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ITU Faculty of Aeronautics and Astronautics
Department of Aeronautical Engineering
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11230 DNK201E Dynamics QUIZ – 2

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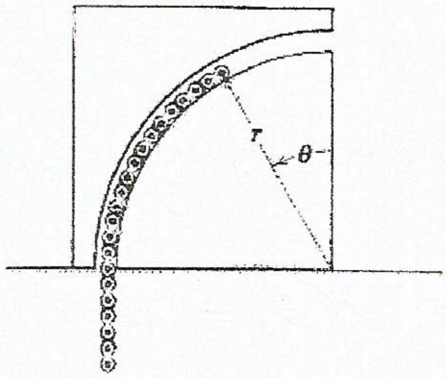
Problem: The flexible bicycle-type chain of length $\pi r/2$ and mass per unit length ρ is released from rest with $\theta = 0$ in the smooth circular channel and falls through the hole in the supporting surface. The velocity of the chain in dimensionless form is

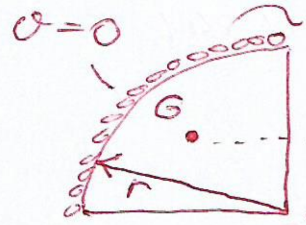


$$v' = \sqrt{2}\sqrt{\frac{1}{\pi}(2+\theta^2) - \left(1-\frac{2\theta}{\pi}\right)\frac{\sin^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{\frac{\pi}{4}-\frac{\theta}{2}}}$$

Calculate the velocity of the chain in decimals for $\theta = 0$, $\theta = 30°$, $\theta = 60°$, and $\theta = 90°$.

Solution: Since chain is of constant length and all points of chain follow same path all must move at same speed. Since no dissipative forces are present conservation of mechanical energy principle can be applied.

At time $t=0 \rightarrow \theta = 0$:



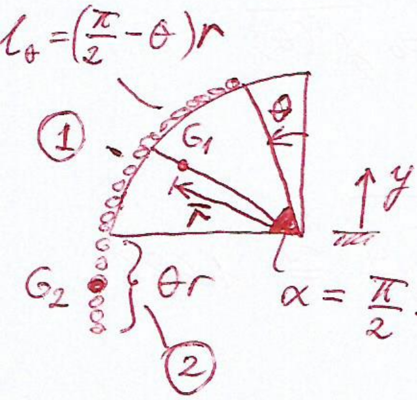
$m = \rho l = \rho \frac{\pi r}{2}$

$h_0 = \frac{2r}{\pi}$ $\uparrow y$

Total mechanical energy:

$U_1 = \cancel{T_1}^{0} + V_1 = m g h_0$

At some other θ position:



$\alpha = \frac{\pi}{2} - \theta$

$$\bar{r} = \frac{r \sin \alpha/2}{\alpha/2}$$

New Center of mass of chain

Elem.	mass	$\bar{y}_i$
1	ρl_θ	$\bar{r}\sin\alpha/2$
2	$\rho\theta r$	$-\theta r/2$

$$m\bar{Y} = m_1 y_1 + m_2 y_2 = \rho\left(\frac{\pi}{2}-\theta\right)r\,\frac{r\sin^2\frac{\alpha}{2}}{\alpha/2} - \rho\theta r\frac{\theta r}{2}$$

$$U_2 = T_2 + V_2 = \frac{1}{2}mv^2 + mg\bar{Y}$$

$$\Rightarrow \rho\frac{\pi r}{2}g\cdot\frac{2r}{\pi} = \frac{1}{2}\underbrace{m}_{\rho\frac{\pi r}{2}}v^2 + \rho g\left(\frac{\pi}{2}-\theta\right)r\,\frac{r\sin^2\alpha/2}{\alpha/2} - \rho g\theta r\frac{\theta r}{2}$$

rearranging the terms and solving for v:

$$v = v(\theta) = \sqrt{2gr}\left[\frac{1}{\pi}(2+\theta^2) - \left(1-\frac{2\theta}{\pi}\right)\frac{\sin^2\left(\frac{\pi}{4}-\frac{\theta}{2}\right)}{\frac{\pi}{4}-\frac{\theta}{2}}\right]^{1/2}$$

nondimensional velocity: $v' = \dfrac{v}{\sqrt{gr}}$

<u>v' values for $\theta = 0, 30°, 60°, 90°$:</u>

$$v'(0) = \sqrt{2}\sqrt{\frac{1}{\pi}\cdot(2+0^2) - \left(1-\frac{2\cdot 0}{\pi}\right)\frac{\sin^2\pi/4}{\pi/4}} = \sqrt{2}\sqrt{\frac{2}{\pi}-\frac{2}{\pi}} = 0$$

$$v'(\pi/6) = \sqrt{2}\sqrt{\frac{1}{\pi}\left(2+\left(\frac{\pi}{6}\right)^2\right) - \left(1-\frac{2\cdot\pi}{6\pi}\right)\frac{\sin^2\pi/6}{\pi/6}} \cong 0.900$$

$$v'(\pi/3) = \sqrt{2}\sqrt{\frac{1}{\pi}\left(2+\left(\frac{\pi}{3}\right)^2\right) - \left(1-\frac{2\pi}{3\pi}\right)\frac{\sin^2\pi/12}{\pi/12}} \cong 1.341$$

$v'(\pi/2)$: $\displaystyle\lim_{\theta\to\frac{\pi}{2}} \frac{\left(1-\frac{2\theta}{\pi}\right)}{\frac{\pi}{4}-\frac{\theta}{2}} = \frac{0}{0} \rightarrow$ indefinite

L'Hospitale rule: $\displaystyle\lim_{\theta\to\theta_0}\frac{f(\theta)}{g(\theta)} = \lim_{\theta\to\theta_0}\frac{f'(\theta)}{g'(\theta)}$

$$\Rightarrow \lim_{\theta\to\frac{\pi}{2}}\frac{\left(1-\frac{2\theta}{\pi}\right)}{\left(\frac{\pi}{4}-\frac{\theta}{2}\right)} = \frac{-2/\pi}{-1/2} = \frac{4}{\pi}$$

$$\Rightarrow v'(\pi/2) = \sqrt{2}\sqrt{\frac{1}{\pi}\left(2+\left(\frac{\pi}{2}\right)^2\right) - \left\{\underbrace{\cancel{\sin^2\left(\frac{\pi}{4}-\frac{\pi}{4}\right)}}_{0}\right\}\frac{4}{\pi}} \cong 1.686$$