

HOMEWORK # 5¹

1. Use computer to plot the following analytical functions between 0 and 1 for $n = 1, 2, 3$.

- (a) $\sin(n\pi x)$
- (b) $\cos(n\pi x)$
- (c) $\tan(n\pi x)$
- (d) $\sinh(n\pi x)$
- (e) $\cosh(n\pi x)$
- (f) $\tanh(n\pi x)$

2. Solve the following homogenous PDE

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad (1)$$

with the nonhomogenous boundary conditions

$$u_x(0, t) = 0, \quad u_x(\pi, t) = 10 \quad (2)$$

and the following initial conditions

$$u(x, 0) = \frac{10x^2}{2\pi} \quad (3)$$

3. Solve the following nonhomogenous PDE

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} + \sin(3\pi x), \quad 0 < x < 1 \quad (4)$$

with the homogenous boundary conditions

$$u(0, t) = u(1, t) = 0, \quad t > 0 \quad (5)$$

and the initial conditions

$$u(x, 0) = \sin(\pi x), \quad 0 < x < 1 \quad (6)$$

4. Use the separation of variables, $u(x, y, t) = X(x)Y(y)T(t)$, to solve the following vibrating membrane problem

$$\frac{\partial^2 u}{\partial t^2} = c^2 \left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] \quad (7)$$

in a domain with $0 < x < 1$ and $0 < y < 1$. The boundary conditions are

$$u(0, y) = 0, \quad u(1, y) = 0 \quad (8)$$

$$u(x, 0) = 0, \quad u_y(x, 1) = 0 \quad (9)$$

and the initial conditions are

$$u(x, y, 0) = f(x, y) \quad (10)$$

$$u_t(x, y, 0) = 0 \quad (11)$$

¹Return date is on 31 May 2012.