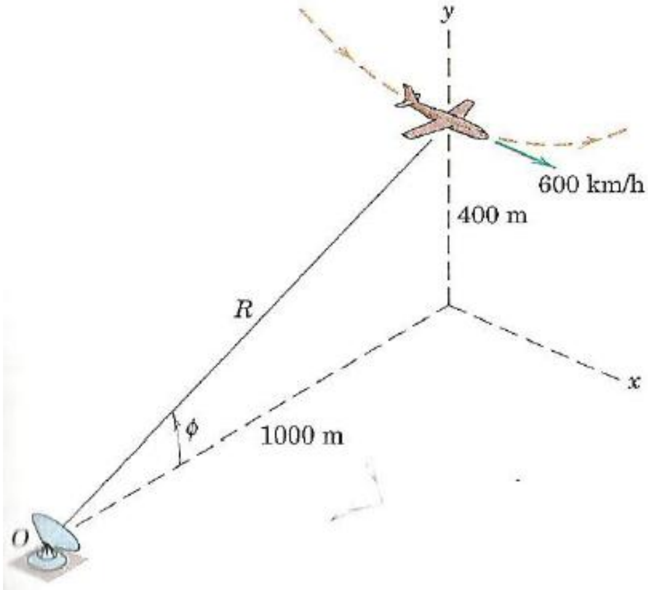
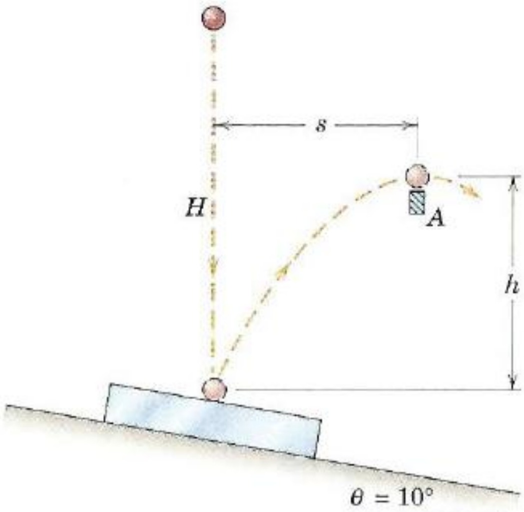
Problem Solutions

<u>Problem 1:</u> At the bottom of a vertical loop in the *x*-*y* plane at an altitude of $400\,m$, the airplane has a speed of $600\,km/h$ with no horizontal *x*-acceleration. The radius of curvature of the loop at the bottom is $1200\,m$.

(a) Find the velocity and acceleration vector of the airplane.
(b) For the radar tracking at *O*, determine the recorded values of $\ddot{R}$ and $\ddot{\phi}$ for this instant.

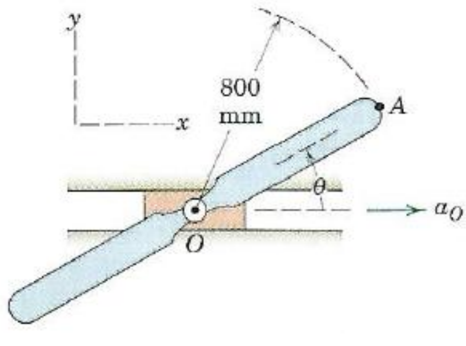


<u>Problem 2:</u> To pass inspection, steel balls designed for use in ball bearings must clear the fixed bar *A* at the top of their rebound when dropped from rest through the vertical distance $H = 900\,mm$ onto the heavy inclined steel plate. If balls which have a coefficient of restitution of less than 0.7 with the rebound plate are to be rejected, determine the position of the bar by specifying h and s. Neglect any friction during impact.

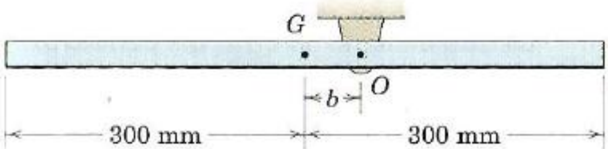


<u>Problem 3:</u> The two rotor blades of $800\,mm$ radius rotate counterclockwise with a constant angular velocity $\omega = \dot{\theta} = 2\,rad/s$ about the shaft at *O* mounted in the sliding block. The acceleration of the block is $a_O = 3\,m/s^2$. Determine the <u>magnitude</u> of the acceleration of the tip *A* of the blade when

(a) $\theta = 0$,
(b) $\theta = 90°$,
(c) $\theta = 180°$.



<u>Problem 4:</u> The uniform 8-kg slender bar is hinged about a horizontal axis through *O* and released from rest in the horizontal position. Determine the distance b from the mass center to *O* which will result in an initial angular acceleration of $16\,rad/s^2$, and find the force *R* on the bar at *O* just after release. $\left(I_G = \frac{1}{12}ml^2\right)$



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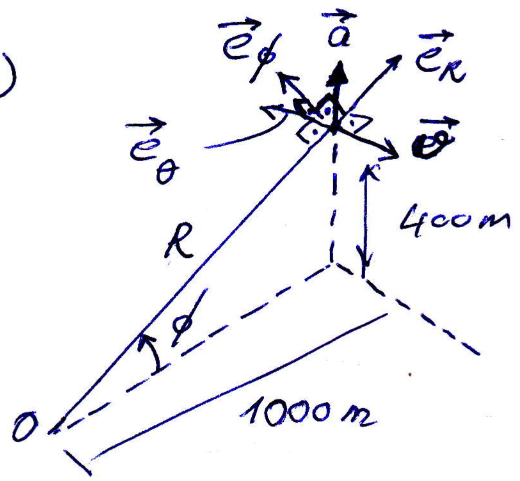
SOLUTION OF PROBLEMS

Problem 1: (a) $\vec{v} = 600/3{,}6\,\vec{i} = 166{,}\bar{6}\,\vec{i}$ m/s

$$\vec{a} = \underbrace{\cancel{\vec{a}_t}}_{0} + \vec{a}_n = (v^2/\rho)\,\vec{j} = (166{,}\bar{6})^2/1200\,\vec{j}$$

$$= 23{,}14\,\vec{j} \text{ m/s}$$

(b)



$$\vec{a} = a_R\vec{e}_R + a_\phi\vec{e}_\phi + \underbrace{\cancel{a_\theta}}_{0}\vec{e}_\theta$$

$$= |\vec{a}|\sin\phi\,\vec{e}_R + |\vec{a}|\cos\phi\,\vec{e}_\phi$$

$$\phi = \arctan(400/1000) = 21.8^\circ$$

$$\theta = 0, \quad R = \sqrt{(400)^2 + (1000)^2} = 1077\text{m}$$

$$\rightarrow \quad a_R = 23{,}14\cdot\sin 21{,}8^\circ = 8{,}59 \text{ m/s}^2$$

$$a_\phi = 23{,}14\cos 21{,}8^\circ = 21{,}48 \text{ m/s}^2$$

$$\rightarrow \begin{cases} a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi = 8.59 \text{ m/s}^2 & (1) \\ a_\phi = R\ddot{\phi} + 2\dot{R}\dot{\phi} + R\dot{\theta}^2\sin\phi\cos\phi = 21{,}48 \text{ m/s}^2 & (2) \end{cases}$$

$$\vec{v} = \dot{R}\vec{e}_R + R\dot{\phi}\vec{e}_\phi + R\dot{\theta}\cos\phi\,\vec{e}_\theta = -166{,}\bar{6}\,\vec{e}_\theta$$

$$\dot{R} = 0, \quad \dot{\phi} = 0 \qquad (3)$$

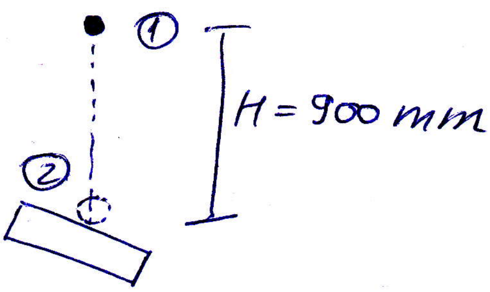
$$\dot{\theta} = -166{,}\bar{6}/(1000) = -0{,}166\bar{6} \text{ rad/s} \qquad (4)$$

(3),(4) → (1),(2): $\ddot{R} = 8{,}59 + 1077\cdot(-0{,}166\bar{6})^2\cos^2 21{,}8$

$$= 34{,}4 \text{ m/s}^2$$

$$\ddot{\phi} = \left(21{,}48 - 1077(-0{,}166\bar{6})^2\sin 21{,}8^\circ\cos 21{,}8^\circ\right)/1077 = 0{,}0104 \ \frac{\text{rad}}{\text{s}^2}$$

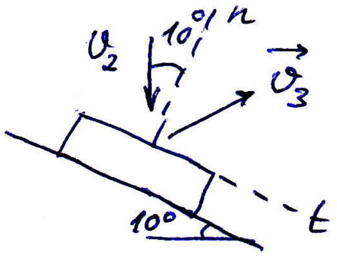
Problem 2:

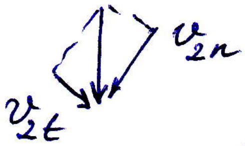


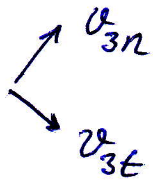
$$E_1 = E_2 \rightarrow mgH = \tfrac{1}{2}mv_2^2$$

$$v_2 = (2gH)^{1/2} = (2\cdot 9.81\cdot 0.9)^{1/2}$$

$$= 4{,}20 \text{ m/s}$$







t-direction:

$$v_{2t} = v_{3t} = 4{,}2 \cdot \sin 10° = 0{,}729 \text{ m/s}$$

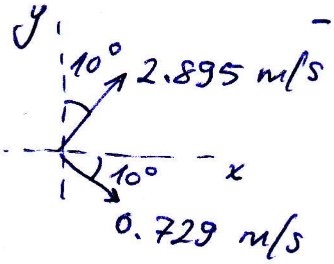
n-direction:

$$e = \frac{\cancel{v'_{ball}} - \cancel{v'_{plate}}^{\,0}}{\cancel{v_{plate}}^{\,0} - v_{ball}}$$

$$\rightarrow \quad e = -\frac{v'_{ball}}{v_{ball}} = -\frac{v_{3n}}{v_{2n}}$$

~~$\rightarrow \quad 0.7 = -v_{3n}/(-0.729) \rightarrow v_{3n} = 0{,}51$ m/s~~

$$0.7 = -\frac{v_{3n}}{-4.2\cos 10°} \longrightarrow v_{3n} = 2{,}895 \text{ m/s}$$



$$v_{3x} = 0.729 \cos 10° + 2{,}895 \sin 10° = 1.22 \text{ m/s}$$

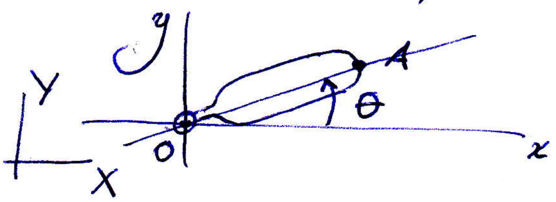
$$v_{3y} = -0.729 \sin 10° + 2{,}895 \cos 10° = 2{,}72 \text{ m/s}$$

$$h_{max} = \frac{v_{3y}^2}{2g} = \frac{(2{,}72)^2}{2 \cdot 9{,}81} = \underline{0{,}377 \text{ m} = 377 \text{ mm}}$$

$$t_{max} = \frac{v_{3y}}{g} = \frac{2.72}{9.81} = 0{,}277 \text{ s}$$

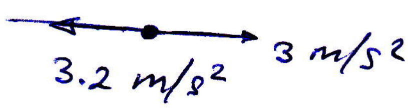
$$s = v_{3x} \cdot t_{max} = 1{,}22 \cdot 0{,}277 = \underline{0{,}337 \text{ m} = 337 \text{ mm}}$$

Problem 3: (a)



$$\vec{a}_A = \vec{a}_O + \cancel{\vec{\alpha}}^{\,0} \times \vec{r}_{OA} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r}_{OA})}_{\text{normal acc.}}$$

$\theta = 0°$:

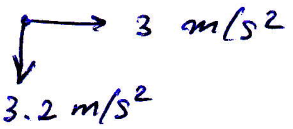


$$\vec{a} = -0{,}2\, \vec{i} \text{ m/s}$$

$$a = |\vec{a}| = 0.2 \text{ m/s}$$

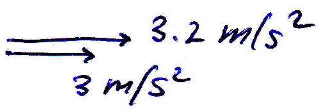
$$(a_{A/O})_n = (v_{A/O})^2 / r_{A/O} = \omega^2 r_{A/O} = 2^2 \cdot 0.8 = 3.2 \text{ m/s}^2$$

(b) $\theta = 90°$:



$$|\vec{a}| = \sqrt{(3)^2 + (3.2)^2} = 4.38 \text{ m/s}^2$$

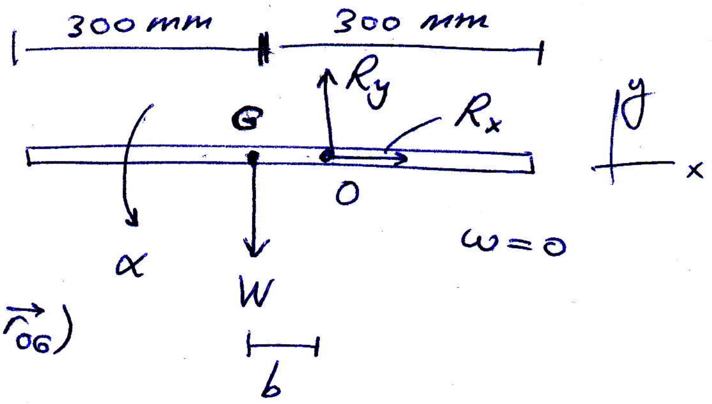
(c) $\theta = 180°$:



$$|\vec{a}| = 3 + 3.2 = 6.2 \text{ m/s}^2$$

Problem 4: rotation about a fixed point.

Free Body Diagram:



Equations of motion:

$$\vec{a}_G = \cancel{\vec{a}_0}^{\,0} + \vec{\alpha} \times \vec{r}_{OG} + \cancel{\vec{\omega} \times (\vec{\omega} \times \vec{r}_{OG})}^{\,0}$$

$$= 16\vec{k} \times (-b\vec{i}) = -16b\,\vec{j}$$

$\Sigma F_x = m a_{Gx}$: $R_x = 0$

$\Sigma F_y = m a_{Gy}$: $R_y - W = -16 b \cdot m$ (1)

$+\circlearrowright \Sigma M_G = I_G \alpha$: $b R_y = \frac{1}{12} m l^2 \cdot \alpha$ (2)

(1), (2) : $\frac{1}{12} \frac{m l^2 \alpha}{b} - mg = -16 b m$

$\longrightarrow$ $b^2 - 0{,}613 b + 0{,}03 = 0$

$\longrightarrow$ $b = \frac{0{,}613 \pm \sqrt{(-0{,}613)^2 - 4 \cdot 1 \cdot 0{,}03}}{2}$

$b_1 = 0{,}559$ m $\boxed{b_2 = 0{,}0536 \text{ m} = 53{,}6 \text{ mm}}$

$$R_y = \frac{8 \cdot 0{,}6^2 \cdot 16}{12 \cdot 0{,}0536} = 71{,}6 \text{ N}$$