Problem 1: (15 p) A particle $P$ moves along a straight line and its velocity varies linearly with time as given in the figure below. At time $t = 0$ the position of the particle is $x_0 = 0$.

(a) Find the functional relationships $a = a(t)$ and $x = x(t)$.
(b) Draw the graphs $a$-$t$ and $x$-$t$ in the interval 0-4s.
(c) Calculate the total distance the particle has travelled in the interval 0-4s.

Problem 2: (20 p) The speed of a car increases uniformly with time from 50 km/h at $A$ to 100 km/h at $B$ during 10 seconds. The radius of curvature of the hump at $A$ is 40 m. If the magnitude of the total acceleration of the car's mass center is the same at $B$ as at $A$, compute the radius of curvature $\rho_B$ of the dip in the road at $B$. The mass center of the car is 0.6 m from the road.

Problem 3: (30 p) The aircraft $A$ with radar detection equipment is flying horizontally at an altitude of 12 km and is increasing its speed at the rate 1.2 m/s each second. Its radar locks onto an aircraft flying in the same direction and in the same vertical plane at an altitude of 18 km. If $A$ has a speed of 1000 km/h at the instant when $\theta = 30^\circ$, determine the values of $\ddot{r}$ and $\ddot{\theta}$ at this same instant if $B$ has a constant speed of 1500 km/h.

Problem 4: (15 p) The 120-g slider has a speed $v = 1.4 \, m/s$ as it passes point $A$ of the smooth guide, which lies in a horizontal plane. Determine the magnitude $R$ of the force which the guide exerts on the slider

(a) just before it passes point $A$ of the guide,
(b) as it passes point $B$.

Problem 5: (20 p) A 5-kg block is attached to a rigid bar of negligible mass which is pivoted at point $O$. The spring of stiffness $k = 700 \, N/m$ is attached to the middle of the bar and is undeformed when the bar is released from rest in the horizontal position. Calculate the speed of the block at $\theta = 30^\circ$. 
Problem 1: (a) \( v(t) = \frac{45 - (-0.5)}{0.5} t - 0.5 = 0.5(t-1) \text{ m/s} \)

\[ a = \frac{dv}{dt} = 0.5 \text{ m/s}^2 \]

\[ \frac{dx}{dt} = v \rightarrow \int dx = \int v dt \Rightarrow x = 0 = [0.25t^2 - 0.5t] \]

\[ x = x(t) = 0.25t^2 - 0.5t \]

(b) 

(c) distance travelled: \( 0.25 + 2.25 = 2.5 \text{ m} \)

Problem 2:

\[ \begin{align*}
\alpha & = \text{at between points A and B} : \alpha = \frac{(100 - 50)/3.6}{10} = 1.38 \text{ m/s}^2 \\
\alpha^2 & = \alpha_A^2 + \alpha_B^2 = \alpha_A^2 + \alpha_A \alpha_B = \alpha_A (\alpha_A + \alpha_B) \\
\alpha_A & = \alpha_B \Rightarrow (\frac{50}{3.6})^2 = (\frac{100}{3.6})^2 \\
& \Rightarrow \rho_A = 163 \text{ m} \\
\end{align*} \]

Problem 3:

\( v_A = 1500 \text{ km/h} = \text{const, } \alpha_A = 0 \)

\( v_B = 1000 \text{ km/h} \)

\( a_A = 1.2 \text{ m/s}^2 \)

\( \gamma = \frac{6}{\sin 30^\circ} = 12 \text{ km} \)
relative motion of B with respect to A:

\[ \vec{a}_{B/A} = \vec{a}_B - \vec{a}_A = 0 - 1.2 \hat{i} = -1.2 \hat{i} \text{ m/s}^2 \]

\[ \vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = 1500 \hat{i} - 1000 \hat{i} = 500 \hat{i} \text{ km/h} \]

\[ \vec{r} = \frac{500}{3.6} \cos 30^\circ \]

\[ = 120.28 \text{ m/s} \]

\[ \vec{v}_r = - \frac{500}{3.6} \sin 30^\circ \]

\[ = -69.44 \text{ m/s} \]

\[ \Theta = \frac{v_r}{r} \]

\[ = - \frac{69.44}{12000} \]

\[ a_r = -1.2 \cos 30^\circ = -1.039 \ \text{ m/s}^2 \]

\[ a_\theta = 1.2 \sin 30^\circ = 0.6 \ \text{ m/s}^2 \]

\[ a_r = \ddot{r} - r \dot{\theta}^2 = -1.039 \]

\[ \ddot{r} = -1.039 + \frac{12000}{(-69.44)^2} = -0.637 \ \text{ m/s}^2 \]

\[ a_\theta = r \ddot{\theta} + 2r \dot{\theta} = 0.6 \]

\[ \dot{\theta} = \frac{0.6 - 2 \cdot \frac{120.28 \cdot (-69.44)}{12000}}{12000} = 1.66 \times 10^{-4} \ \text{ rad/s} \]

**Problem 4:**

(a) \( A \)

\[ W = mg \]

\[ \sum F_j = ma_y = 0 \]

\[ N = R \]

\[ N - W = 0 \implies N = W = R \]

\[ R = mg = 0.12 \cdot 9.81 = 1.1772 \ \text{ N} \]

(b) \( L \)

\[ \text{at conservation of mechanical energy}: \]

\[ v_B = v_A = 1.4 \ \text{ m/s} \]

\[ \dot{\theta} = 0 \]

\[ a_{\theta} \text{ at } \theta = \frac{v_{\theta}^2}{\rho} = \frac{1.4^2}{0.2} = 9.8 \ \text{ m/s}^2 \]

\[ F_n = \frac{m \omega}{L} = 0.12 \cdot 9.8 = 1.176 \ \text{ N} \]

\[ R = \sqrt{F_n^2 + N^2} \]

\[ N = W = 1.1772 \ \text{ N} \]

\[ \implies 1.664 \ \text{ N} \]
Problem 5:

\[ \begin{align*}
\theta &= 30^\circ \\
BC &= 2 \times 0.5 \cos 75^\circ \\
&= 0.2588 \text{ m}
\end{align*} \]

\[ d^2 = (0.2588)^2 + (0.8)^2 - (2)(0.8)(0.2588) \cos 165^\circ \]

\[ d = 1.052 \text{ m} \]

Spring deformation \[ s = 1.052 - 0.8 = 0.252 \text{ m} \]

No non-conservative forces present

\[ \text{conservation of mechanical energy} \]

\[ \frac{1}{2} m v_2^2 - m g h + \frac{1}{2} k s^2 = 0 \]

\[ \frac{1}{2} (5) 0_2^2 - (5)(9.81)(1)(\sin 30^\circ) + \frac{1}{2} \times 700 \times 0.252^2 = 0 \]

\[ v_2 = \sqrt{0.9588} \text{ m/s} \]