Name – Surname:

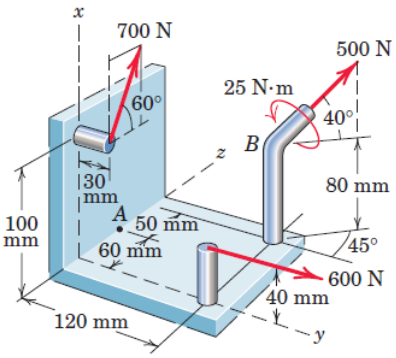
No.:

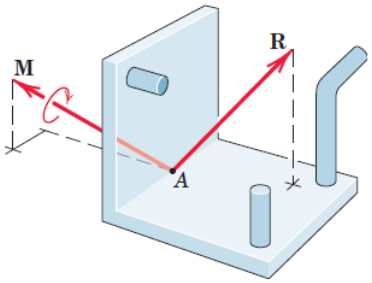
Istanbul Technical University – Faculty of Aeronautics and Astronautics **22.11.2019**
2019-2020 Fall Term
15549 STA201E Statics

Quiz – 7 - SOLUTION

Problem:

Replace the two forces and the negative wrench by a single force **R** applied at A and the corresponding couple **M**.





Solution. The resultant force has the components

$$[R_x = \Sigma F_x] \quad R_x = 500 \sin 40^\circ + 700 \sin 60^\circ = 928 \text{ N}$$

$$[R_y = \Sigma F_y] \quad R_y = 600 + 500 \cos 40^\circ \cos 45^\circ = 871 \text{ N}$$

$$[R_z = \Sigma F_z] \quad R_z = 700 \cos 60^\circ + 500 \cos 40^\circ \sin 45^\circ = 621 \text{ N}$$

Thus,
$$\mathbf{R} = 928\mathbf{i} + 871\mathbf{j} + 621\mathbf{k} \text{ N}$$

and
$$R = \sqrt{(928)^2 + (871)^2 + (621)^2} = 1416 \text{ N} \qquad \textit{Ans.}$$

The couple to be added as a result of moving the 500-N force is

1 $$[\mathbf{M} = \mathbf{r} \times \mathbf{F}] \quad \mathbf{M}_{500} = (0.08\mathbf{i} + 0.12\mathbf{j} + 0.05\mathbf{k}) \times 500(\mathbf{i} \sin 40^\circ + \mathbf{j} \cos 40^\circ \cos 45^\circ + \mathbf{k} \cos 40^\circ \sin 45^\circ)$$

where **r** is the vector from A to B.

The term-by-term, or determinant, expansion gives

$$\mathbf{M}_{500} = 18.95\mathbf{i} - 5.59\mathbf{j} - 16.90\mathbf{k} \text{ N·m}$$

2 The moment of the 600-N force about A is written by inspection of its x- and z-components, which gives

$$\mathbf{M}_{600} = (600)(0.060)\mathbf{i} + (600)(0.040)\mathbf{k}$$
$$= 36.0\mathbf{i} + 24.0\mathbf{k} \text{ N·m}$$

The moment of the 700-N force about A is easily obtained from the moments of the x- and z-components of the force. The result becomes

$$\mathbf{M}_{700} = (700 \cos 60^\circ)(0.030)\mathbf{i} - [(700 \sin 60^\circ)(0.060) + (700 \cos 60^\circ)(0.100)]\mathbf{j} - (700 \sin 60^\circ)(0.030)\mathbf{k}$$
$$= 10.5\mathbf{i} - 71.4\mathbf{j} - 18.19\mathbf{k} \text{ N·m}$$

3 Also, the couple of the given wrench may be written

$$\mathbf{M}' = 25.0(-\mathbf{i} \sin 40^\circ - \mathbf{j} \cos 40^\circ \cos 45^\circ - \mathbf{k} \cos 40^\circ \sin 45^\circ)$$
$$= -16.07\mathbf{i} - 13.54\mathbf{j} - 13.54\mathbf{k} \text{ N·m}$$

Therefore, the resultant couple on adding together the **i**-, **j**-, and **k**-terms of the four **M**'s is

4 $$\mathbf{M} = 49.4\mathbf{i} - 90.5\mathbf{j} - 24.6\mathbf{k} \text{ N·m}$$

and
$$M = \sqrt{(49.4)^2 + (90.5)^2 + (24.6)^2} = 106.0 \text{ N·m} \qquad \textit{Ans.}$$

Helpful Hints

1 *Suggestion:* Check the cross-product results by evaluating the moments about A of the components of the 500-N force directly from the sketch.

2 For the 600-N and 700-N forces it is easier to obtain the components of their moments about the coordinate directions through A by inspection of the figure than it is to set up the cross-product relations.

3 The 25-N·m couple vector of the *wrench* points in the direction opposite to that of the 500-N force, and we must resolve it into its x-, y-, and z-components to be added to the other couple-vector components.

4 Although the resultant couple vector **M** in the sketch of the resultants is shown through A, we recognize that a couple vector is a free vector and therefore has no specified line of action.