in a cloud of n particles surrounded by an infinite expanse of fluid, the velocity of each sphere under creeping flow conditions is equal to the velocity of an isolated particle, unchanged by interactions between particles.

Keh and Chen (1990) considered the axisymmetric thermocapillary motion of two spherical droplets in a constant applied temperature gradient along their line of centers under creeping flow conditions. It was shown that for the thermocapillary motion of two identical liquid droplets, both migrate faster than the velocity they would posses if isolated. For the case of two gas bubbles with equal radii, there was no particle interaction for all separation distance.

Keh and Chen (1992) investigated the axisymmetric thermocapillary motion of a chain of spherical droplets along their line of centers in a quasi-steady limit of conservation of energy and momentum by a combined analytical-numerical study. For the case of two droplets, the migration velocity of each drop were confirmed. For the special case of multiple gas bubbles, it was demonstrated that the migration velocity of each bubble is unaffected by the presence of the others if all the bubbles have identical radii.

Zhang and Davis (1992) examined the pairwise collision rate of small spherical drops undergoing thermocapillary migration in a dilute dispersion under creeping flow conditions by using a trajectory analysis. It was found that increases in the viscosity and/or thermal conductivity of the drop fluid decrease the collision efficiency, described as the effects of the drop interaction on collision rate, due to the effects of hydrodynamic and thermocapillary interactions.

Strape (1992) analytically examined the interaction between bubbles in the zero Marangoni and Reynolds number case. He also assumed that the Capillary number is negligible so that the bubbles are always spherical. He has given the trajectories for the two-bubble case in these limits. He also found that for a statistically homogeneous cloud of bubbles, the bubble collision rate increases with the standard deviation of the bubble size distribution.

Wei and Subramanian (1993) theoretically investigated the quasi-static thermocapillary migration of a chain of two and three spherical bubbles in an unbounded fluid with a uniform temperature gradient, at the limit of vanishing Reynolds and Marangoni number. They explored the flow topology and identified reverse flow wakes.

Keh and Chen (1993) considered the more general problem of droplet interactions in thermocapillary migration. They also solved this general problem in the limit of zero Reynolds and Marangoni numbers and showed that the terminal velocity of a drop is not affected by the presence of other drops if they all are equal in size. They have also examined the effect of volume fractions on the average thermocapillary migration velocities in a bounded dilute suspension.

The interaction between bubbles and drops and plane surfaces has been the subject of other investigations. Meyyapan *et al.* (1981) investigated theoretically the slow axisymmetric thermocapillary migration of a spherical gas bubble normal to a solid plane surface and a free liquid surface. Their calculation showed that the effect of the plane surface is to decrease the bubble's migration velocity. They also demonstrated that the distance at which the bubble starts to be affected by the presence of the plane surface is much smaller than for a gravity driven motion. They explained this behavior in terms of the decay rates of the disturbance velocity fields.

A gas bubble in a temperature gradient with an arbitrary orientation with respect to the plane surface, was studied by Meyyapan and Subramanian (1987), extending the work of Meyyapan *et al.* (1981). Their results show that the presence of the planar surface always reduces the migration velocity of the bubble. The highest reduction is observed when the bubble moves normal to the plane surface.

Ascoli and Leal (1990) considered the thermocapillary migration of a deformable drop moving normal to a planar wall and found that the deformation increases with increasing effective Capillary number.

Chen and Keh (1990) examined the migration of a drop towards a planar surface under creeping flow conditions. It was found that for the motion of a droplet normal to a solid plane, the effect of the plane surface is to reduce the migration velocity of the droplet. For the case of droplet migrating toward a free surface, the droplet velocity can be either greater or smaller than that which would exist in the absence of the plane surface, depending on the relative thermal conductivity of the droplet and its relative distance from the plane.

Chen *et al.* (1991) examined the steady, creeping, thermocapillary migration of a spherical fluid particle in a tube owing to an imposed axial temperature gradient under conditions of axisymmetry, negligible thermal convection and an insulated tube wall. They studied the influence of wall-fluid particle hydrodynamic and thermal interactions in determining the thermocapillary migration velocity.

1.3 Current Work

The literature on thermal migration is more extensive for single bubbles or drops than for the interaction of many bubbles and drops. In most previous work, it has been assumed that the bubbles do not deform and that convective transfer can be neglected. Here, we present results for both single and several bubbles and drops by solving the full governing equations numerically in two and three dimensions. In our computations, we do not impose any restriction on the shape of the bubbles, although we find that the bubbles remain nearly spherical in most cases.

The full Navier-Stokes equations, as well as the energy equation for the temperature distribution, are solved for the fluid inside and outside of the bubbles by a Front Tracking/Finite Difference Method. The material properties of the bubble fluid and the ambient fluid are different, and we assume temperature dependent surface tension. We explore the dependence of the thermal migration velocity and the deformation on the various non-dimensional parameters.

The definition of the physical problem and the mathematical formulation, as well as the governing parameters, are covered in Chapter II. Chapter II also includes a description of the numerical method used to solve the governing equations and the validation of the two and three dimensional code.

In Chapter III, we start with the rise of a single bubble and present the effect of the various governing parameters on the migration velocity and the deformation of the bubble. Then, we move to the interaction of two, two-dimensional bubbles and study the effect of the initial condition of bubbles on their interaction. The interaction between two bubbles or two drops is then explored in detail by two and three dimensional simulations in the rest of Chapter III.

We present the behavior of large numbers of bubble systems in Chapter IV. In the first part of Chapter IV, we consider simulation of a cloud of equal-sized bubbles. The evolution of six and sixteen equal-sized bubbles is explored by two-dimensional simulation. Then, the interactions of nine bubbles is investigated by fully three-dimensional simulations. Similar to the mono-dispersed case, the behavior of a polydispersed system is explored in the rest of Chapter IV. First, the interaction of six and sixteen unequal-sized two-dimensional bubbles is presented, followed by fully three-dimensional computation of nine, unequal-sized bubbles.

Chapter V contains the conclusions, and suggestions for future work on the thermocapillary migration of bubbles and drops.