**HOMEWORK # 3**[1]

1. Use the Laplace transform to solve the initial value problem

$$y'' + 3y' + 2y = sin(2t) \tag{1}$$

where $y(0) = 2$ and $y'(0) = -1$.

2. Find the canonical form of the following partial differential equation.

$$y^2 u_{xx} - 2xyu_{xy} + x^2 u_{yy} = \frac{y^2}{x} u_x + \frac{x^2}{y} u_y \tag{2}$$

3. Compute the characteristic curves of the following wave equation

$$\frac{\partial^2 u}{\partial t^2} - a^2 \frac{\partial^2 u}{\partial x^2} = 0 \tag{3}$$

and draw them on an $x - t$ coordinate system.

4. Find the eigenvlues and eigenfunction of the following Sturm-Liouville system

$$y'' + \lambda y = 0, \qquad 0 < x < 1 \tag{4}$$

with the boundary conditions

$$hy(0) - y'(0) = 0 \tag{5}$$
$$y'(1) = 0 \tag{6}$$

and $h > 0$.

5. Use the separation of variables, $u(x, y) = X(x)Y(y)$, to solve the following partial differential equation

$$\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \tag{7}$$

in a domain with $0 < x < 1$ and $0 < y < 1$. The boundary conditions are

$$u(0, y) = 0, \qquad u_x(1, y) = 0 \tag{8}$$
$$u(x, 0) = 0, \qquad u(x, 1) = 1 \tag{9}$$

[1] Return date is on 4 May 2012.