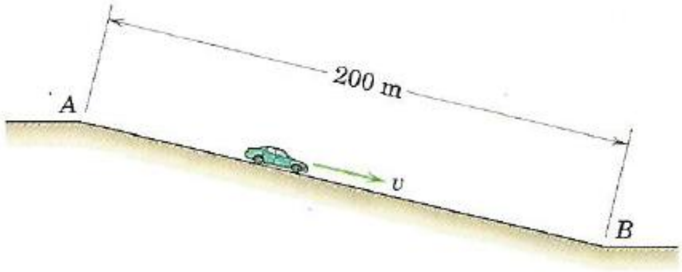
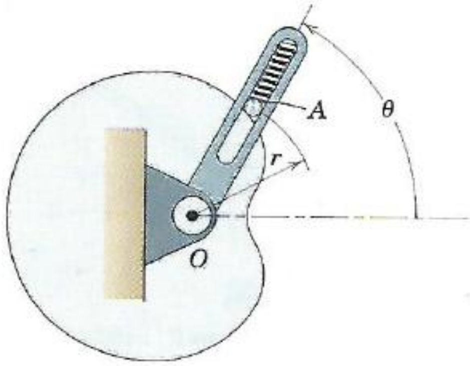
ITU Faculty of Aeronautics and Astronautics **09.11.2015**
Department of Aeronautical Engineering
2015-2016 Fall Term
11230 DNK201E Dynamics MIDTERM EXAM – 1

<u>**Problem 1:**</u> The driver of a car, which is initially at rest at the top *A* of the grade, releases the brakes and coasts down the grade with an acceleration in meters per second squared given by $a = 0.981 - 0.013v^2$, where v is the velocity in meters per second. Determine the velocity v_B at the bottom *B* of the grade.

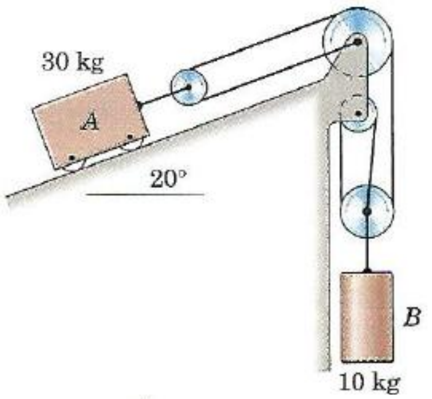


<u>**Problem 2:**</u> The cam is designed so that the center of the roller *A* which follows the contour moves on a limaçon defined by $r = b - c\cos\theta$, where $b > c$. If the cam does not rotate,

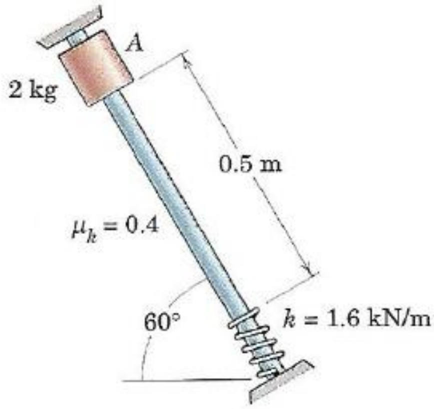


(a) determine the magnitude of the total acceleration of *A* in terms of θ if the slotted arm revolves with a constant counterclockwise angular rate $\dot{\theta} = \omega$.
(b) If $b = 100\text{ mm}$, $c = 75\text{ mm}$, and $\omega = 40\text{ rev/min}$, calculate the magnitude of the total acceleration of *A*.

<u>**Problem 3:**</u> Consider the mass and pulley system given below with $m_A = 30\text{ kg}$ and $m_B = 10\text{ kg}$. Neglect all friction and the mass of the pulleys and determine the accelerations of bodies *A* and *B* upon release from rest. Also state in which direction the masses will move.

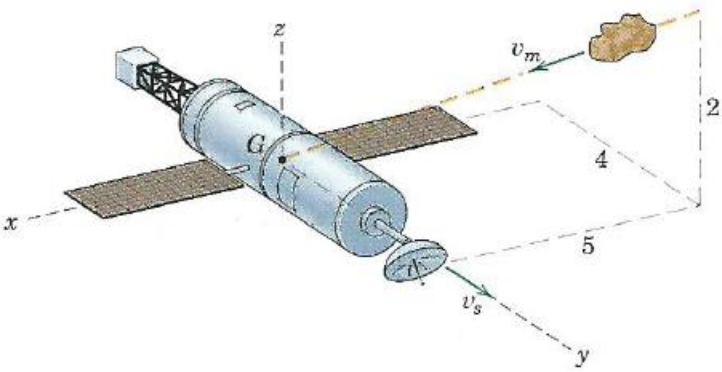


<u>**Problem 4:**</u> The 2-kg collar is released from rest at *A* and slides down the inclined fixed rod in the vertical plane. The coefficient of kinetic friction is 0.4. Calculate



(a) the velocity v of the collar as it strikes the spring and
(b) the maximum deflection x of the spring.

<u>**Problem 5:**</u> A 1000-kg spacecraft is travelling in deep space with a speed of $v_s = 2000\text{ m/s}$ when a 10-kg meteor moving with a velocity $\vec{v}_m$ of magnitude 5000 m/s in the direction shown strikes and becomes embedded in the spacecraft. Determine the final velocity $\vec{v}$ of the mass center *G* of the spacecraft. Calculate the angle β between $\vec{v}$ and the initial velocity $\vec{v}_s$ of the spacecraft.



11230 DNK DYNAMICS MIDTERM EXAM - 1 SOLUTION
2015 - 2016 FALL TERM
09 NOVEMBER 2015

Problem 1:

$$a\,ds = v\,dv \longrightarrow \int_0^{200m} ds = \int_0^{v_B} \frac{v\,dv}{0.981 - 0.013v^2}$$

$$\left[s\right]_{s=0}^{s=200m} = \left[-\frac{1}{0.026}\ln\left(0.981 - 0.013v^2\right)\right]_{v=0}^{v=v_B}$$

$$200 = -\frac{1}{0.026}\ln\left\{\frac{0.981 - 0.013v_A^2}{0.981}\right\}$$

$$\Rightarrow \boxed{v_B = 8.66 \text{ m/s}}$$

Problem 2: (a) Polar coordinates : $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{n}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{n}_\theta$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \quad , \quad \dot{\theta} = \omega = \text{const} \longrightarrow \ddot{\theta} = \dot{\omega} = 0$$

$$r = b - c\cos\theta \longrightarrow \dot{r} = +c\dot{\theta}\sin\theta$$

$$\ddot{r} = +c\ddot{\theta}\sin\theta + c\dot{\theta}^2\cos\theta$$

$$a_r = +c\underbrace{\cancel{\ddot{\theta}}}_{0}\cos\theta + c\dot{\theta}^2\cos\theta - (b - c\cos\theta)\cdot\dot{\theta}^2$$

$$= -b\dot{\theta}^2 + 2c\dot{\theta}^2\cos\theta$$

$$a_\theta = 2\cdot c\dot{\theta}\sin\theta\cdot\dot{\theta}^2 = 2c\dot{\theta}^2\sin\theta$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \left\{(-b\dot{\theta}^2 + 2c\dot{\theta}^2\cos\theta)^2 + (2c\dot{\theta}^2\sin\theta)^2\right\}^{1/2}$$

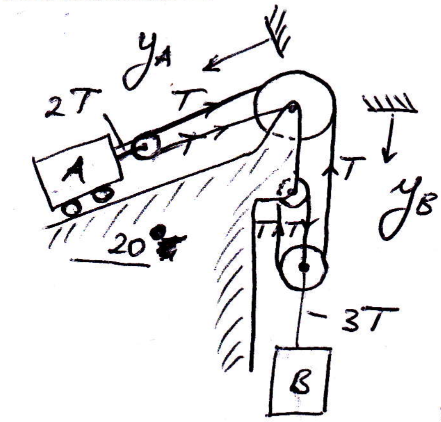
$$= \dot{\theta}^2\left\{b^2 - 4bc\cos\theta + 4c^2\cos^2\theta + 4c^2\sin^2\theta\right\}^{1/2}$$

$$= \boxed{\omega^2\sqrt{b^2 - 4bc\cos\theta + 4c^2} = a}$$

(b) $$a(\theta = \pi/2) = \left(40\cdot\frac{2\pi}{60}\right)^2\sqrt{(0.1)^2 - 4(0.1)(0.075)\cdot\underbrace{\cancel{\cos\frac{\pi}{2}}}_{0} + 4\cdot(0.075)^2}$$

$$= \boxed{3.16 \text{ m/s}^2 = a}$$

Problem 3:



since friction is neglected T will be constant throughout rope.
Length of rope through pulleys is constant, therefore:

$$2y_A + 3y_B = \text{const} \rightarrow 2a_A + 3a_B = 0 \quad (1)$$

Eqs. of motion of A & B:

$$m_A g \sin 20° - 2T = m_A a_A \quad (2) \qquad m_B g - 3T = m_B a_B \quad (3)$$

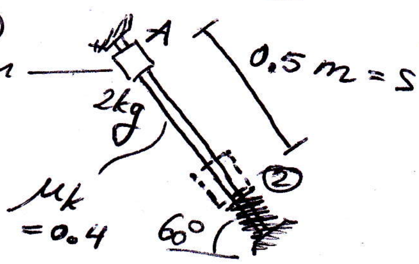
(2), (3) : $2m_B g - 3m_A g \sin 20° = 2m_B a_B - 3m_A a_A \quad (4)$

(1), (4) : $a_A = 1.023 \rightarrow$

$a_B = -0.682 \rightarrow$

$$\boxed{\begin{array}{l} a_A = 1.023\ m/s^2 \text{ down} \\ a_B = 0.682\ m/s^2 \text{ up} \end{array}}$$

Problem 4: (a) change of mechanical energy = work done by non-conservative forces

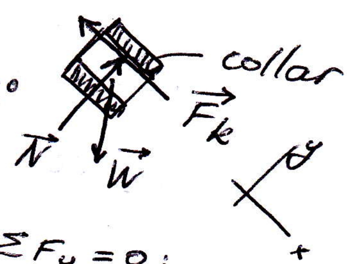


$$U_{12} = \Delta(T+V) = (T_2 + V_2) - (\cancel{T_1}_0 + \cancel{V_1}_0)$$

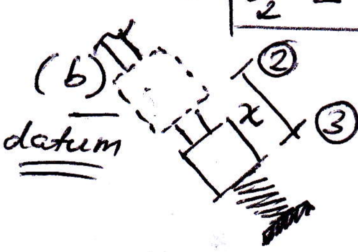
$$-\mu_k \cdot N \cdot s = \frac{1}{2} m v_2^2 - mg s \sin 60°$$

$$-0.4 \cdot 2 \cdot 9.81 \cdot \cos 60° \cdot 0.5 = \frac{1}{2} \cdot 2 \cdot v_2^2 - 2 \cdot 9.81 \cdot 0.5 \cdot \sin 60°$$

$$\rightarrow \boxed{v_2 = 2.56\ m/s}$$



(b)



$$U_{23} = \cancel{T_3}_0 + V_3 - (T_2 + \cancel{V_2}_0)$$

$$-\mu_k N \cdot x = -mgx \sin 60° - \frac{1}{2} m v_2^2 + \frac{1}{2} k x^2$$

$\Sigma F_y = 0$:

$N - W\cos 60° = 0$

$F_k = \mu_k \cdot N$

$$-0.4 \cdot 2 \cdot 9.81 \cdot \cos 60° \cancel{x} = -2 \cdot 9.81 \cdot x \sin 60° - \frac{1}{2} \cdot 2 \cdot 2.56^2 + \frac{1}{2} \cdot 1600 \cdot x^2$$

$$\rightarrow 800x^2 - 13.06x - 6.55 = 0 \Longrightarrow \boxed{x \cong 99\ mm}$$

Problem 5:

$$\vec{v}_m = v_m \vec{\lambda}_m = 5000 \cdot (5\vec{i} - 4\vec{j} - 2\vec{k}) / \sqrt{(5)^2 + (-4)^2 + (-2)^2}$$

$$= 3726{,}7\vec{i} - 2981{,}4\vec{j} - 1490{,}7\vec{k}\ m/s$$

cons. of lin. momentum: $M_S \vec{V}_S + m_m \vec{v}_m = (M_S + m_m)\vec{v}$

$$\rightarrow (1000)(2000\vec{j}) + (10)(3726{,}7\vec{i} - 2981{,}4\vec{j} - 1490{,}7\vec{k}) = (1010)\vec{v}$$

$$\boxed{\vec{v} = 36.9\vec{i} + 1951\vec{j} - 14.75\vec{k}\ m/s}$$

Angle between $\vec{v}$ and $\vec{v}_S$:

$$\vec{v} \cdot \vec{v}_S = |\vec{v}||\vec{v}_S| \cos\beta \rightarrow \cos\beta = \frac{2000 \cdot 1951}{2000 \cdot 1951.4} \rightarrow \boxed{\beta \cong 1.16°}$$