MUKAVEMET

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17. Theories of failure or yield criteria

(1) Maximum shearing stress theory
(2) Octahedral shearing stress theory
(3) Maximum normal stress theory – **for brittle materials**.

Maximum shearing stress theory or Tresca Criterion

This theory says that:

Yielding occurs when the maximum shear stress in the material reaches the value of the shear stress at yielding in a uniaxial tension (or compression) test.

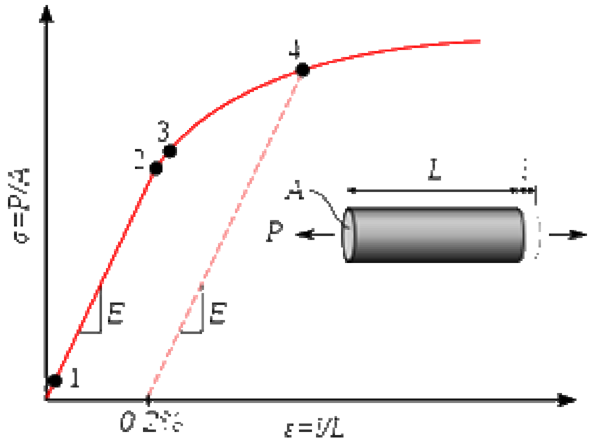
Maximum shearing stress under general state of stress is

$$\tau_{max} = max(\tau_1, \tau_2, \tau_3)$$

where ;

$$\text{where} \quad \tau_1 = \left|\frac{\sigma_2 - \sigma_3}{2}\right|;\ \tau_2 = \left|\frac{\sigma_1 - \sigma_3}{2}\right|;\ \tau_3 = \left|\frac{\sigma_1 - \sigma_2}{2}\right|$$

Definition



Typical yield behavior for non-ferrous alloys.
1: True elastic limit
2: Proportionality limit
3: Elastic limit
4: Offset yield strength

It is often difficult to precisely define yielding due to the wide variety of stress– strain curves exhibited by real materials. In addition, there are several possible ways to define yielding:[1]

True elastic limit

The lowest stress at which dislocations move. This definition is rarely used, since dislocations move at very low stresses, and detecting such movement is very difficult.

Proportionality limit

Up to this amount of stress, stress is proportional to strain (Hooke's law), so the stress-strain graph is a straight line, and the gradient will be equal to the elastic modulus of the material.

Elastic limit (yield strength)

Beyond the elastic limit, permanent deformation will occur. The lowest stress at which permanent deformation can be measured. This requires a manual load-unload procedure, and the accuracy is critically dependent on equipment and operator skill. For elastomers, such as rubber, the elastic limit is much larger than the proportionality limit. Also, precise strain measurements have shown that plastic strain begins at low stresses.[2][3]

Offset yield point (proof stress)
Offset yield point (proof stress)

This is the most widely used strength measure of metals, and is found from the stress-strain curve as shown in the figure to the right. A plastic strain of 0.2% is usually used to define the offset yield stress, although other values may be used depending on the material and the application. The offset value is given as a subscript, e.g., $R_{p0.2}$=310 MPa. In some materials there is essentially no linear region and so a certain value of strain is defined instead. Although somewhat arbitrary, this method does allow for a consistent comparison of materials.

Upper yield point and lower yield point

Some metals, such as mild steel, reach an upper yield point before dropping rapidly to a lower yield point. The material response is linear up until the upper yield point, but the lower yield point is used in structural engineering as a conservative value. If a metal is only stressed to the upper yield point, and beyond, luders bands can develop.[4]

Yield criterion

A yield criterion, often expressed as yield surface, or yield locus, is an hypothesis concerning the limit of elasticity under any combination of stresses. There are two interpretations of yield criterion: one is purely mathematical in taking a statistical approach while other models attempt to provide a justification based on established physical principles. Since stress and strain are tensor qualities they can be described on the basis of three principal directions, in the case of stress these are denoted by σ_1, σ_2, and σ_3.

The following represent the most common yield criterion as applied to an isotropic material (uniform properties in all directions). Other equations have been proposed or are used in specialist situations.

[edit] Isotropic yield criteria

Maximum Principal Stress Theory - Yield occurs when the largest principal stress exceeds the uniaxial tensile yield strength. Although this criterion allows for a quick and easy comparison with experimental data it is rarely suitable for design purposes.

$$\sigma_1 \leq \sigma_y$$

Maximum Principal Strain Theory - Yield occurs when the maximum principal strain reaches the strain corresponding to the yield point during a simple tensile test. In terms of the principal stresses this is determined by the equation:

$$\sigma_1 - \nu(\sigma_2 + \sigma_3) \leq \sigma_y.$$

Maximum Shear Stress Theory - Also known as the Tresca yield criterion, after the French scientist Henri Tresca. This assumes that yield occurs when the shear stress τ exceeds the shear yield strength τ_y:

$$\tau = \frac{\sigma_1 - \sigma_3}{2} \le \tau_{ys}.$$

Total Strain Energy Theory - This theory assumes that the stored energy associated with elastic deformation at the point of yield is independent of the specific stress tensor. Thus yield occurs when the strain energy per unit volume is greater than the strain energy at the elastic limit in simple tension. For a 3-dimensional stress state this is given by:

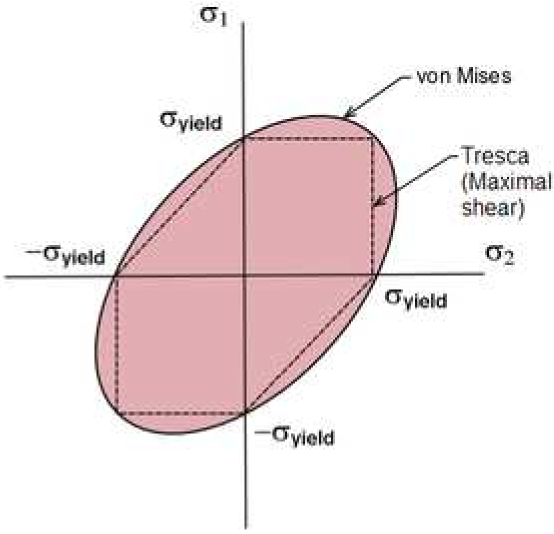
$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3) \le \sigma_y^2.$$

Distortion Energy Theory - This theory proposes that the total strain energy can be separated into two components: the *volumetric* (hydrostatic) strain energy and the *shape* (distortion or shear) strain energy. It is proposed that yield occurs when the distortion component exceeds that at the yield point for a simple tensile test. This is generally referred to as the Von Mises yield criterion and is expressed as:

$$\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \le \sigma_y^2.$$

Based on a different theoretical underpinning this expression is also referred to as **octahedral shear stress theory**.

Von Mises criterion for different stress conditions



Projection of the von Mises yield criterion into the σ_1, σ_2 plane

In the case of **uniaxial stress** or **simple tension**, $\sigma_1 \neq 0$, $\sigma_3 = \sigma_2 = 0$, the von Mises criterion reduces to

$\sigma_1 = \sigma_y$.

Therefore, the material starts to yield, when $\sigma_1$reaches the *yield strength* of the material σ_y, which is a characteristic material property. In practice, this parameter is indeed determined in a tensile test satisfying the uniaxial stress condition.

It is also convenient to define an **Equivalent tensile stress** or **von Mises stress**, σ_v, which is used to predict yielding of materials under **multiaxial loading conditions** using results from simple uniaxial tensile tests. Thus, we define

$$\begin{aligned}\sigma_v &= \sqrt{3J_2} \\ &= \sqrt{\frac{(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{11}-\sigma_{33})^2+6(\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2)}{2}} \\ &= \sqrt{\frac{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2}{2}} \\ &= \sqrt{\tfrac{3}{2}\, s_{ij}s_{ij}}\end{aligned}$$

where $s_{ij}$are the components of the stress deviator tensor $\boldsymbol{\sigma}^{dev}$:

$$\boldsymbol{\sigma}^{dev} = \boldsymbol{\sigma} - \frac{1}{3}(\boldsymbol{\sigma}\cdot\mathbf{I})\,\mathbf{I}$$.

In this case, yielding occurs when the equivalent stress, σ_v, reaches the yield strength of the material in simple tension, σ_y. As an example, the stress state of a steel beam in compression differs from the stress state of a steel axle under torsion, even if both specimen are of the same material. In view of the stress tensor, which fully describes the stress state, this difference manifests in six degrees of freedom, because the stress tensor has six independent components. Therefore, it is difficult to tell which of the two specimens is closer to the yield point or has even reached it. However, by means of the von Mises yield criterion, which depends solely on the value of the scalar von Mises stress, i.e., one degree of freedom, this comparison is straightforward: A larger von Mises value implies that the material is closer to the yield point.

In the case of **pure shear stress**, $\sigma_{12} = \sigma_{21} \neq 0$, while all other $\sigma_{ij} = 0$, von Mises criterion becomes:

$$\sigma_{12} = k = \frac{\sigma_y}{\sqrt{3}}.$$

This means that, at the onset of yielding, the magnitude of the shear stress in pure shear is $\sqrt{3}$times lower than the tensile stress in the case of simple tension. The von Mises yield criterion for pure shear stress, expressed in principal stresses, is

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 = 6\sigma_{12}^2$$

In the case of **plane stress**, $\sigma_3 = 0$, the von Mises criterion becomes:

$$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = 3k^2 = \sigma_y^2$$

This equation represents an ellipse in the plane $\sigma_1 - \sigma_2$, as shown in the Figure above