

Seventh Edition

VECTOR MECHANICS FOR ENGINEERS: STATICS

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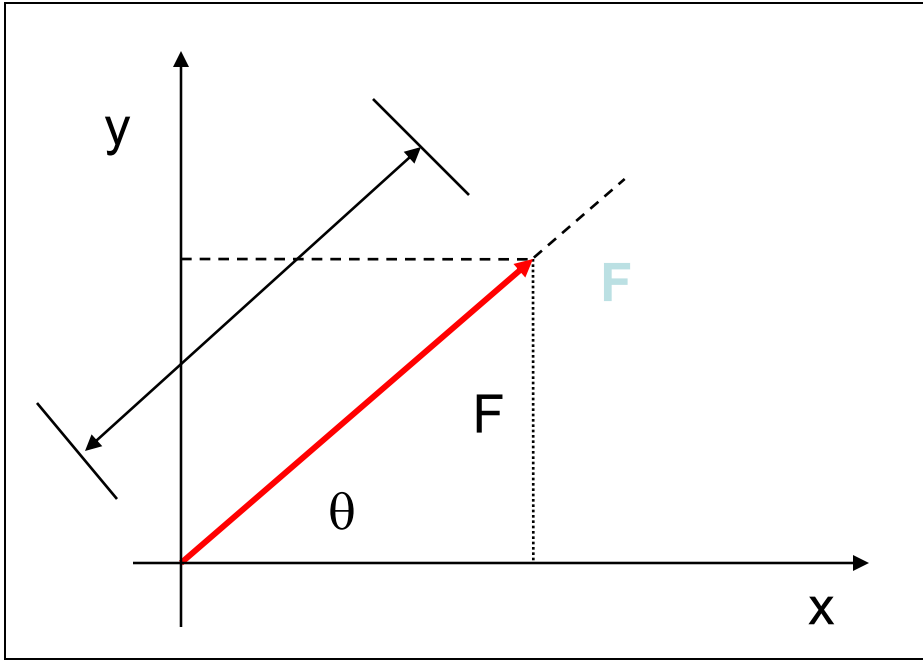
Web: <http://atlas.cc.itu.edu.tr/~acarh>

2. MADDESEL NOKTALARIN STATIĞI



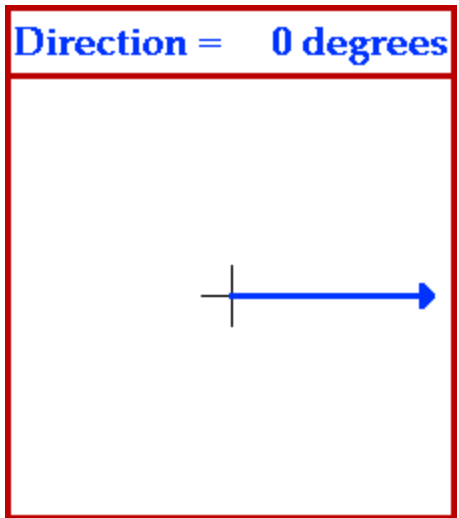
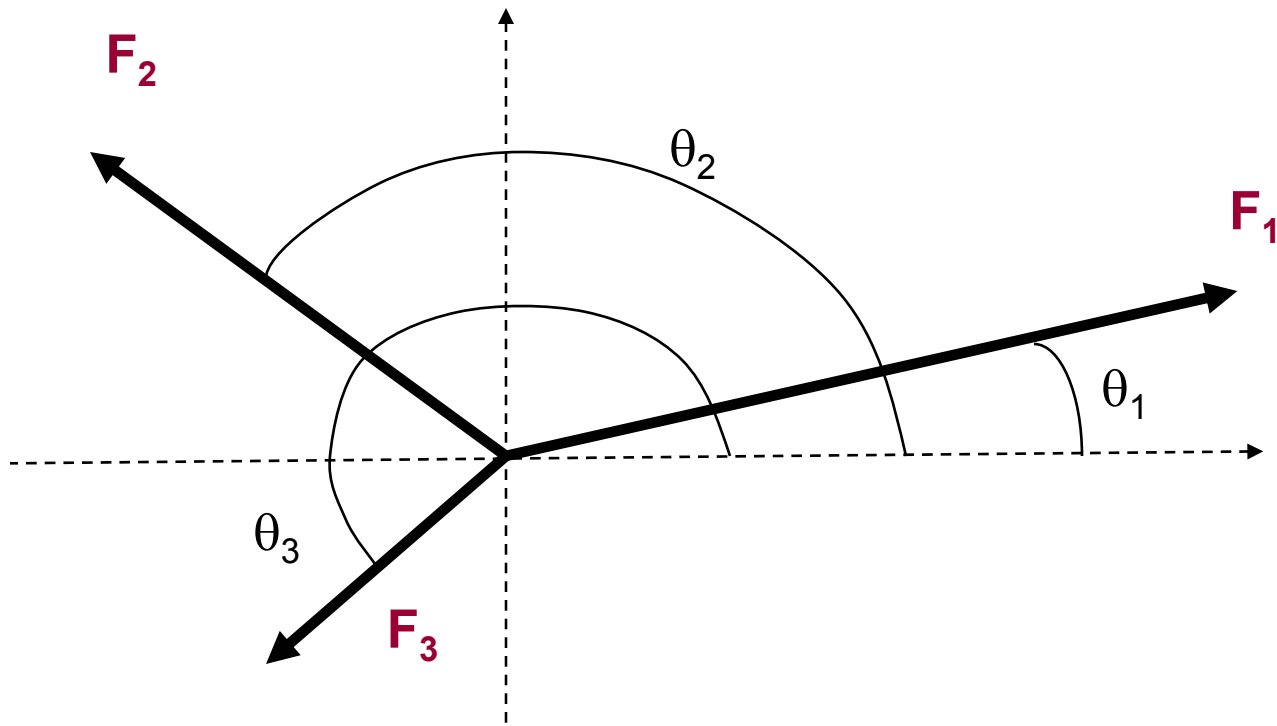
DÜZLEM KUVVETLER SİSTEMİ

- KUVVET VEKTÖREL BİR BÜYÜKLÜKTÜR



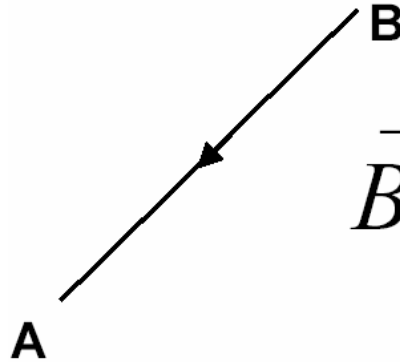
BİLİNMESİ GEREKENLER

- UYGULAMA NOKTASI
- ŞİDDETİ
- YÖNÜ

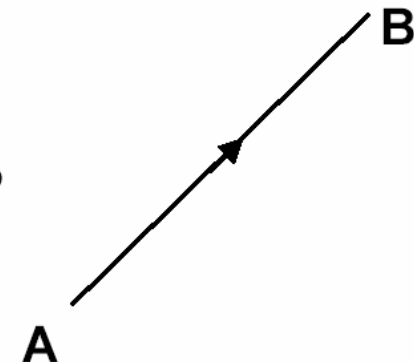


VEKTÖR ÖZELLİKLERİ

NEGATİF VEKTÖR:

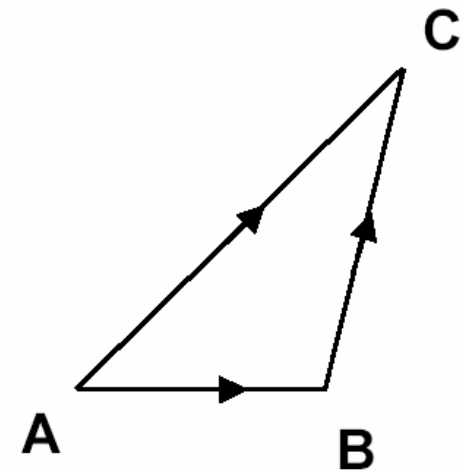


$$\vec{BA} = -\vec{AB}$$

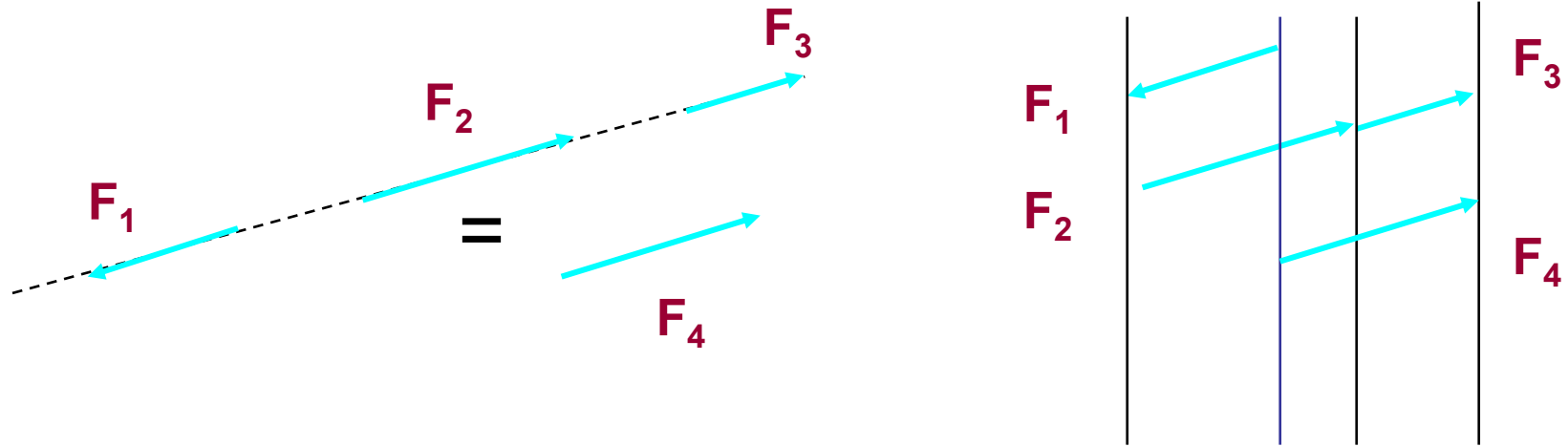


TOPLAMA:

$$\vec{AC} = \vec{AB} + \vec{BC}$$

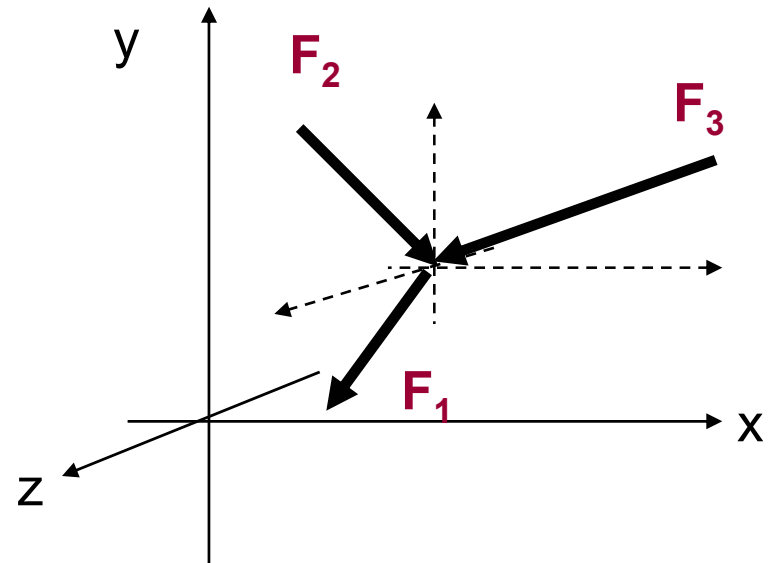
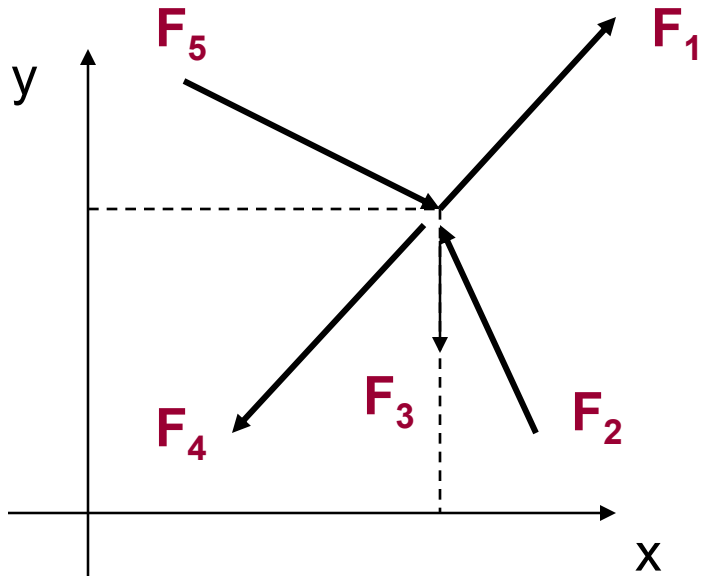


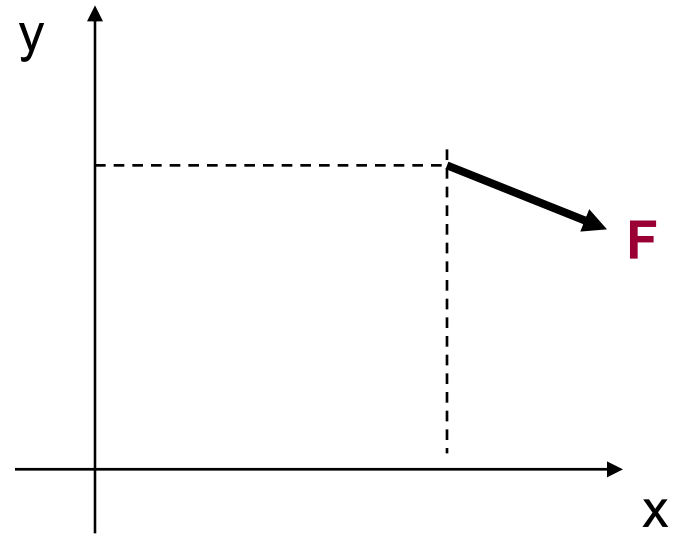
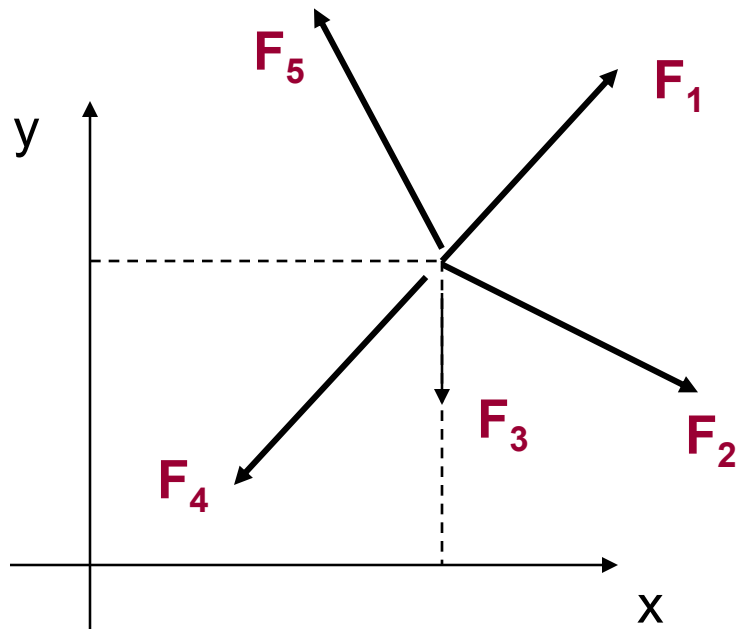
TESİR ÇİZGİSİ AYNI OLAN VEKTÖRLERİN TOPLANMASI:



$$-F_1 + F_2 + F_3 = F_4$$

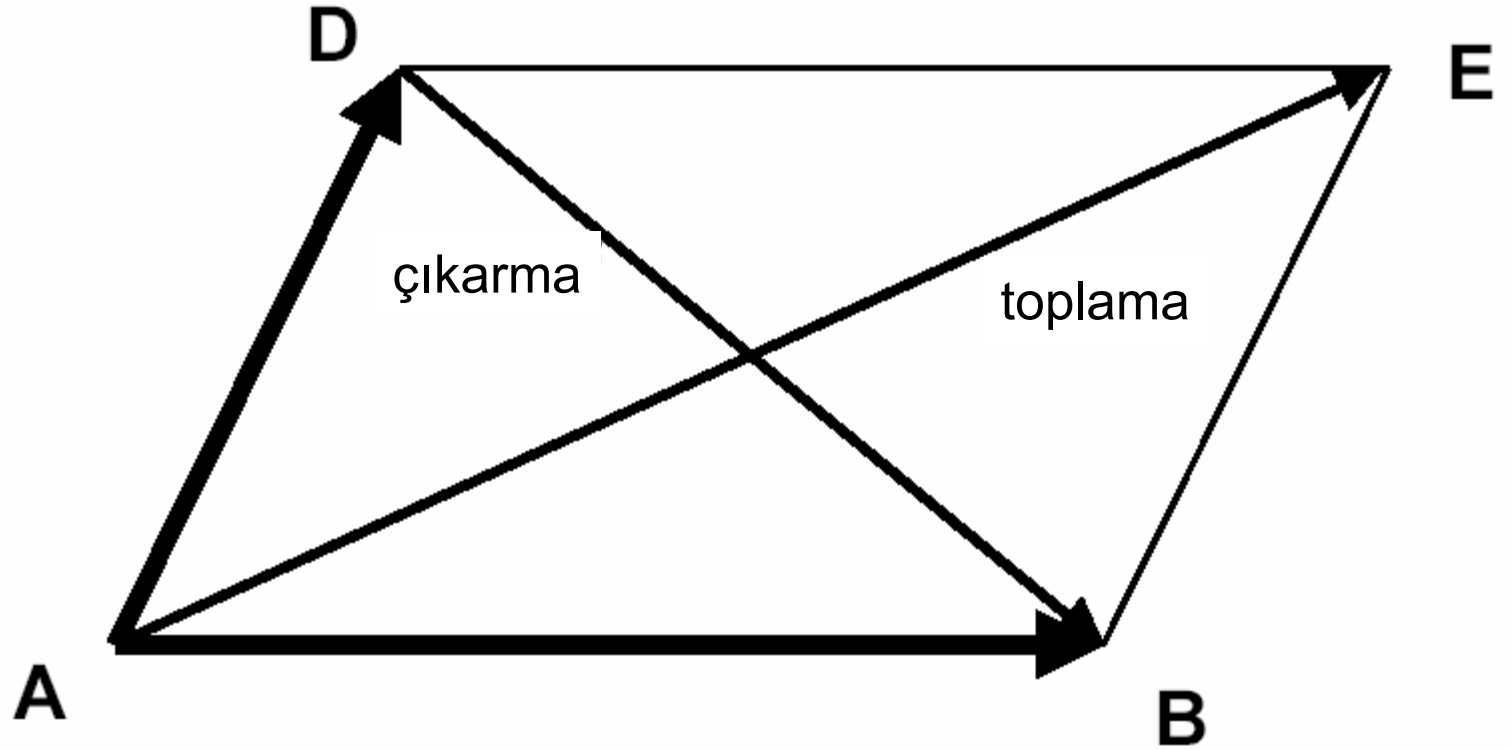
AYNI NOKTAYA TESİR EDEN VEKTÖRLERİN TOPLANMASI:





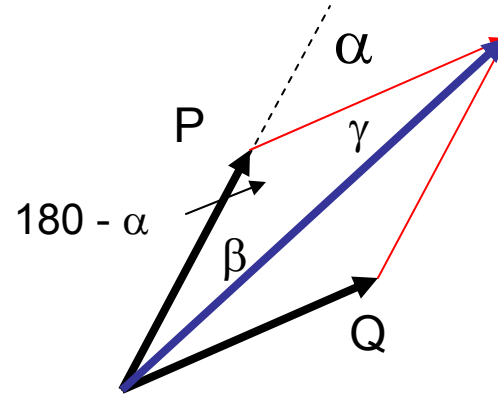
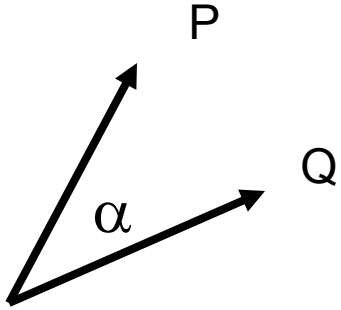
$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5 = \vec{F}$$

PARALEL KENAR YASASI:



$$\vec{AB} + \vec{AD} = \vec{AB} + \vec{BE} = \vec{AE}$$

$$\vec{AB} - \vec{AD} = \vec{DB}$$



COSİNÜS TEOREMİ:

$$R^2 = P^2 + Q^2 - 2PQ\cos(180-\alpha)$$

$$R^2 = P^2 + Q^2 + 2PQ\cos(\alpha)$$

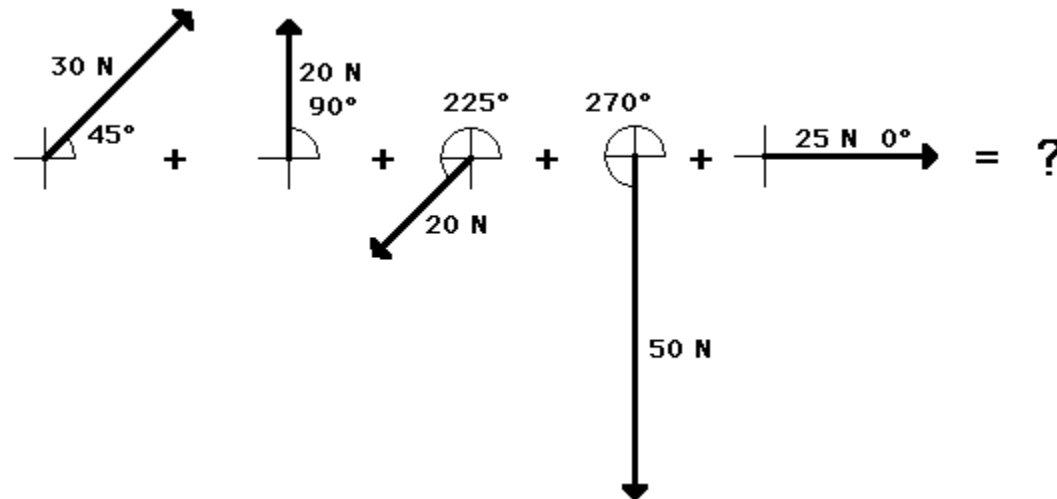
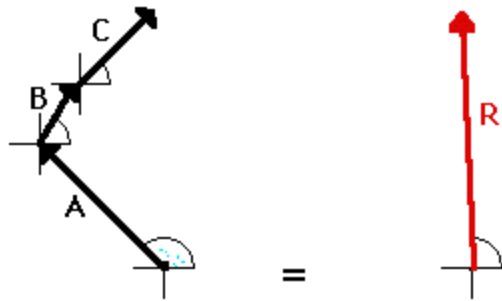
SİNÜS TEOREMİ:

$$\frac{P}{\sin \gamma} = \frac{Q}{\sin \beta} = \frac{R}{\sin \alpha}$$

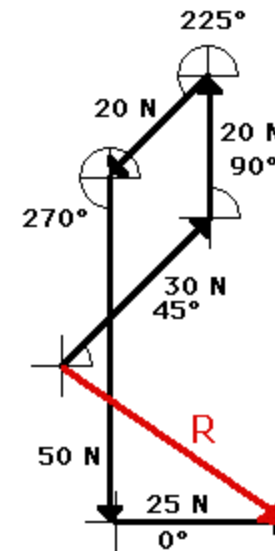


$$8 \text{ N}, 0^\circ + 6 \text{ N}, 0^\circ = 14 \text{ N}, 0^\circ$$

İKİDEN FAZLA VEKTÖRÜN TOPLANMASI

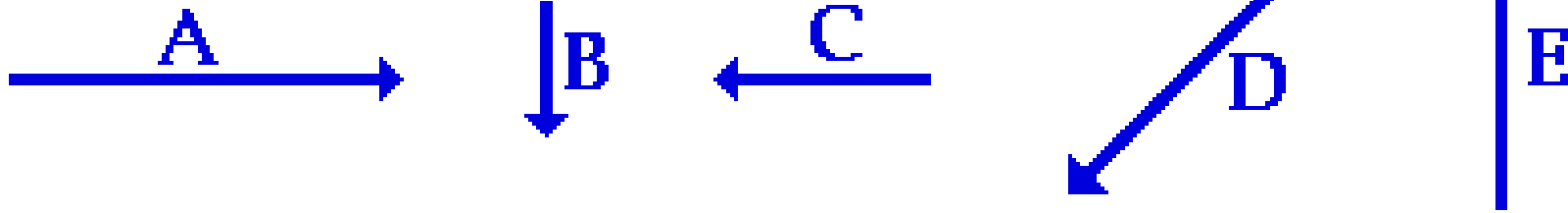


SCALE: 1 cm = 10 N

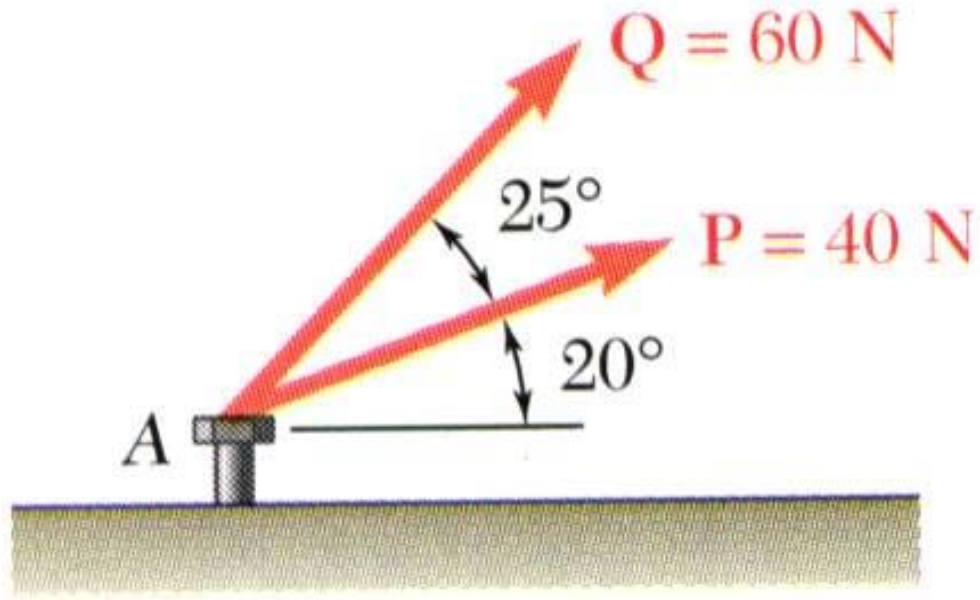


R=39.4 N, 324°

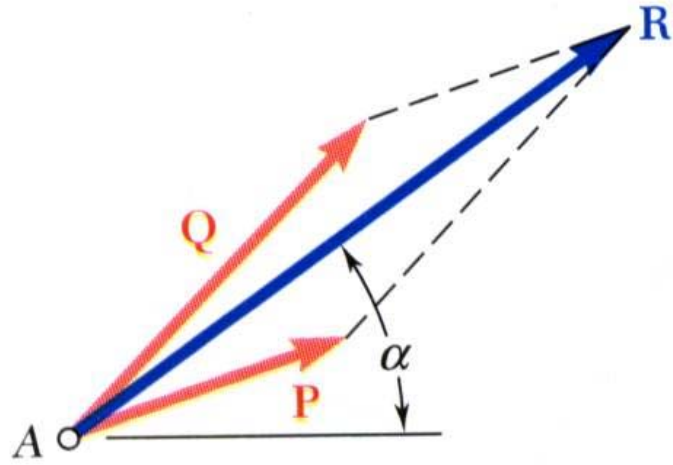
VEKTÖRLERİN TOPLANMASI



Örnek Problem 2.1



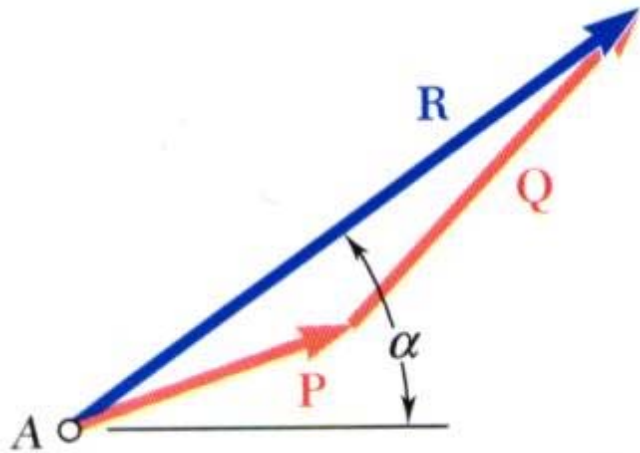
Şekildeki somuna A noktasından iki kuvvet etkimektedir.
Bu kuvvetlerin bileşkesini bulunuz.



- Grafik çözüm - paralelkenar yöntemi:

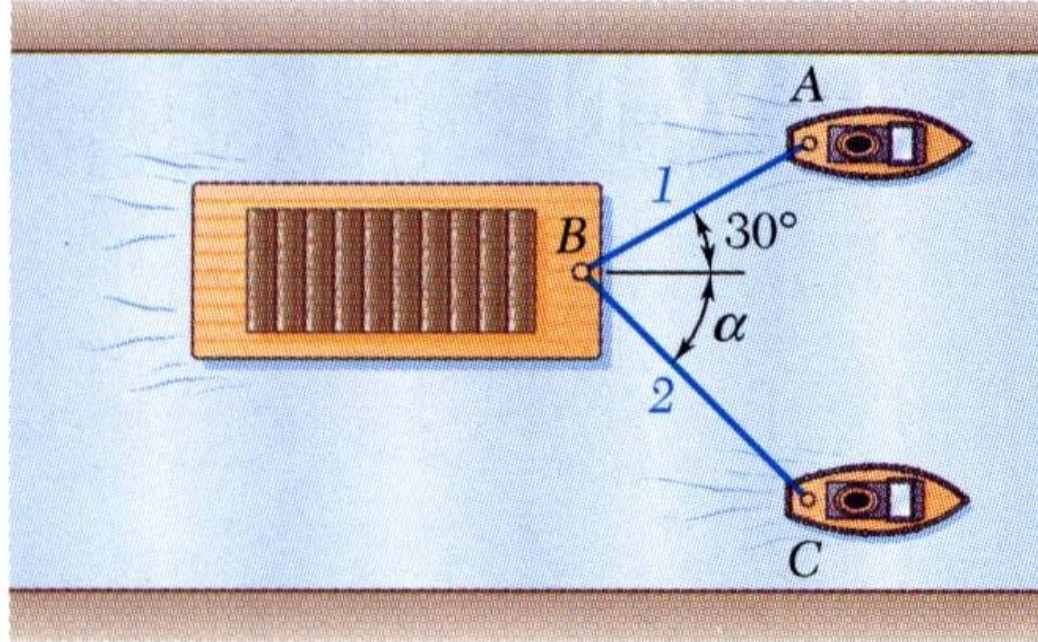
$$\mathbf{R} = 98 \text{ N} \quad \alpha = 35^\circ$$

- Trigonometrik çözüm – cos ve sin teoremleri:



$$\mathbf{R} = 98 \text{ N} \quad \alpha = 35^\circ$$

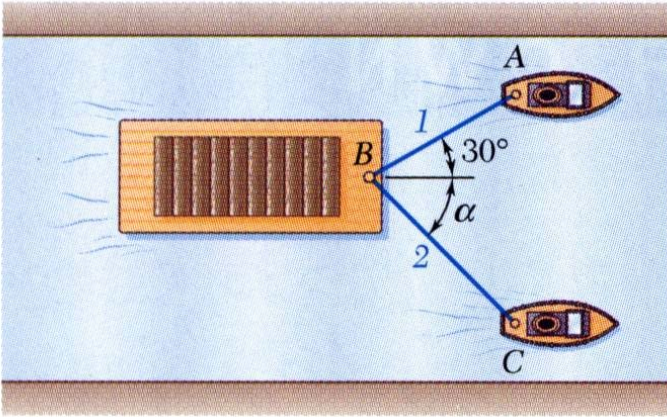
Örnek Problem 2.2



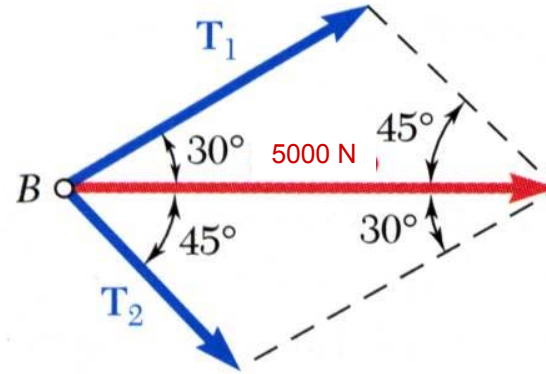
Şekildeki mavna iki römorkör ile çekilmektedir. Römorkörlerin uyguladığı kuvvetin bileşkesi 5000 N ise

- $\alpha = 45^\circ$ olduğu durumda her iki halattaki kuvveti,
- 2 nolu halattaki kuvvetin minimum olması için α açısını bulunuz.

(a)

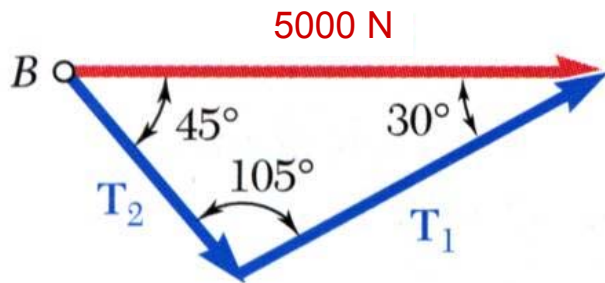


- Grafik çözüm – Paralelkenar yöntemi



$$T_1 = 3700 \text{ N} \quad T_2 = 2600 \text{ N}$$

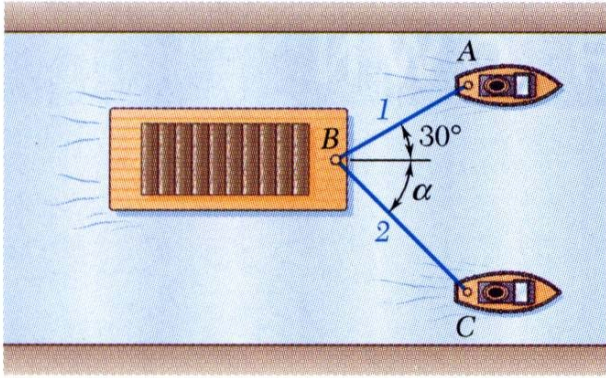
- Trigonometrik çözüm – Sinüs kuralı



$$\frac{T_1}{\sin 45^\circ} = \frac{T_2}{\sin 30^\circ} = \frac{5000 \text{ N}}{\sin 105^\circ}$$

$$T_1 = 3660 \text{ N} \quad T_2 = 2590 \text{ N}$$

(b)



- 2. halattaki minimum kuvvet için üçgen kuralı kullanılarak değişik α açıları için çözüm yapılır:

- 2. halattaki minimum kuvvet T_1 ve T_2 birbirine dik olduğu zaman oluşur:

$$T_2 = (5000 N) \sin 30^\circ$$

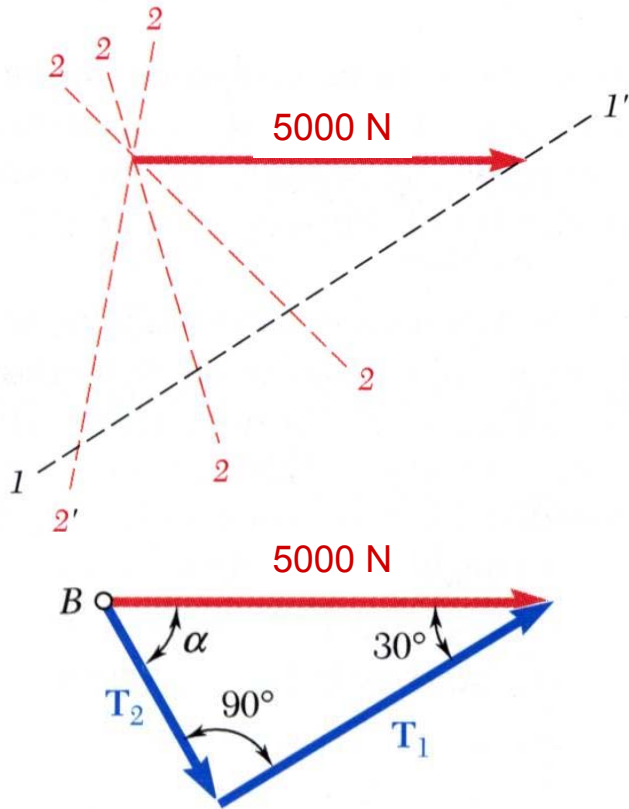
$$T_2 = 2500 N$$

$$T_1 = (5000 N) \cos 30^\circ$$

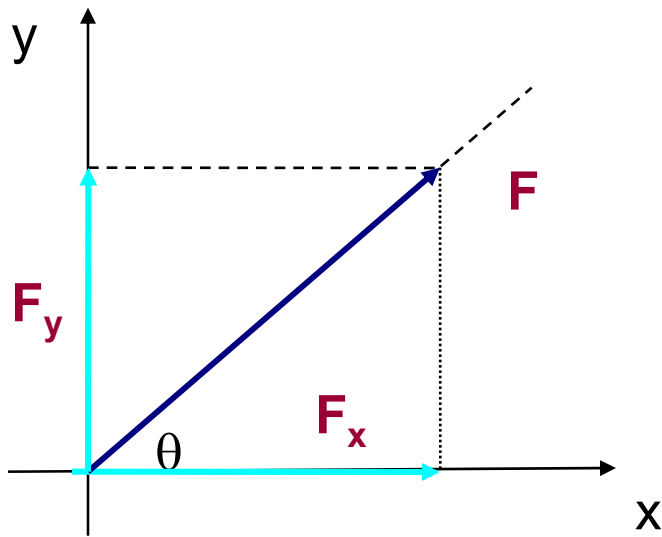
$$T_1 = 4330 N$$

$$\alpha = 90^\circ - 30^\circ$$

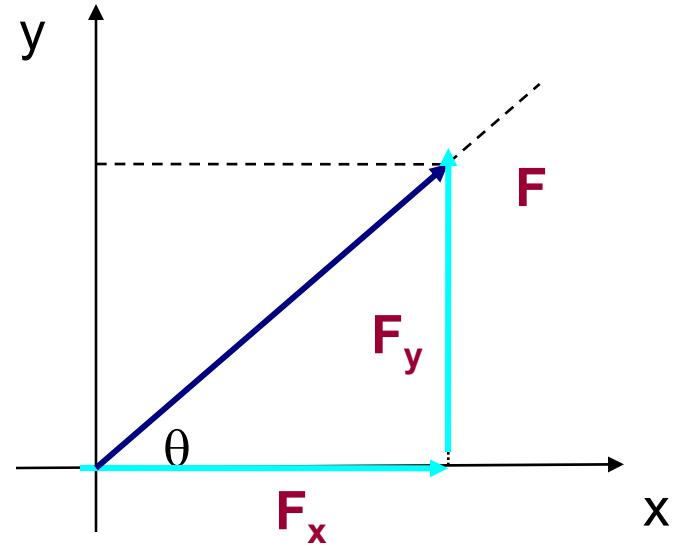
$$\alpha = 60^\circ$$



BİR KUVVETİN DİK BİLEŞENLERİNE AYRILMASI



$$\mathbf{F} = (F_x, F_y) = F_x \mathbf{i} + F_y \mathbf{j}$$

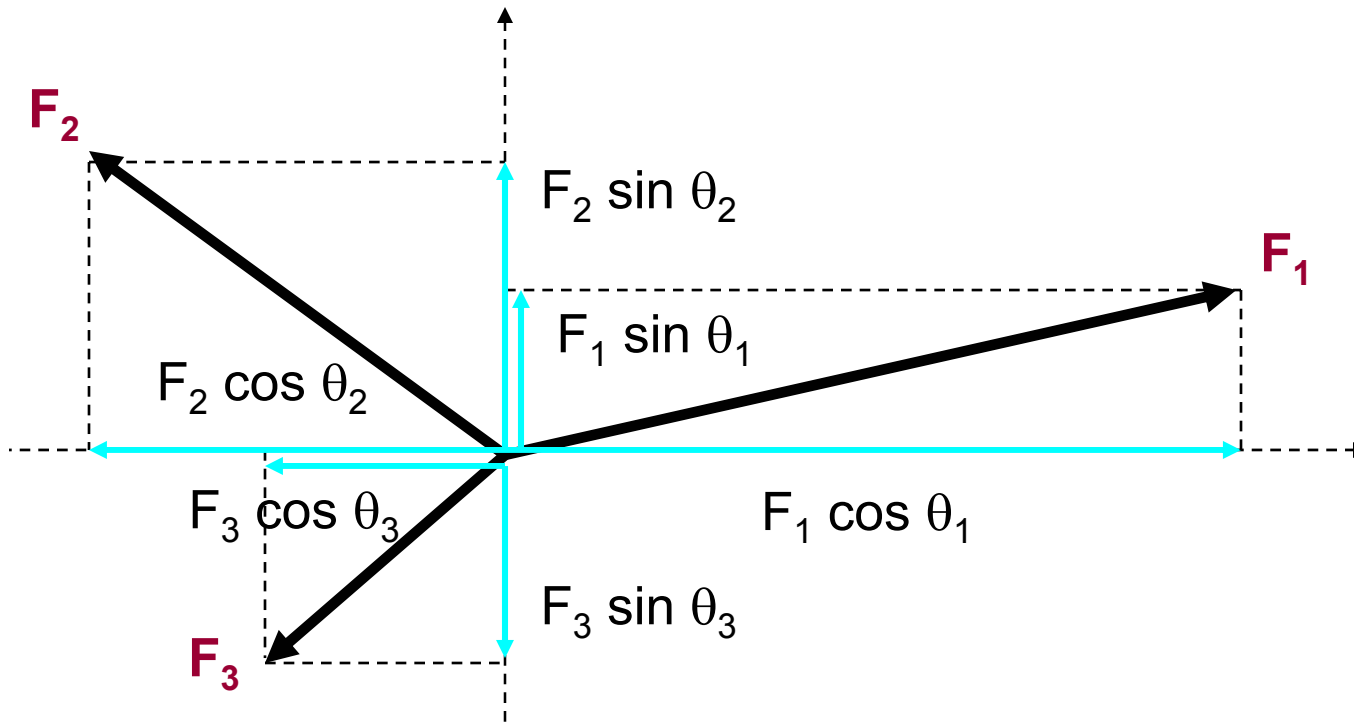


$$F_x = F \cos \theta$$

$$\tan \theta = F_y / F_x$$

$$F_y = F \sin \theta$$

$$F = (F_x^2 + F_y^2)^{1/2}$$

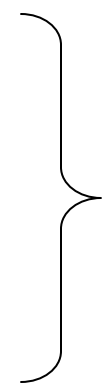


$$\mathbf{F}_1 = F_{1x} \mathbf{i} + F_{1y} \mathbf{j}$$

$$\mathbf{F}_2 = F_{2x} \mathbf{i} + F_{2y} \mathbf{j}$$

$$\vdots$$

$$\mathbf{F}_n = F_{nx} \mathbf{i} + F_{ny} \mathbf{j}$$

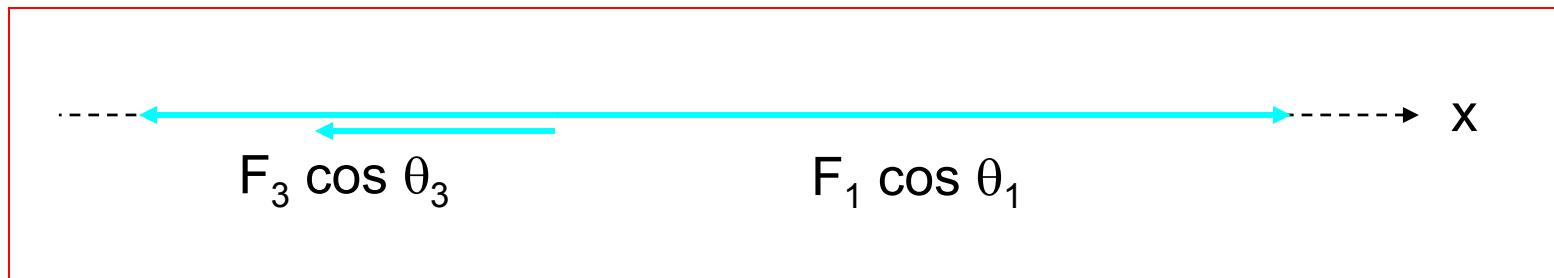
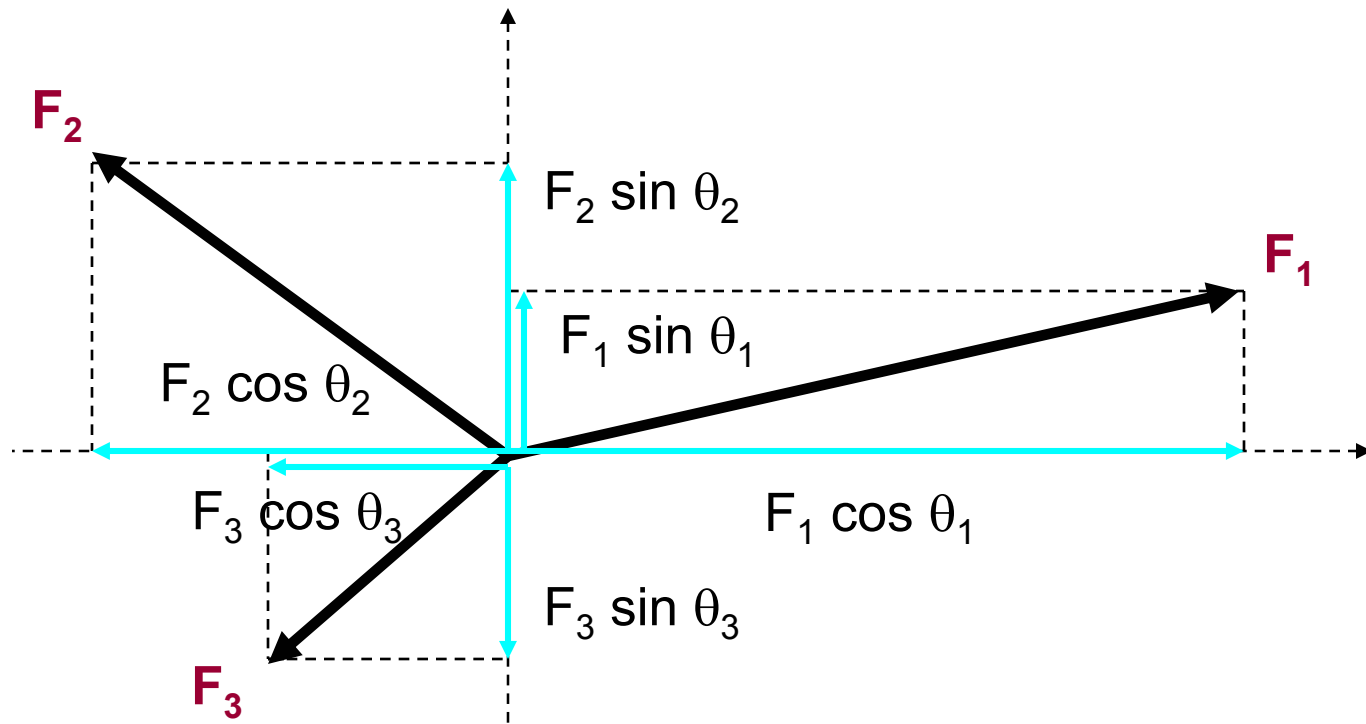


$$\mathbf{R} = (\sum F_{ix} \mathbf{i})_0^{n+} + (\sum F_{iy} \mathbf{j})_0^n = 0$$

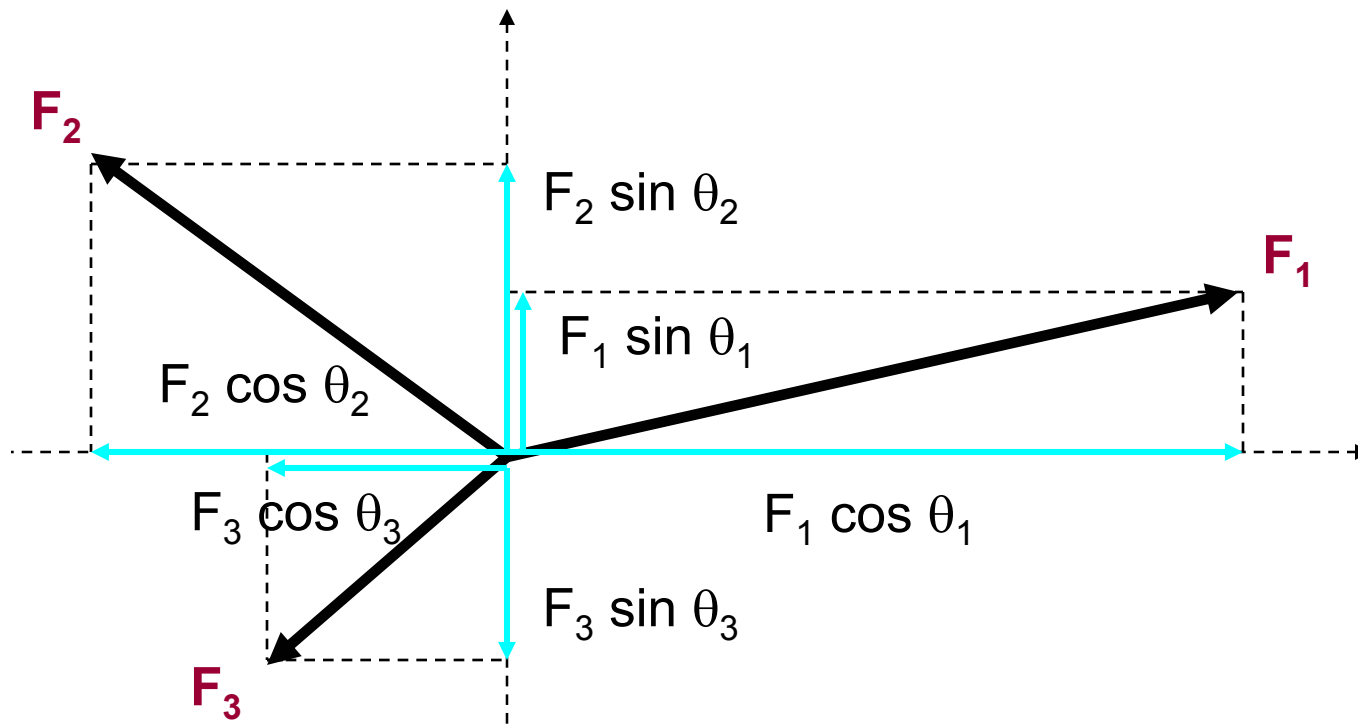


$$R_x = (\sum F_{ix})$$

$$R_y = (\sum F_{iy})$$



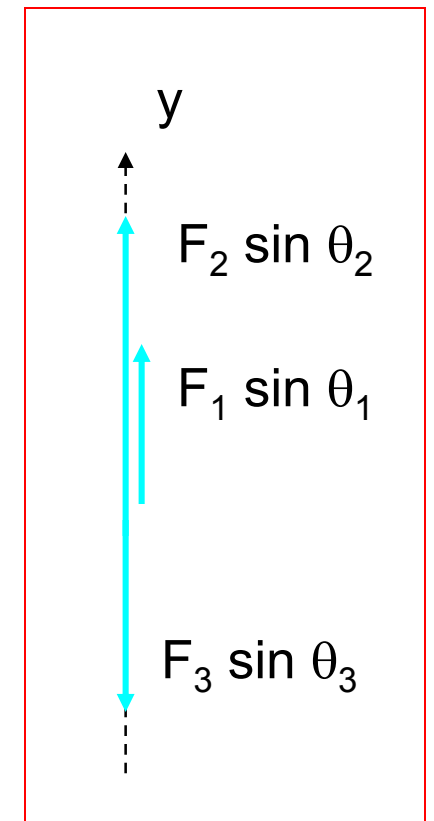
$$F_x = F_1 \cos \theta_1 + F_2 \cos \theta_2 + F_3 \cos \theta_3$$



$$F_y = F_1 \sin \theta_1 + F_2 \sin \theta_2 + F_3 \sin \theta_3$$

$$F = (F_x^2 + F_y^2)^{1/2}$$

$$\theta = \tan^{-1} (F_y / F_x)$$



MADDESEL NOKTANIN DENGESİ

$$\Sigma M = 0 \text{ (moment)}$$

$$\Sigma F = 0 \text{ (kuvvet)}$$

$$\mathbf{F}_1 = F_{1x} \mathbf{i} + F_{1y} \mathbf{j}$$

$$\mathbf{F}_2 = F_{2x} \mathbf{i} + F_{2y} \mathbf{j}$$

⋮

$$\mathbf{F}_n = F_{nx} \mathbf{i} + F_{ny} \mathbf{j}$$

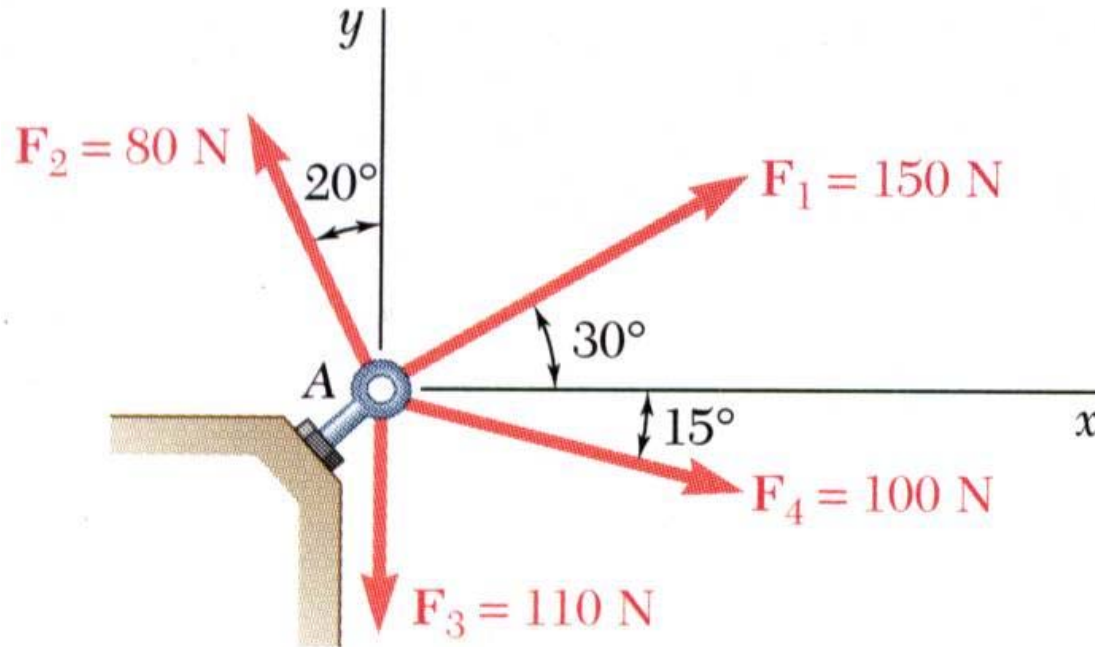
$$\mathbf{R} = (\Sigma F_{ix} \mathbf{i}) + (\Sigma F_{iy} \mathbf{j}) = 0$$



$$(\Sigma F_{ix}) = 0$$

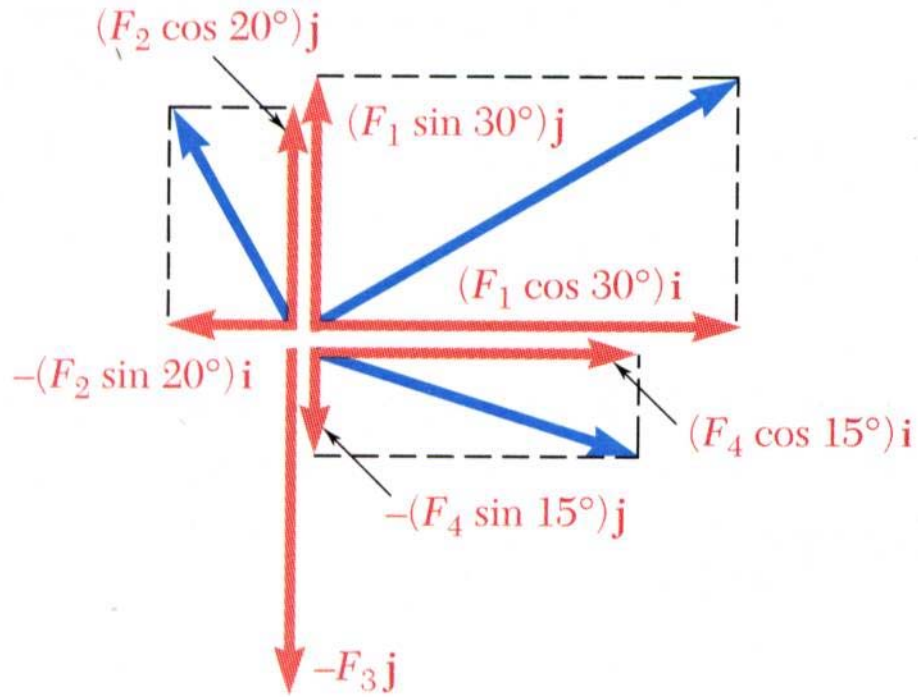
$$(\Sigma F_{iy}) = 0$$

Örnek Problem 2.3



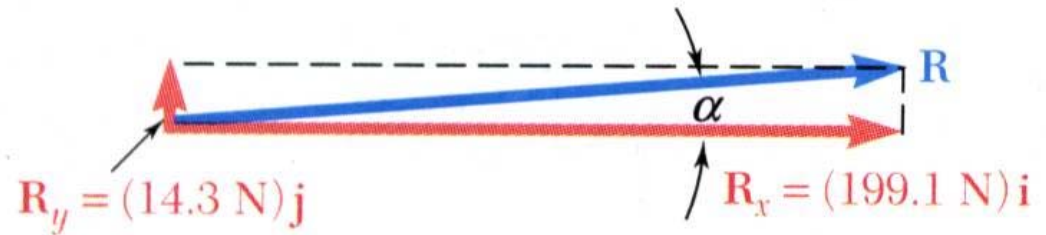
Şekildeki somuna A noktasında dört kuvvet etkimektedir. Bileşke kuvveti bulunuz.

Her kuvvetin dik bileşenleri hesaplanır:



kuvvet	şid	x - bil	y - bil
\vec{F}_1	150	+129.9	+75.0
\vec{F}_2	80	-27.4	+75.2
\vec{F}_3	110	0	-110.0
\vec{F}_4	100	+96.6	-25.9

$$R_x = +199.1 \quad R_y = +14.3$$



- Bileşkenin şiddeti ve yönü:

$$R = \sqrt{199.1^2 + 14.3^2}$$

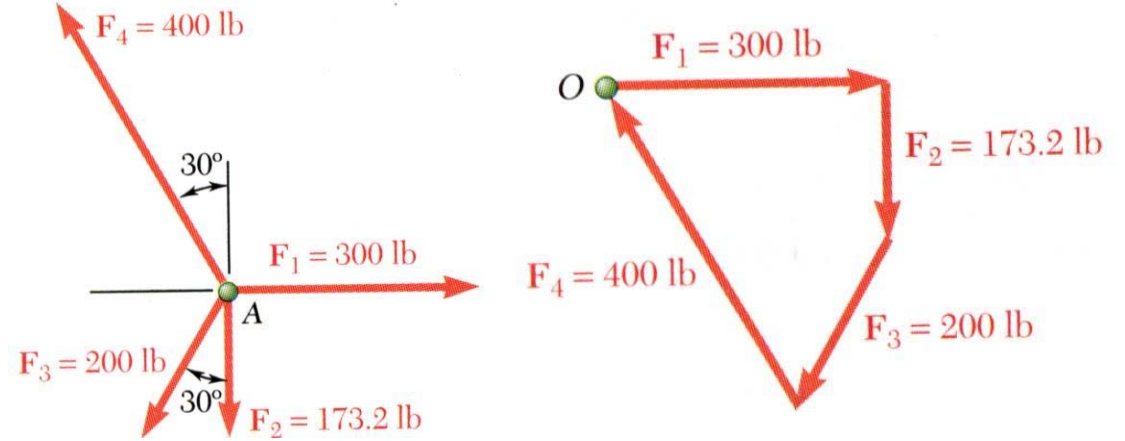
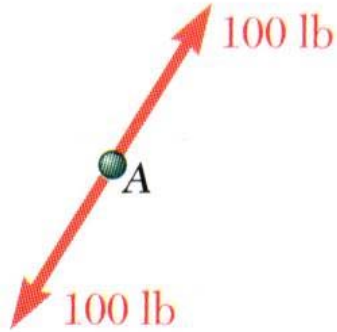
$$R = 199.6 \text{ N}$$

$$\tan \alpha = \frac{14.3 \text{ N}}{199.1 \text{ N}}$$

$$\alpha = 4.1^\circ$$

Parçacıkların Dengesi

- Parçacığa etkiyen kuvvetlerin bileşkesi sıfır ise parçacık dengededir.
- *Newton'un 1. Kanunu*: Bileşke kuvvet sıfır ise, parçacık başlangıçtaki hareketini korur. Dengede ise dengede kalır, belirli bir hızı varsa aynı hızda devam eder.



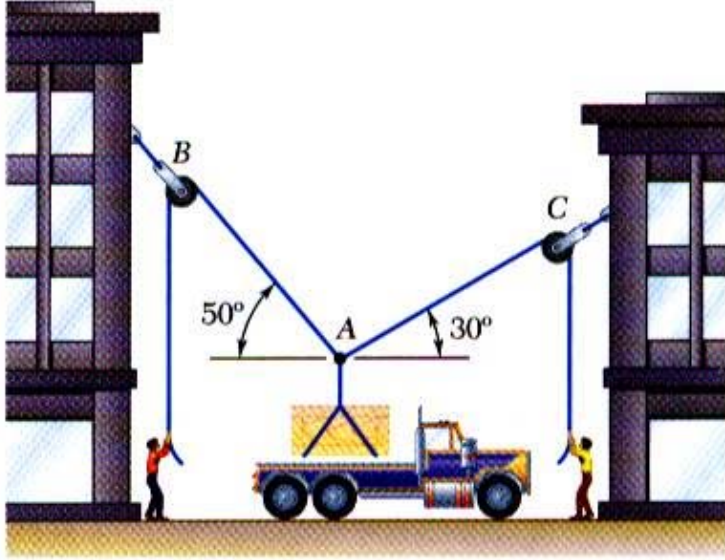
- Parçacığa iki kuvvet etkiyor :
 - eşit şiddetli
 - aynı tesir çizgisi
 - zıt yönlü

- Parçacık ikiden fazla kuvvet etkisi altında:
 - grafik çözüm zorlaşır.
 - cebirsel çözüm uygundur.

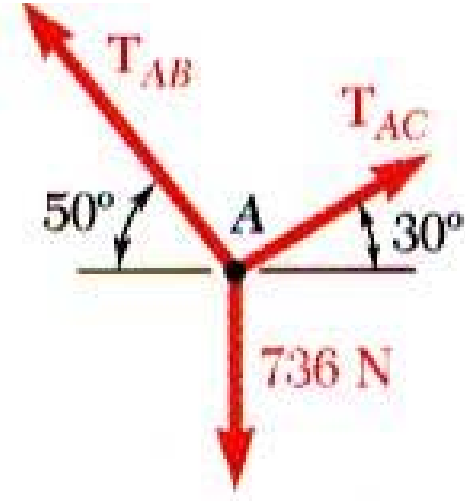
$$\vec{R} = \sum \vec{F} = 0$$

$$\sum F_x = 0 \quad \sum F_y = 0$$

Serbest Cisim Diyagramı

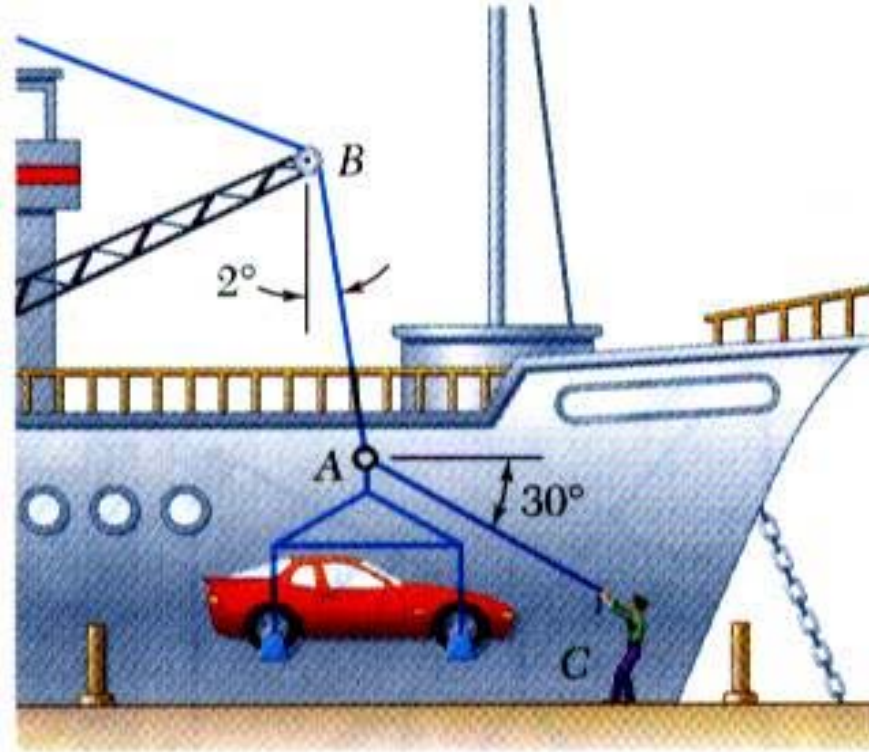


Uzay Diyagramı: Problemin fiziksel durumunu gösteren resim

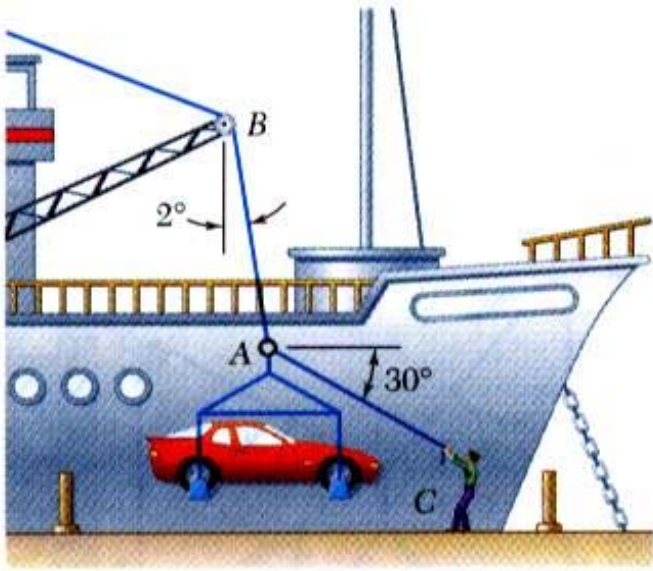


Serbest Cisim Diyagramı: Seçilen elemana etkiyen kuvvetleri gösteren çizim.

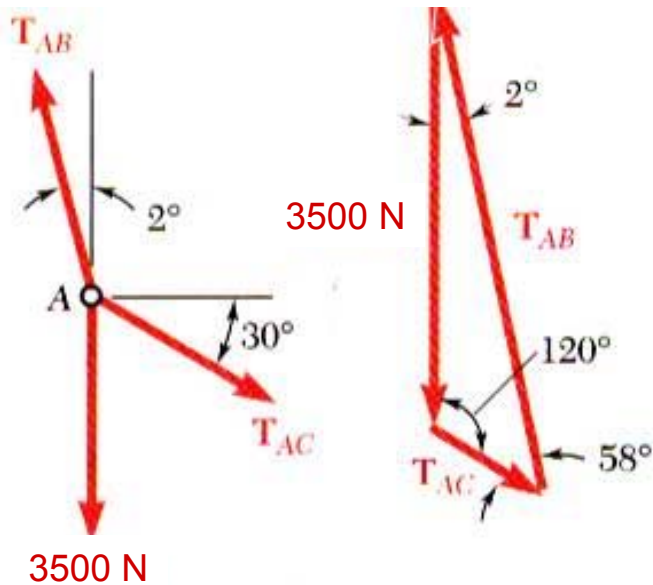
Örnek Problem 2.4



Bir gemiye araç yüklemesi sırasında 3500 N 'luk bir araba kablolar ile kaldırılmaktadır. AC kablosundaki kuvveti bulunuz.



Serbest cisim diyagramı

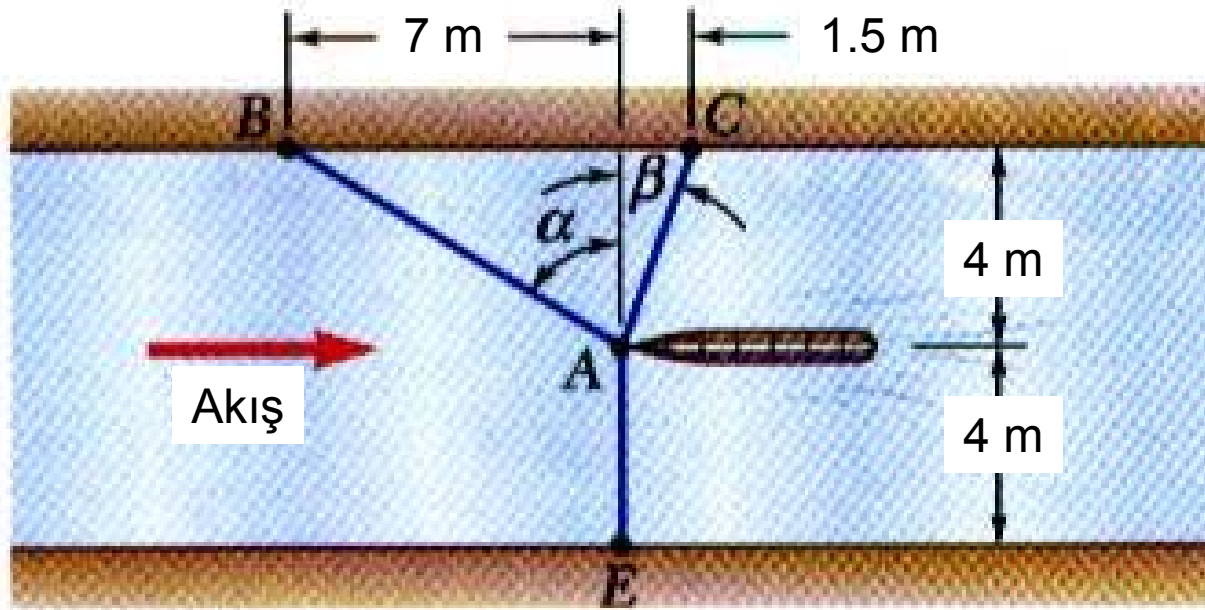


$$\frac{T_{AB}}{\sin 120^\circ} = \frac{T_{AC}}{\sin 2^\circ} = \frac{3500 \text{ N}}{\sin 58^\circ}$$

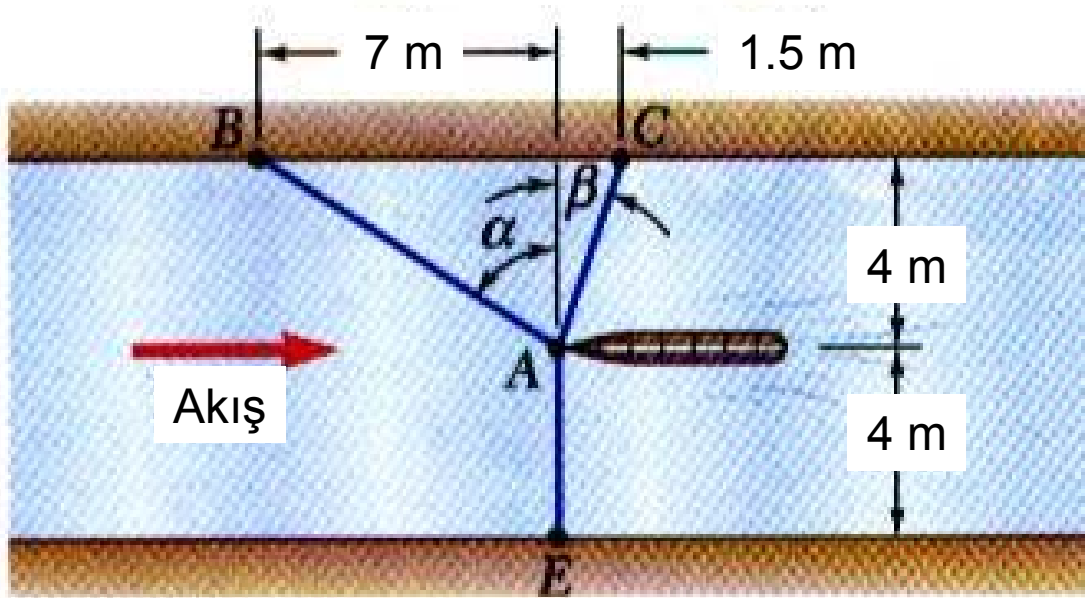
$$T_{AB} = 3570 \text{ N}$$

$$T_{AC} = 144 \text{ N}$$

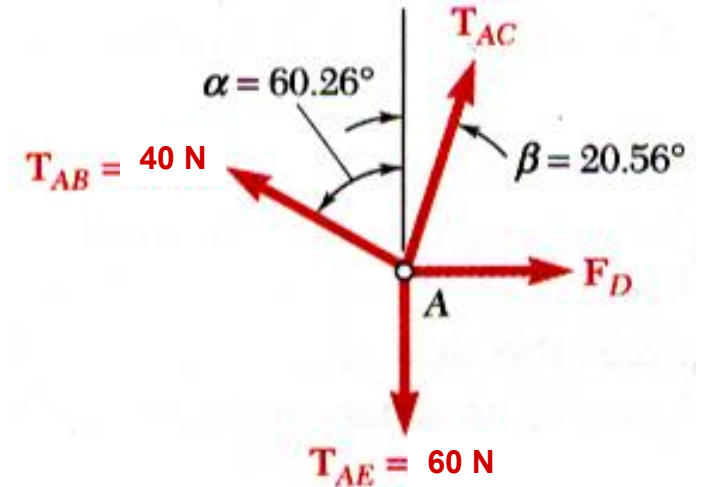
Örnek Problem 2.6



Bir botun sürüklenme kuvvetinin hesaplanabilmesi için su kanalı kullanılmaktadır. Bot 3 kablo ile desteklenmiştir. AB kablosunda 40 N, AE kablosunda ise 60 N kuvvet olduğu biliniyorsa, botun sürüklenme kuvvetini ve AC kablosundaki kuvveti bulunuz.



Botun serbest cisim diyagramı:



- Denge için kuvvet toplamının sıfıra eşit olması gerekir:

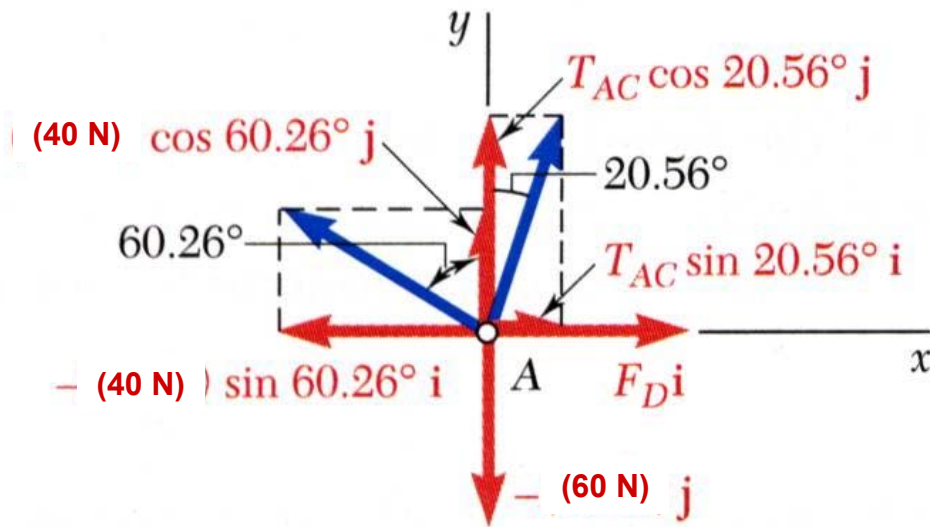
$$\vec{R} = \vec{T}_{AB} + \vec{T}_{AC} + \vec{T}_{AE} + \vec{F}_D = 0$$

$$\tan \alpha = \frac{7 \text{ m}}{4 \text{ m}} = 1.75$$

$$\alpha = 60.25^\circ$$

$$\tan \beta = \frac{1.5 \text{ m}}{4 \text{ m}} = 0.375$$

$$\beta = 20.56^\circ$$



$$\begin{aligned}\vec{T}_{AB} &= -(40 \text{ N})\sin 60.26^\circ \vec{i} + (40 \text{ N})\cos 60.26^\circ \vec{j} \\ &= -(34.73 \text{ N})\vec{i} + (19.84 \text{ N})\vec{j}\end{aligned}$$

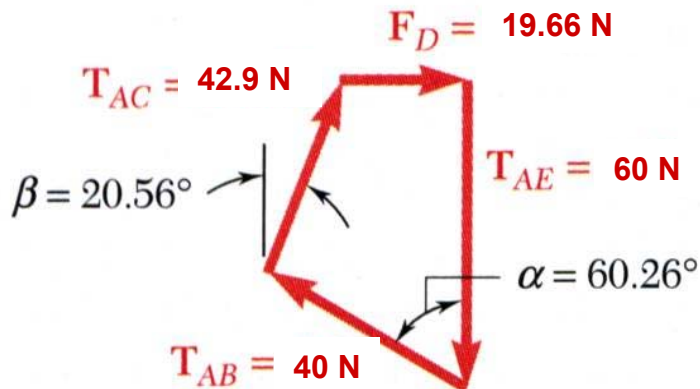
$$\begin{aligned}\vec{T}_{AC} &= T_{AC} \sin 20.56^\circ \vec{i} + T_{AC} \cos 20.56^\circ \vec{j} \\ &= 0.3512 T_{AC} \vec{i} + 0.9363 T_{AC} \vec{j}\end{aligned}$$

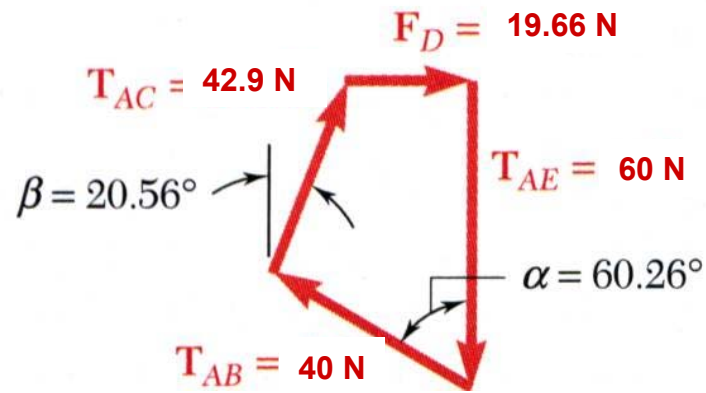
$$\vec{T} = -(60 \text{ N})\vec{j}$$

$$\vec{F}_D = F_D \vec{i}$$

$$\vec{R} = 0$$

$$\begin{aligned}&= (-34.73 + 0.3512 T_{AC} + F_D)\vec{i} \\ &\quad + (19.84 + 0.9363 T_{AC} - 60)\vec{j}\end{aligned}$$





$$\vec{R} = 0$$

$$= (-34.73 + 0.3512T_{AC} + F_D)\vec{i} + (19.84 + 0.9363T_{AC} - 60)\vec{j}$$

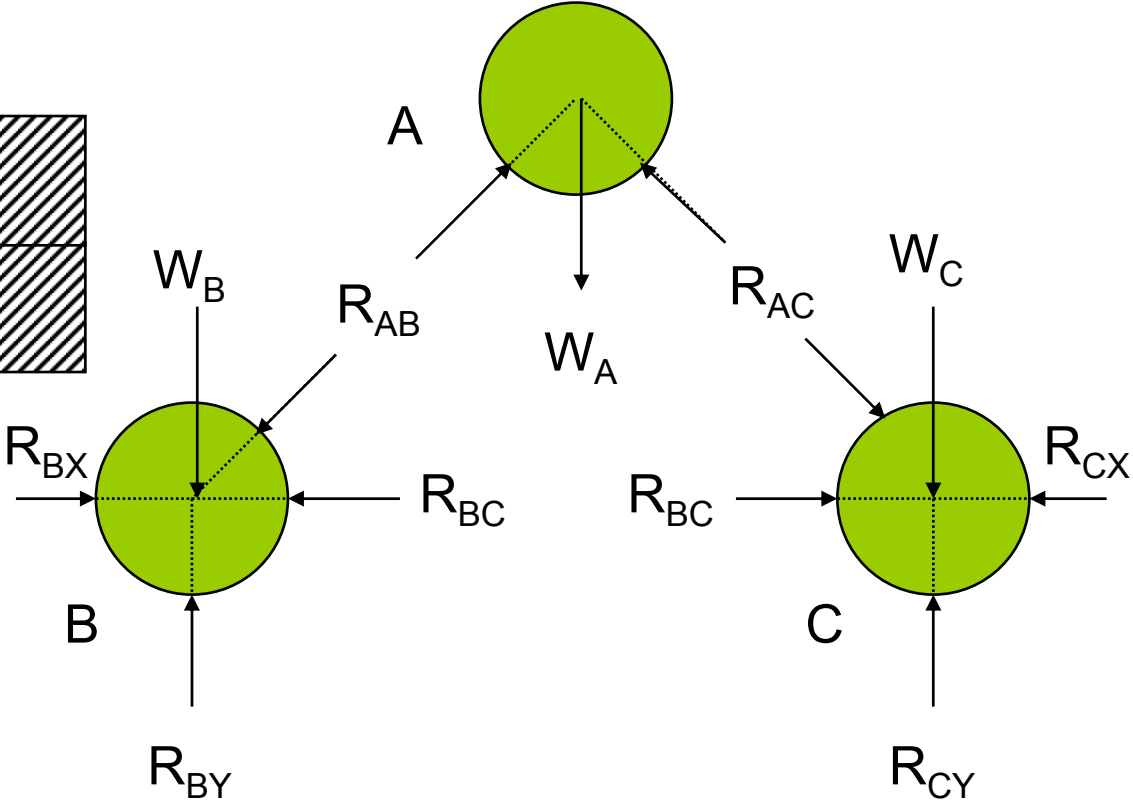
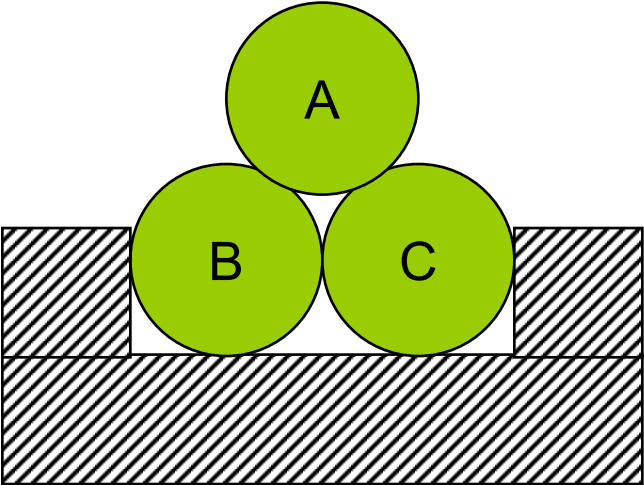
$$(\sum F_x = 0) \quad 0 = -34.73 + 0.3512T_{AC} + F_D$$

$$(\sum F_y = 0) \quad 0 = 19.84 + 0.9363T_{AC} - 60$$

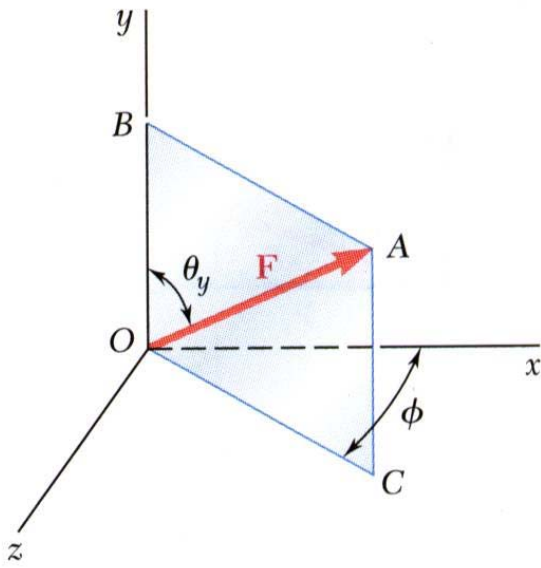
$$T_{AC} = +42.9 \text{ N}$$

$$F_D = +19.66 \text{ N}$$

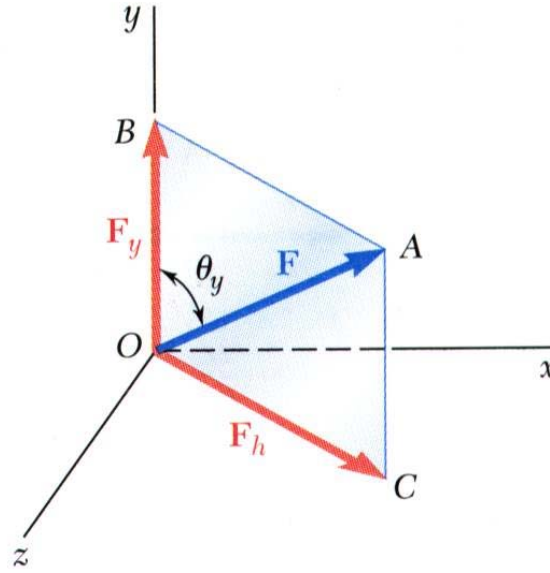
Serbest Cisim Diyagramları



Uzayda Dik Bileşenler



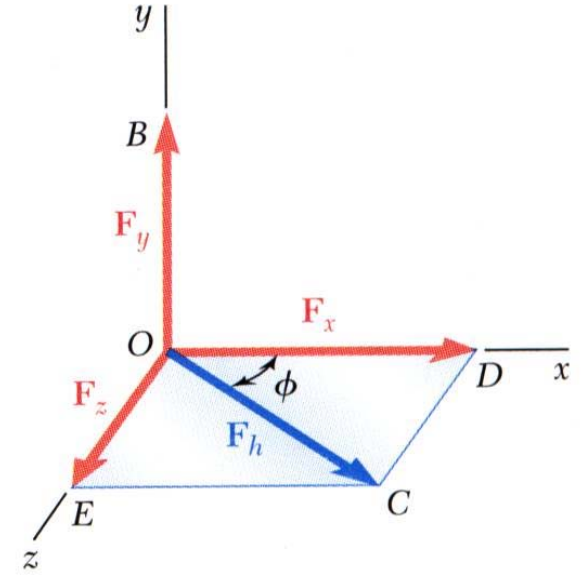
- Vektör $OBAC$ düzlemindedir.



- Düşey ve yatay bileşenleri:

$$F_y = F \cos \theta_y$$

$$F_h = F \sin \theta_y$$



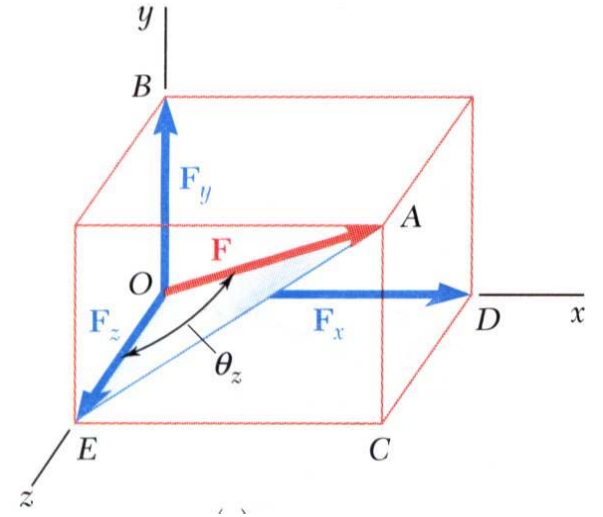
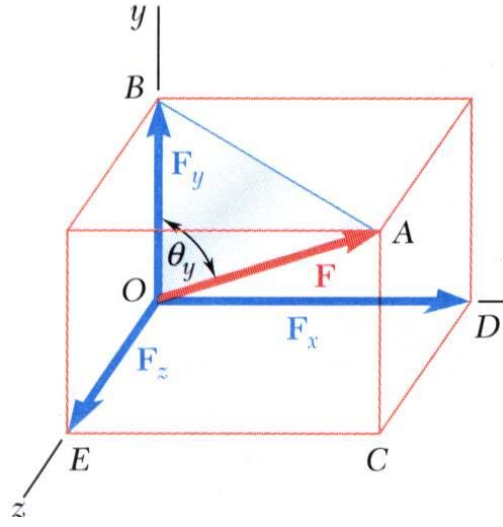
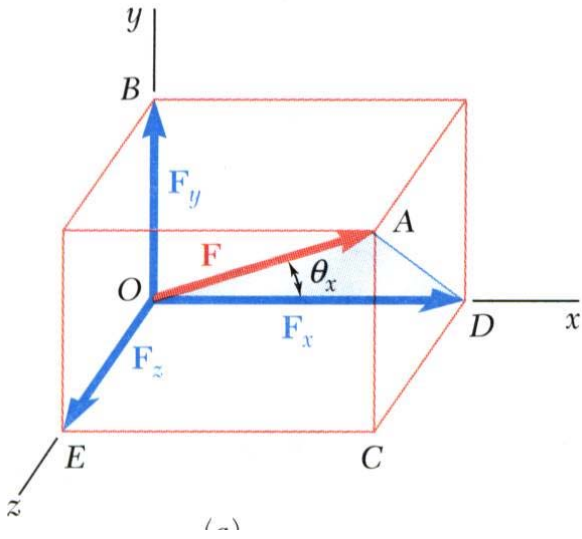
- Yatay bileşenlerin dik bileşenleri

$$F_x = F_h \cos \phi$$

$$= F \sin \theta_y \cos \phi$$

$$F_z = F_h \sin \phi$$

$$= F \sin \theta_y \sin \phi$$



- F vektörünün dik eksenlerle yaptığı açı biliniyorsa:

$$F_x = F \cos \theta_x \quad F_y = F \cos \theta_y \quad F_z = F \cos \theta_z$$

$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$

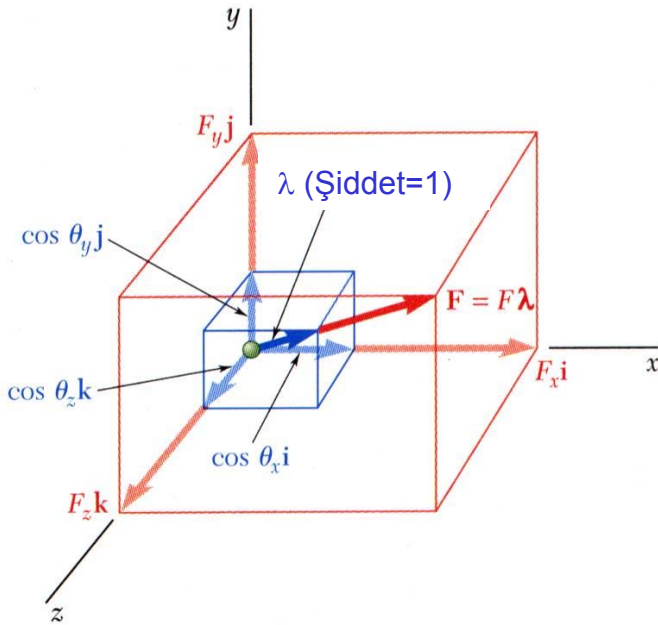
$$= F (\cos \theta_x \vec{i} + \cos \theta_y \vec{j} + \cos \theta_z \vec{k})$$

$$= F \vec{\lambda}$$

$$\vec{\lambda} = \cos \theta_x \vec{i} + \cos \theta_y \vec{j} + \cos \theta_z \vec{k}$$

- $\vec{\lambda}$ tesir çizgisi üzerindeki birim vektördür.

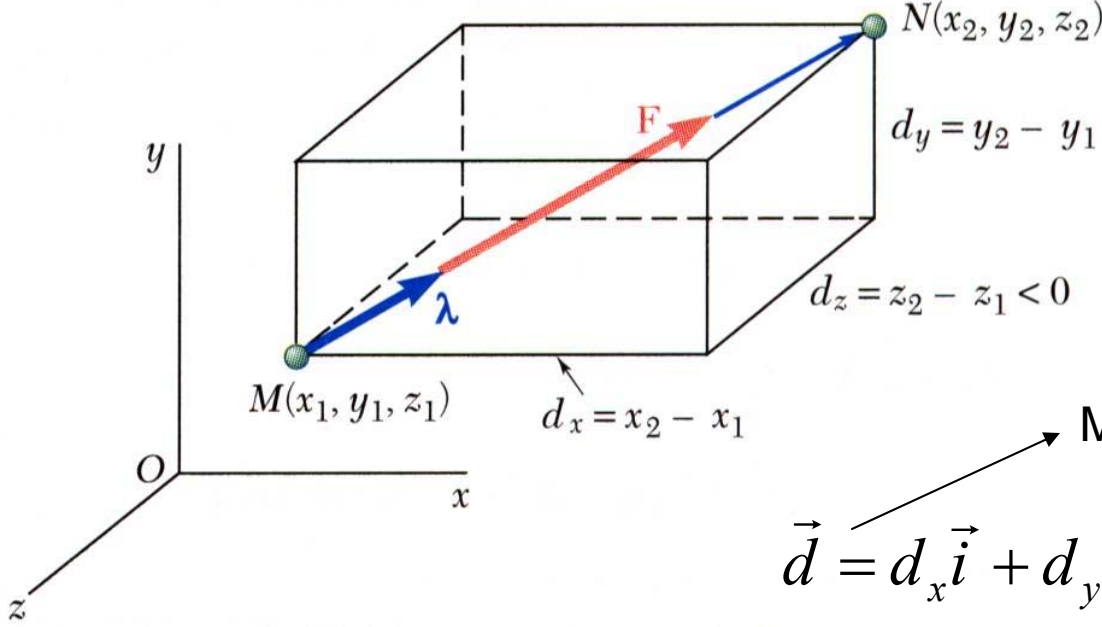
$\cos \theta_x$, $\cos \theta_y$ ve $\cos \theta_z$: doğrultman cosinüsleri



Tesir çizgisi üzerindeki iki nokta ile tanımlanan kuvvetin yönü

$$M(x_1, y_1, z_1) \text{ ve } N(x_2, y_2, z_2)$$

↓
Noktalar



M ve N noktalarını birleştiren vektör

$$\vec{d} = d_x \vec{i} + d_y \vec{j} + d_z \vec{k}$$

$$d_x = x_2 - x_1 \quad d_y = y_2 - y_1 \quad d_z = z_2 - z_1$$

$$\vec{\lambda} = \frac{1}{d} (d_x \vec{i} + d_y \vec{j} + d_z \vec{k})$$

$$\vec{F} = F \vec{\lambda}$$

$$F_x = \frac{F d_x}{d} \quad F_y = \frac{F d_y}{d} \quad F_z = \frac{F d_z}{d}$$