Istanbul Technical University – Faculty of Aeronautics and Astronautics **04.10.2019**
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15549 STA201E Statics

Quiz – 1 - SOLUTION

Problem: Consider the two vectors $\vec{A} = \vec{\imath} - \vec{\jmath} + 2\vec{k}$, and $\vec{B} = 5\vec{\imath} + 3\vec{\jmath} - 2\vec{k}$. Find,

- **(a)** the magnitude of both vectors,
- **(b)** angle between the vectors,
- **(c)** unit vectors in directions of the vectors,
- **(d)** a unit vector that is perpendicular to both vectors, and
- **(e)** the dot product of the vectors.

Solution:

(a) $A = |\vec{A}| = \sqrt{(1)^2 + (-1)^2 + (2)^2} = \sqrt{6} = 2.449$
$B = |\vec{B}| = \sqrt{(5)^2 + (3)^2 + (-2)^2} = \sqrt{38} = 6.164$

(b) $\vec{A} \cdot \vec{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$
$\cos\theta = [(1)(5) + (-1)(3) + (2)(-2)]/[(2.449)(6.1644)] \rightarrow \theta = 97.61°$

(c) $\vec{\lambda}_A = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{\imath} - \vec{\jmath} + 2\vec{k}}{2.449} = 0.408\vec{\imath} - 0.408\vec{\jmath} + 0.816\vec{k}$

$\vec{\lambda}_B = \dfrac{\vec{B}}{|\vec{B}|} = \dfrac{5\vec{\imath} + 3\vec{\jmath} - 2\vec{k}}{6.164} = 0.811\vec{\imath} - 0.486\vec{\jmath} + 0.324\vec{k}$

(d) The vector that is the vector product of the two vectors will be perpendicular to both vectors $\vec{V} = \vec{A} \times \vec{B}$, therefore a unit vector on the line of action of the cross product will also be perpendicular to them.

$$\vec{V} = \vec{A} \times \vec{B} = \left(\vec{\imath} - \vec{\jmath} + 2\vec{k}\right) \times \left(5\vec{\imath} + 3\vec{\jmath} - 2\vec{k}\right) = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ 1 & -1 & 2 \\ 5 & 3 & -2 \end{vmatrix} = -4\vec{\imath} + 12\vec{\jmath} + 8\vec{k}$$

(e) $\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = (1)(5) + (-1)(3) + (2)(-2) = -2$