PET-342E Reservoir Engineering-I
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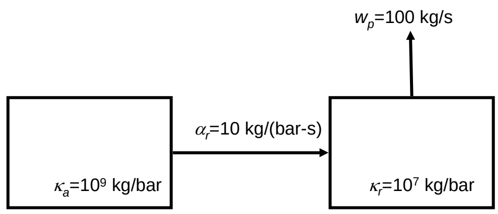
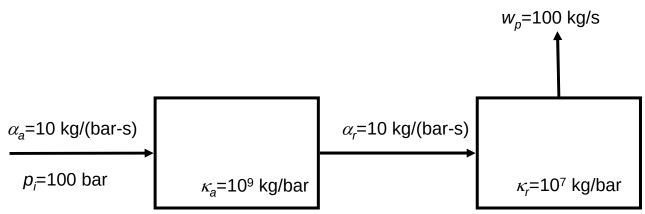
Project#2
2007-2008
İTÜ

Given: 18 Apr 2008
Due : 2 May 2008

Problem -1 (40 points) : Consider the two tank open and two tank closed systems given in the figure below:





Using the analytical solutions, compute the pressure behavior of the reservoir with time for both systems. You should compute the pressure behavior for 10000 days with a time step of 1 day. Hence you need to make 10000 computations for each system. Plot the pressure behavior of both open and closed system on the same graph. In other words you will need to prepare a pressure vs. time graph comparing the closed and the open system.

Prepare the same plot on a semi-log graph. Comment on the results that you have obtained from both plots. In your comment include why (physically) the pressures show such behavior and compare the two cases.

Problem-2 (60 points) : On a 2-tank closed system we wish to perform a sensitivity regarding the time steps we use for numerically computing the pressure. For this purpose construct a plot of the pressure behavior with time comparing the analytical solution and the numerical solution. For this purpose construct the plot for 1000 days. The plot should contain the analytical response with time steps of 1 days. On the same graph also plot the numerical solutions with time steps of 0.1 days, 1 day, 10 days and 100 days. In other words your graph should contain 5 curves, one from the analytical solution and 4 from the numerical solutions with various time steps. Comment on your results. Are there any differences between the curves? If so what are the reasons?

Analytical solutions for a 2-tank closed system

$$p_i - p(t) = \frac{w_p}{(\kappa_a + \kappa_r)} t + \frac{w_p}{\alpha_r} \left(\frac{\kappa_a}{\kappa_a + \kappa_r} \right)^2 \left[1 - \exp\left(-\alpha_r \frac{\kappa_a + \kappa_r}{\kappa_a \kappa_r} t \right) \right]$$

Analytical solutions for a 2-tank open system

$$p_i - p(t) = \frac{w_p}{\kappa_r} \left[\frac{d}{\mu_1 \mu_2} + \frac{\mu_1 - d}{\mu_1 (\mu_2 - \mu_1)} \exp(-\mu_1 t) + \frac{\mu_2 - d}{\mu_2 (\mu_1 - \mu_2)} \exp(-\mu_2 t) \right]$$

Where

$$d = \frac{\alpha_a + \alpha_r}{\kappa_a}$$

$$\mu_1 = \frac{\left[\frac{(\alpha_a + \alpha_r)}{\kappa_a} + \frac{\alpha_r}{\kappa_r} \right] + \sqrt{\left[\frac{(\alpha_a + \alpha_r)}{\kappa_a} + \frac{\alpha_r}{\kappa_r} \right]^2 - 4 \frac{\alpha_a \alpha_r}{\kappa_a \kappa_r}}}{2}$$

$$\mu_2 = \frac{\left[\frac{(\alpha_a + \alpha_r)}{\kappa_a} + \frac{\alpha_r}{\kappa_r} \right] - \sqrt{\left[\frac{(\alpha_a + \alpha_r)}{\kappa_a} + \frac{\alpha_r}{\kappa_r} \right]^2 - 4 \frac{\alpha_a \alpha_r}{\kappa_a \kappa_r}}}{2}$$

Numerical solution for a 2-tank closed system:

$$\begin{bmatrix} -\left(\alpha_r + \frac{\kappa_r}{dt} \right) & \alpha_r \\ \alpha_r & -\left(\alpha_r + \frac{\kappa_a}{dt} \right) \end{bmatrix} \begin{bmatrix} p_r^{n+1} \\ p_a^{n+1} \end{bmatrix} = \begin{bmatrix} -\frac{p_r^n \kappa_r}{dt} + w_p \\ -\frac{p_a^n \kappa_a}{dt} \end{bmatrix}$$

Where the superscript n and n+1 represent the time levels at n and n+1 respectively. p_r is the reservoir pressure and p_a is the aquifer pressure. dt is given as follows:

$$dt = t^{n+1} - t^n$$

The units of the variables are as follows:

α : kg(bar-s)
κ : kg/bar
w_p : kg/s
p : bar
t : sec