A METHOD TO GUARANTEE THE CONTINUITY OF GPS IN URBAN ENVIRONMENTS

Fatih Çenesiz[1],Halis Saka[2],Selman Nas[3],Cihangir Özşamlı[4]

[1]Department of Computer Engineering,Gebze Institue of Technology, Kocaeli, Turkey
[2]Department of Geodesy and Fotogrametry, Gebze Institue of Technology, Kocaeli, Turkey
[3]Department of Astronoutical Engineering, Istanbul Tecnical University, Istanbul, Turkey
[4]Department of Geodesy and Fotogrametry, Istanbul Tecnical University, Istanbul, Turkey

The Global Positioning System (GPS) is the most famous navigation system in today's transportation world. It supplies any moving object with highly accurate position information in all over the world. But the performance of GPS decreases in urban areas where it is not always possible to track the necessary number of satellites to calculate the 3-D position. This paper outlines the new analytical approaches to overcome the discontinuity problems and as a result to increase the performance of GPS in metropolitan cities. To be able to compute the 4 unknowns of the system (3-D position parameters, and the receiver clock bias) in case 4 satellites are not available, the system is supplied with additional external information.

1.INTRODUCTION

In navigation solution we have four unknowns to be solved. These are the position of the receiver antenna which is generally expressed in Earth Centered Earth Fixed (ECEF) coordinate system, and the receiver clock offset. In order to solve such a system we have to have at least four linear equations. This is possible only if the receiver can get signals from at least four satellites simultaneously. When more than four satellites visible, various statistical techniques can be used to solve the system for the best results. The two famous techniques are Least Square Adjustment (LSA) and Kalman Filtering (KF).

When the situation is reverse, that is less than four satellites are visible, the complete solution can not be achieved for four unknowns. In such a situation two possibilities exist. One is that the receiver can not determine its position and the other is the position of the receiver can still be solved approximately if some assumptions are made using the previous available data.

If the number of visible satellites are three, then one of the unknowns must be known so that the solution can be obtained for other three. While a vehicle is moving on the street the instantenaous changes in altitude are not significant. That means we can fix the altitude (or height) for small time intervals and solve the 3-unknown system. Alternatively using a precise clock and analyzing the different geometries of three satellites an approximate solution can be obtained [M. Sturza, 1983].

If only two satelites are visible, then we should make a further assumption over the 3-satellite situation. If the receiver clock is stable enough (i.e quartz crystal clock), then we can model it for small time intervals using statistical techniques such as Kalman Filter or some extrapolation methods. Then by fixing the altitudes and using this clock model we can solve the remained two unknowns (lattitude and longtitude) that is the 2-D position of the receiver.

2. PROBLEM DEFINITION

In an urban environment, it is not always possible for a moving vehicle to see four or more satellites. This is because of the tall buildings and narrow streets. The Figure 1 shows the situation clearly. The number of satellites visible to the vehicle is directly related with the elevation angle α. Assuming that the vehicle is at the center of the street, α is expressed as **arctan(h/s)**.
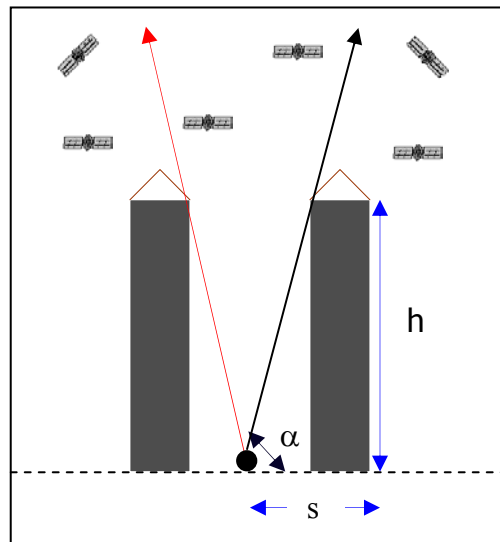
Figure 1: Relation between the elevation angle (α) and the visible satellites.

As the value of α increases, in other words the ratio between the height of buildings and the width of streets gets larger, the number of visible satellites decreases. Such a condition produces a potential problem especially for the Intelligent Transportation Systems (ITS). Table 1 lists the values of the visible satellites for some elevation angles.

Table 1: Elevation Angle and Satellite Visibility (G. Turetzky, 1997)

Elevation Angle	Visible Satellites (min)	Visible Satellites (max)
0 - 10	6	12
10 - 20	5	9
20 - 30	3	7
30 - 40	2	6
40 - 50	1	4
50 - 60	0	4
60 - 70	0	1
70 - 80	0	1
80 - 90	0	1

3. FORMULATIONS

Measured pseudoranges from a satellite to the user receiver is written in Cartesian coordinate system as follows:

$$S_r^s = \sqrt{(x_R - x^S) + (y_R - y^S) + (z_R - z^S)} + cdt \tag{1}$$

where (x^S, y^S, z^S) are the coordinates of the satellite s whic is computed from the ephemeris data, and (x_R, y_R, z_R, dt) are the unknowns for the receiver [E. D. Kaplan, 1996].In order to take some parameters as constraints in the solution this equation should be converted to the geodetic coordinates (φ, λ, h). Conversion equations are given as:

$$x_R = a \cos \lambda_R / \sqrt{1 + (1 - e^2) \tan^2 \varphi_R} + h \cos \lambda_R \cos \varphi_R \tag{2}$$

$$y_R = a \sin \lambda_R / \sqrt{1 + (1 - e^2) \tan^2 \varphi_R} + h \sin \lambda_R \cos \varphi_R \tag{3}$$

$$z_R = a(1 - e^2) \sin \varphi_R / \sqrt{1 - e^2 \sin^2 \varphi_R} + h \sin \varphi_R \tag{4}$$

Substituting (2), (3) and (4) in equation (1) and taking the partial derivatives as

$$\frac{\partial S}{\partial \varphi_R} = \frac{\partial S}{\partial x_R}\frac{\partial x_R}{\partial \varphi_R} + \frac{\partial S}{\partial y_R}\frac{\partial y_R}{\partial \varphi_R} + \frac{\partial S}{\partial z_R}\frac{\partial z_R}{\partial \varphi_R}$$

$$\frac{\partial S}{\partial \lambda_R} = \frac{\partial S}{\partial x_R}\frac{\partial x_R}{\partial \lambda_R} + \frac{\partial S}{\partial y_R}\frac{\partial y_R}{\partial \lambda_R} + \frac{\partial S}{\partial z_R}\frac{\partial z_R}{\partial \lambda_R}$$

$$\frac{\partial S}{\partial h_R} = \frac{\partial S}{\partial x_R}\frac{\partial x_R}{\partial h_R} + \frac{\partial S}{\partial y_R}\frac{\partial y_R}{\partial h_R} + \frac{\partial S}{\partial z_R}\frac{\partial z_R}{\partial h_R}$$

$$\frac{\partial S}{\partial cdt_R} = 1$$

The equation (1) can be linearized by using the Taylor Series expansion. When four measurement from four satellites are done, the coefficient matrix A is takes the following form

$$A = \begin{bmatrix} \dfrac{\partial S^1}{\partial \varphi_R} & \dfrac{\partial S^1}{\partial \lambda_R} & \dfrac{\partial S^1}{\partial h_R} & \dfrac{\partial S^1}{\partial cdt_R} \\[2ex] \dfrac{\partial S^2}{\partial \varphi_R} & \dfrac{\partial S^2}{\partial \lambda_R} & \dfrac{\partial S^2}{\partial h_R} & \dfrac{\partial S^2}{\partial cdt_R} \\[2ex] \dfrac{\partial S^3}{\partial \varphi_R} & \dfrac{\partial S^3}{\partial \lambda_R} & \dfrac{\partial S^3}{\partial h_R} & \dfrac{\partial S^3}{\partial cdt_R} \\[2ex] \dfrac{\partial S^4}{\partial \varphi_R} & \dfrac{\partial S^4}{\partial \lambda_R} & \dfrac{\partial S^4}{\partial h_R} & \dfrac{\partial S^4}{\partial cdt_R} \end{bmatrix}$$

When we apply LSA the covariance matrix is obtained as

$$D_{xx} = \sigma_0^2 \begin{bmatrix} q_{\varphi\varphi} & q_{\varphi\lambda} & q_{\varphi h} & q_{\varphi cdt} \\ q_{\lambda\varphi} & q_{\lambda\lambda} & q_{\lambda h} & q_{\lambda cdt} \\ q_{h\varphi} & q_{h\lambda} & q_{hh} & q_{hcdt} \\ q_{cdt\varphi} & q_{cdt\lambda} & q_{cdth} & q_{cdtcdt} \end{bmatrix}$$

If we have four or more satellites visible we can solve the four unknowns (lattitude,longtitude, height and clock bias). But when the number of satellites decreases to three that is we have

four unknowns and three equations, we should take one of the unknowns as a known value and solve the other triple.

In such a case the height (h) can be taken as the average value of the working environment or can be taken as the last computed value when the more satellites were available.

In this condition the covariance matrix takes the form

$$D_{xx} = \sigma_0^2 \begin{bmatrix} q_{\varphi\varphi} & q_{\varphi\lambda} & q_{\varphi cdt} \\ q_{\lambda\varphi} & q_{\lambda\lambda} & q_{\lambda cdt} \\ q_{cdt\varphi} & q_{cdt\lambda} & q_{cdt} \end{bmatrix}$$

In the case where only two satellites are being visible, further assumption is necessary. Using a precise receiver clock (by 'precise' we mean the clock behaivour of the receiver is accurate enough in time) we can get the solutions for the lattitude and the longtitude.The main problem with the two satellite model is the geometrical distribution of the satellites. In some cases it does not give a reasonable solution.

GPS calculation is a solution naturaly based on a weak geometry. The known points (satellites) are spread over the visinity of sky. Geometrical distribution of the satellites with respect to the receiver employs an important role in solution.The accuracy of the calculated position depends on the solution matrix . This is named as 'Dilutions of Precisions (DOP)'. The DOP factors can be shown as

$$\text{GDOP (Geometric DOP)} \quad = \sqrt{\sigma_\varphi^2 + \sigma_\lambda^2 + \sigma_h^2 + \sigma_{cdt}^2}$$

$$\text{PDOP (Position DOP)} \quad = \sqrt{\sigma_\varphi^2 + \sigma_\lambda^2 + \sigma_h^2}$$

$$\text{HDOP (Horizontal DOP)} \quad = \sqrt{\sigma_\varphi^2 + \sigma_\lambda^2}$$

$$\text{VDOP (Vertical DOP)} \quad = \sigma_h$$

Horizontal positions can be computed more accurately when the height and/or time is constrained. Such a constraint can be useful under poor geometry. Height can be constrained using an external measurement unit such as a barometer [G. Lachapelle,1996], or using an topographic surface model of the working area.

4. APPLICATION AND RESULTS

In the application we used the data collected by Astech Z12 GPS receiver for 40 epochs. We analyzed the situations where five, three and two satellites were visible.

The first step is solving the system when five satellites are visible. This is the normal LSA solution of the four unknowns. We take this solution as the basis for comparing with the solutions made in three and two satellite situations. See Figure 2
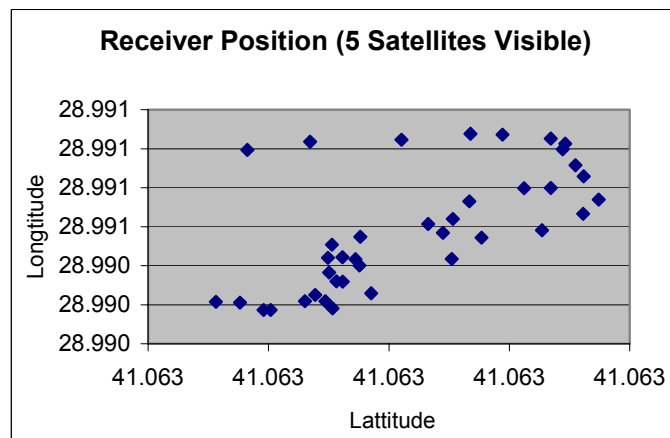

.**Figure 2: The receiver position (five satellites)**

The next step is to analyze the system when three satellites are available. This time the heights (h) are taken as constraints in the model and the system is solved for the three unknowns (φ, λ, dt). See Figure 3 for the results.
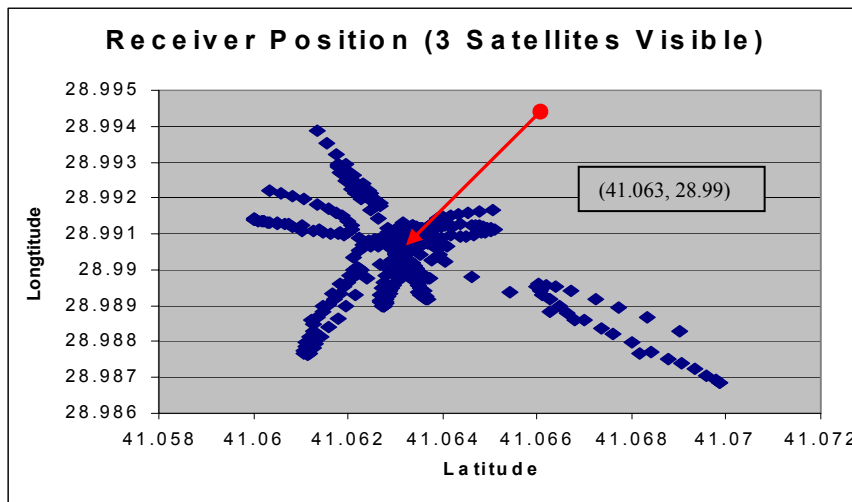

Figure 3: The receiver position (3 satellites & heights fixed)

In Figure 3 there are many solution points. This is because we take all 3 combinations of 5 satellites. For each combination there are 40 epochs. So there are C(5,3) * 40 = 400 points in the figure. We do this type of calculation in order to analyze the effect of the satellite geometry. As seen in the Figure 3, there are some points far away from the true position. These points are obtained from those satellites whose geometrical distribution is not well enough.

After that, we try to solve the system when only two satellites are visible. To do this, first we should look at the clock behaviour of the receiver. From the results obtained in the first step (using five satellites) we analyze the clock offsets for each epoch and using a curve-fitting method we map them to a model. Figure 4 shows the results.
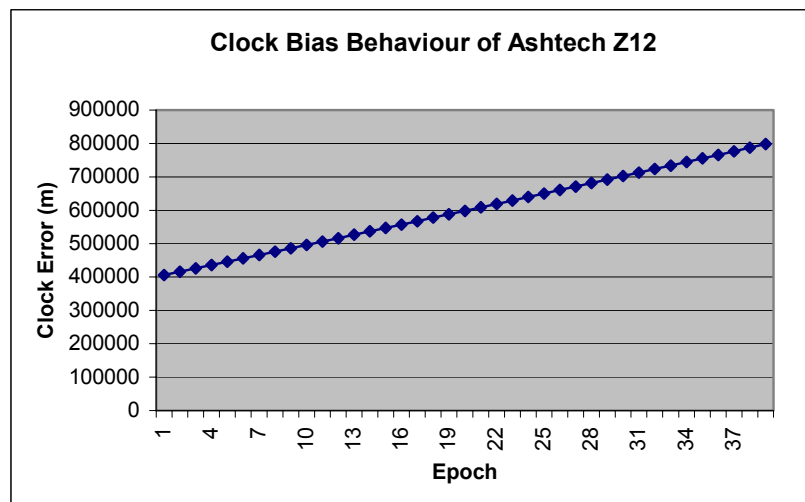

Figure 4: Receiver clock bias for 40 epochs

In Figure 4, clock offsets are shown in meters meaning that they are multiplied with the speed of light (c). It is seen that the clock of Astech Z12 has linear-like error behaviour in time. We first map these values to a linear function but wee see that the residuals are not small enough, so we use the quadratic function

$$y = 396004.63 + 9903.4292 * x + 10.308041 * x^2$$

where x is the epoch number, and y is the corresponding clock offset.

When the number of visible satellites decreases to two, we use the function above as the estimator of the clock offset. As in the case of three satellites condition, we again form the all two combinations of five satellites and solve the unknowns by using all these combinations. See Figure 5 for details.
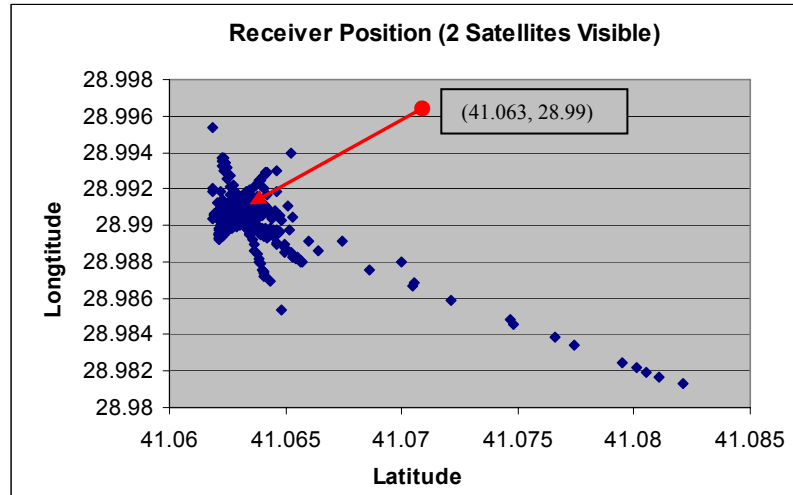
Figure 5: The receiver position (Two satellites)

Again, some points are seen to be far away from the exact solution point. These points belong to the satellite pairs (PRN: 26,27 and PRN: 16,19), which are geometrically ill positioned. Table 2 shows the azimuth and elevation angles of these pairs for the first and the last epochs.
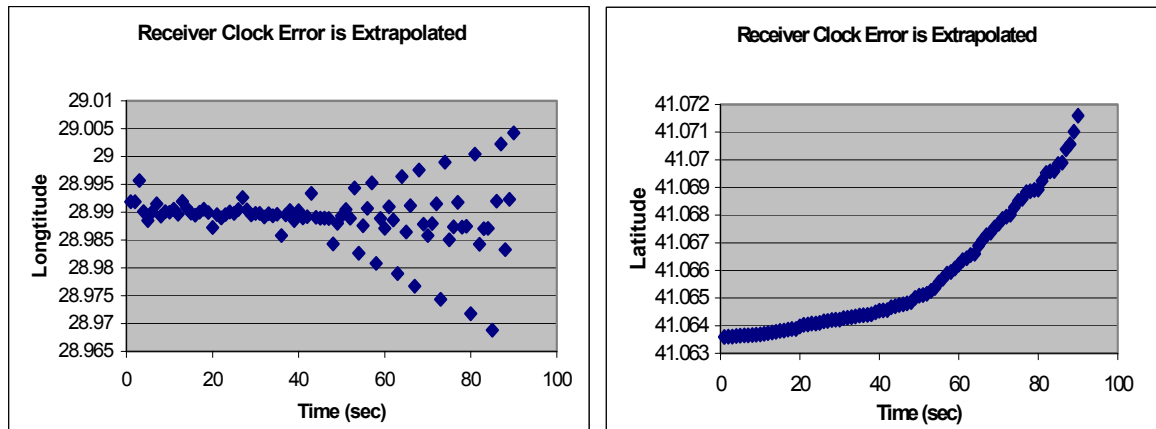
Table 2: Azimuth & Elevation angles

PRN's	Epoch #	Azimuth	Elevation
26	1	307.95	18.564
27	1	32.838	67.89
26	40	309.067	22.198
27	40	40.87	64.826
16	1	257.759	49.507
19	1	67.663	33.721
16	40	251.468	47.824
19	40	70.384	30.397

Note that the receiver clock model can not be used more than a short time interval. To make this point more clear, we plot the latitudes and longitudes solved by this model in Figure 6(a) and Figure 6(b). The model generates reasonable solutions up to 45-50 seconds. After that point, it can not produce good values because of the accumulation of the error in each epoch.

Figure 6: Receiver position when clock error is modelled. (a) Longitudes. (b) Lattitudes

Receiver Clock Error is Extrapolated (left chart: Longtitude vs Time (sec))

Receiver Clock Error is Extrapolated (right chart: Latitude vs Time (sec))

5.CONCLUSION

Maintaining the continuity of the GPS in urban environments is one of the main problems of GPS system because the number of visible satellites may not be enough to solve the four unknowns of the receiver (x_R, y_R, z_R, dt). To make the GPS more powerful in such conditions, some assumptions can be made on some unknowns. When three satellites are visible, assuming the heights are fixed we can still solve the receiver position. This can be applied by either taking the last computed height as a known or using an external device such as a barometer to measure the height. The results have shown that taking the height as known in the calculation, even in the case of four or more satellites, yields to more accurate positions than the case when it is taken as unknown. If only two satellites are visible, then by modelling the receiver clock offset for short time intervals (max. 1 minute) we can still reach the solution with a reasonable performance.

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