PROJECT # 1

The two-dimensional wave equation on a domain $[-1,1] \times [-1,1]$ is given by

$$\frac{\partial u}{\partial t} + a_1 \frac{\partial u}{\partial x_1} + a_2 \frac{\partial u}{\partial x_2} = 0 \tag{1}$$

with $a_1 = 1$ and $a_2 = 0$. The analytical solution is given by

$$u(x_1, x_2, t) = e^{-100((0.5+x_1-a_1 t)^2 + (x_2 - a_2 t)^2)} \tag{2}$$

Use the analytical solution at $t = 0$ as the initial condition. For the boundary conditions, use the time dependent analytical solutions at $(x_1, -1)$, $(x_1, +1)$ and $(-1, x_2)$. For outflow, there is no boundary condition and you have to solve the PDE.

Use second order accurate spatial approximations with the following numerical methods to find the solution at $t = 1$

1. Euler explicit finite difference method
2. Euler implicit finite difference method
3. Crank-Nicolson finite difference method

The error function is given by

$$\text{Error} = \frac{\|u_{i,j} - u_{analytic}\|_2}{\sqrt{i_{max} j_{max}}} \tag{3}$$

Employ uniform $i_{max} \times j_{max} = 41 \times 41$, 81×81 and 121×121 grid resolutions with time steps $CFL = 0.005$, $CFL = 0.1$, $CFL = 1$ and $CFL = 2$. Then compute the numerical solutions at $t = 1$ and compare them with the analytical one in the same graph. Finally, draw the error versus mesh and time resolutions and determine the spatial and temporal convergence rates. Finally, compare the rate of convergence with the truncation error.

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