HOMEWORK # 4[1]

1. [25 points] Find the eigenvlues and eigenfunction of the following Saturm-Liouville equation

$$y^{''} + \lambda y = 0 \tag{1}$$

with the boundary conditions

$$\begin{aligned} y(0) - y^{'}(0) &= 0 & (2)\\ y(1) + y^{'}(1) &= 0 & (3) \end{aligned}$$

2. [25 points] Use the separation of variables, $u(x, y) = X(x)Y(y)$, to solve the following partial differential equation

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{4}$$

in a domain with $0 < x < 1$ and $0 < y < 1$. The boundary conditions are

$$\begin{aligned} u(x, 0) = f_1(x), \qquad u(0, y) = 0 \quad (5)\\ u(x, 1) = f_2(x), \qquad u(1, y) = 0 \quad (6) \end{aligned}$$

where

$$f_1(x) = f_2(x) = \begin{cases} 2x, & 0 < x < \frac{1}{2} \\ 2 - 2x, & \frac{1}{2} < x < 1 \end{cases} \tag{7}$$

3. [25 points] Solve the boundary value problem (BVP)

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \tag{8}$$

in a square enclosure $[0, 1] \times [0, 1]$, subject to

$$\begin{aligned} \psi(x, 0) &= 0 & (9)\\ \psi(x, 1) &= x - x^2 & (10)\\ \psi(0, y) &= 0 & (11)\\ \psi(1, y) &= 0 & (12) \end{aligned}$$

4. [25 points] Use the separation of variables, $u(r, \theta) = R(r)\Theta(\theta)$, to solve the following partial differential equation

$$\nabla^2 u = \frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0 \tag{13}$$

[1] Return date is on 11 May 2012.

in a domain with $1 < r < 2$ and $0 < \theta < \pi$. The boundary conditions are

$$u(1,\theta) = 0, \qquad u(2,\theta) = 1 \tag{14}$$

$$u(r,0) = 0, \qquad u(r,\pi) = 0 \tag{15}$$