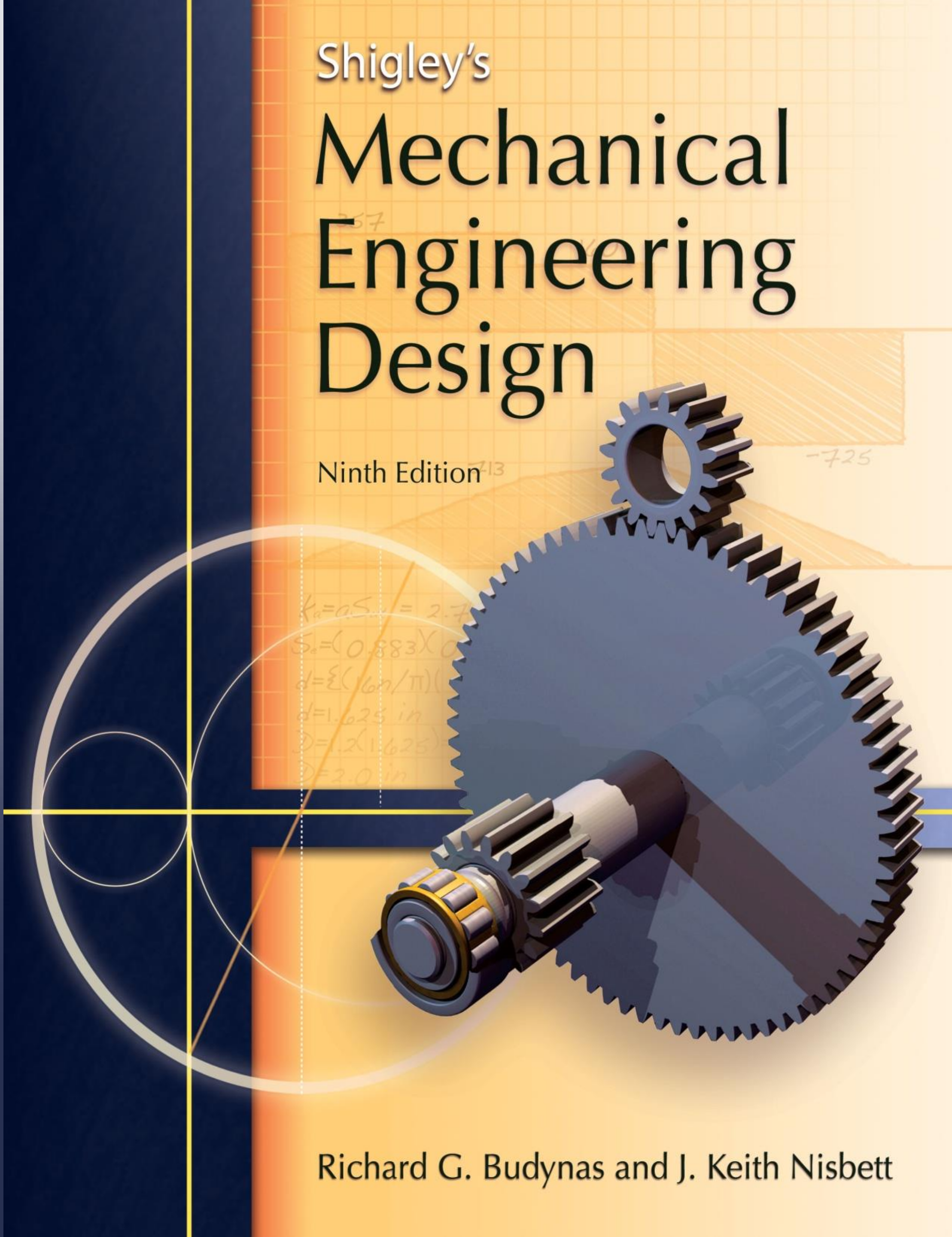
Lecture Slides

Chapter 8

Screws, Fasteners, and the Design of Nonpermanent Joints



Chapter Outline

8–1	Thread Standards and Definitions	410
8–2	The Mechanics of Power Screws	414
8–3	Threaded Fasteners	422
8–4	Joints—Fastener Stiffness	424
8–5	Joints—Member Stiffness	427
8–6	Bolt Strength	432
8–7	Tension Joints—The External Load	435
8–8	Relating Bolt Torque to Bolt Tension	437
8–9	Statically Loaded Tension Joint with Preload	440
8–10	Gasketed Joints	444
8–11	Fatigue Loading of Tension Joints	444
8–12	Bolted and Riveted Joints Loaded in Shear	451

Reasons for Non-permanent Fasteners

- Field assembly
- Disassembly
- Maintenance
- Adjustment

Thread Standards and Definitions

- *Pitch* – distance between adjacent threads. Reciprocal of threads per inch
- *Major diameter* – largest diameter of thread
- *Minor diameter* – smallest diameter of thread
- *Pitch diameter* – theoretical diameter between major and minor diameters, where tooth and gap are same width

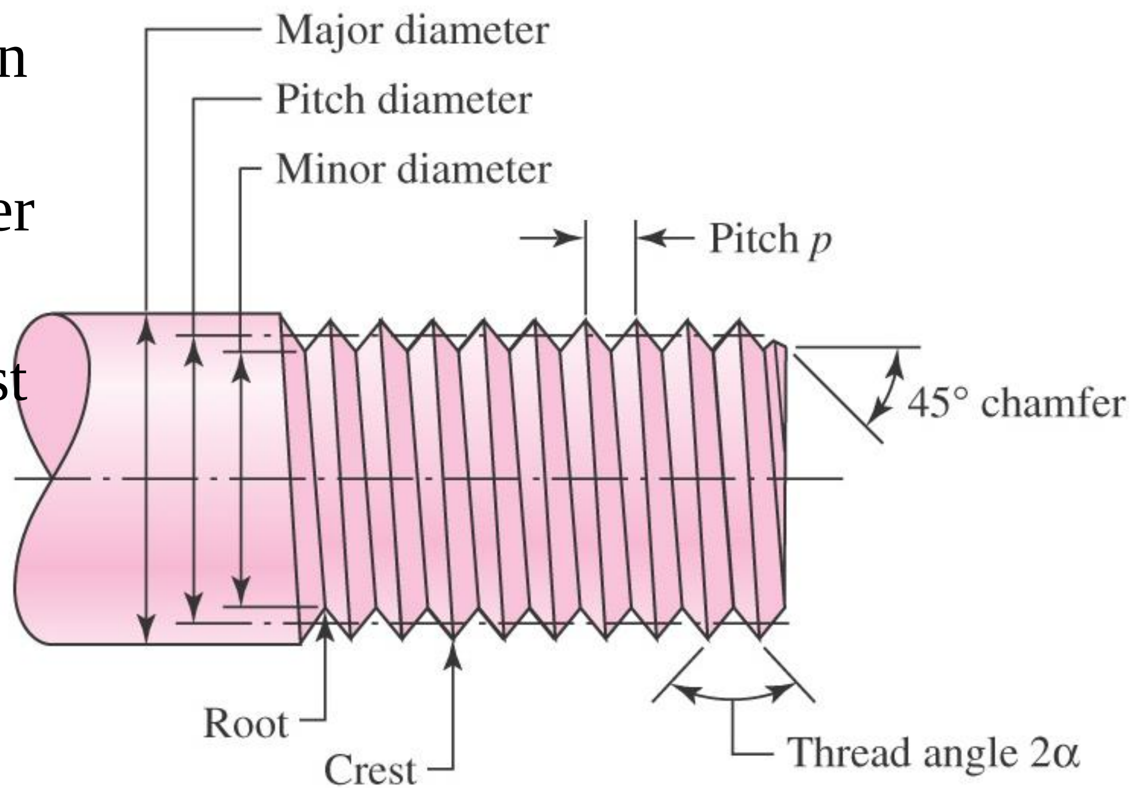


Fig. 8–1

Standardization

- The *American National* (*Unified*) thread standard defines basic thread geometry for uniformity and interchangeability
- American National (Unified) thread
  - UN normal thread
  - UNR greater root radius for fatigue applications
- Metric thread
  - M series (normal thread)
  - MJ series (greater root radius)

Standardization

- Coarse series UNC
  - General assembly
  - Frequent disassembly
  - Not good for vibrations
  - The "normal" thread to specify
- Fine series UNF
  - Good for vibrations
  - Good for adjustments
  - Automotive and aircraft
- Extra Fine series UNEF
  - Good for shock and large vibrations
  - High grade alloy
  - Instrumentation
  - Aircraft

Standardization

- Basic profile for metric M and MJ threads shown in Fig. 8–2
- Tables 8–1 and 8–2 define basic dimensions for standard threads

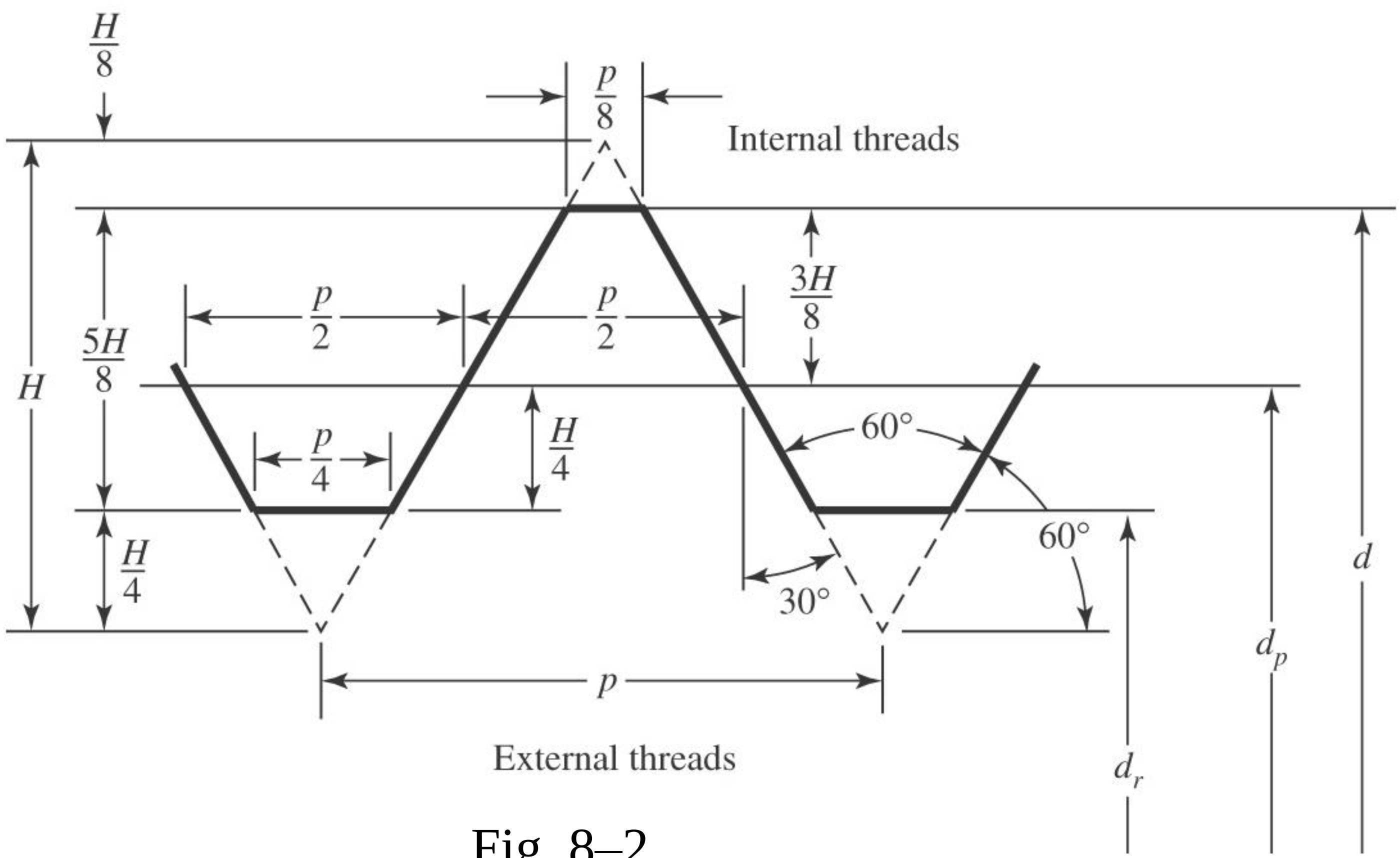


Fig. 8–2

Diameters and Areas for Metric Threads

Table 8–1

Diameters and Areas of Coarse-Pitch and Fine-Pitch Metric Threads.*

Nominal Major Diameter d mm	Coarse-Pitch Series: Pitch p mm	Coarse-Pitch Series: Tensile-Stress Area A_t mm^2	Coarse-Pitch Series: Minor-Diameter Area A_r mm^2	Fine-Pitch Series: Pitch p mm	Fine-Pitch Series: Tensile-Stress Area A_t mm^2	Fine-Pitch Series: Minor-Diameter Area A_r mm^2
1.6	0.35	1.27	1.07			
2	0.40	2.07	1.79			
2.5	0.45	3.39	2.98			
3	0.5	5.03	4.47			
3.5	0.6	6.78	6.00			
4	0.7	8.78	7.75			
5	0.8	14.2	12.7			
6	1	20.1	17.9			
8	1.25	36.6	32.8	1	39.2	36.0
10	1.5	58.0	52.3	1.25	61.2	56.3
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157
20	2.5	245	225	1.5	272	259
24	3	353	324	2	384	365
30	3.5	561	519	2	621	596
36	4	817	759	2	915	884
42	4.5	1120	1050	2	1260	1230
48	5	1470	1380	2	1670	1630
56	5.5	2030	1910	2	2300	2250
64	6	2680	2520	2	3030	2980

Diameters and Areas for Unified Screw Threads

Table 8–2

Size Designation	Nominal Major Diameter in	Coarse Series—UNC Threads per Inch N	Coarse Series—UNC Tensile-Stress Area A_t, in^2	Coarse Series—UNC Minor-Diameter Area A_r, in^2	Fine Series—UNF Threads per Inch N	Fine Series—UNF Tensile-Stress Area A_t, in^2	Fine Series—UNF Minor-Diameter Area A_r, in^2
0	0.0600				80	0.001 80	0.001 51
1	0.0730	64	0.002 63	0.002 18	72	0.002 78	0.002 37
2	0.0860	56	0.003 70	0.003 10	64	0.003 94	0.003 39
3	0.0990	48	0.004 87	0.004 06	56	0.005 23	0.004 51
4	0.1120	40	0.006 04	0.004 96	48	0.006 61	0.005 66
5	0.1250	40	0.007 96	0.006 72	44	0.008 80	0.007 16
6	0.1380	32	0.009 09	0.007 45	40	0.010 15	0.008 74
8	0.1640	32	0.014 0	0.011 96	36	0.014 74	0.012 85
10	0.1900	24	0.017 5	0.014 50	32	0.020 0	0.017 5
12	0.2160	24	0.024 2	0.020 6	28	0.025 8	0.022 6
$\frac{1}{4}$	0.2500	20	0.031 8	0.026 9	28	0.036 4	0.032 6
$\frac{5}{16}$	0.3125	18	0.052 4	0.045 4	24	0.058 0	0.052 4
$\frac{3}{8}$	0.3750	16	0.077 5	0.067 8	24	0.087 8	0.080 9
$\frac{7}{16}$	0.4375	14	0.106 3	0.093 3	20	0.118 7	0.109 0
$\frac{1}{2}$	0.5000	13	0.141 9	0.125 7	20	0.159 9	0.148 6
$\frac{9}{16}$	0.5625	12	0.182	0.162	18	0.203	0.189
$\frac{5}{8}$	0.6250	11	0.226	0.202	18	0.256	0.240
$\frac{3}{4}$	0.7500	10	0.334	0.302	16	0.373	0.351
$\frac{7}{8}$	0.8750	9	0.462	0.419	14	0.509	0.480
1	1.0000	8	0.606	0.551	12	0.663	0.625
$1\frac{1}{4}$	1.2500	7	0.969	0.890	12	1.073	1.024
$1\frac{1}{2}$	1.5000	6	1.405	1.294	12	1.581	1.521

Tensile Stress Area

- The tensile stress area, A_t , is the area of an unthreaded rod with the same tensile strength as a threaded rod.
- It is the effective area of a threaded rod to be used for stress calculations.
- The diameter of this unthreaded rod is the average of the pitch diameter and the minor diameter of the threaded rod.

Square and Acme Threads

- Square and Acme threads are used when the threads are intended to transmit power

Fig. 8–3

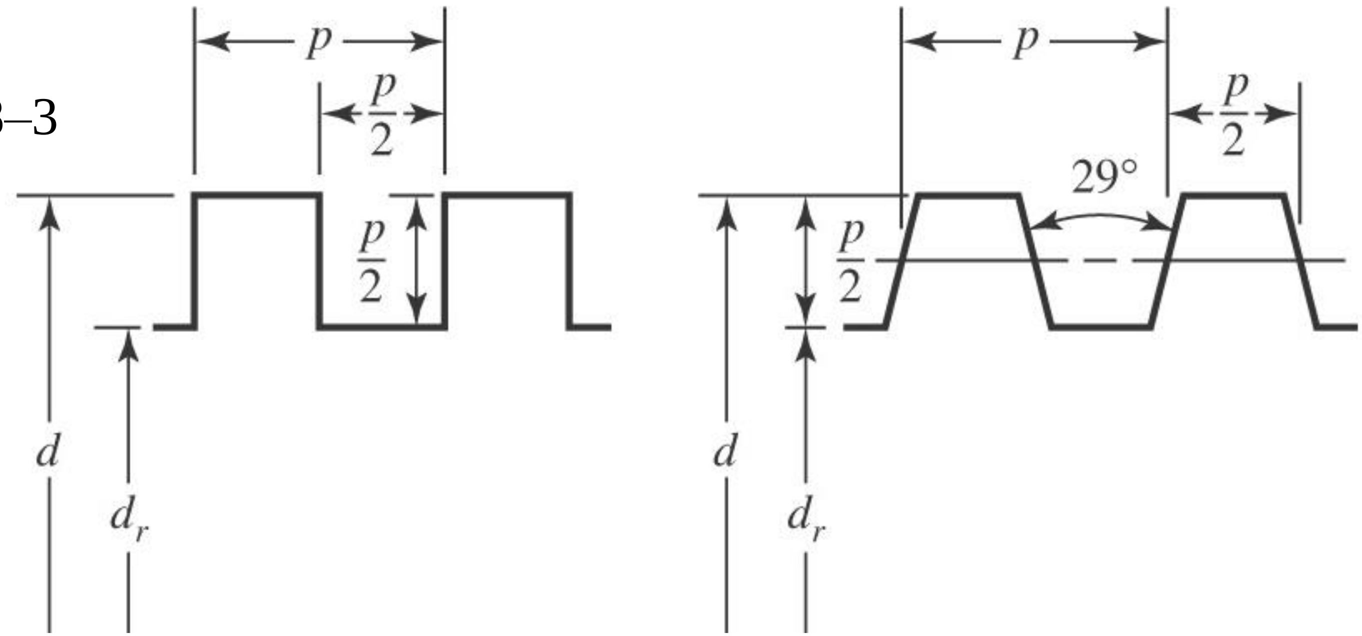


Table 8-3 Preferred Pitches for Acme Threads

d, in	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2	$2\frac{1}{2}$	3
p, in	$\frac{1}{16}$	$\frac{1}{14}$	$\frac{1}{12}$	$\frac{1}{10}$	$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$

Mechanics of Power Screws

- *Power screw*
  - Used to change angular motion into linear motion
  - Usually transmits power
  - Examples include vises, presses, jacks, lead screw on lathe

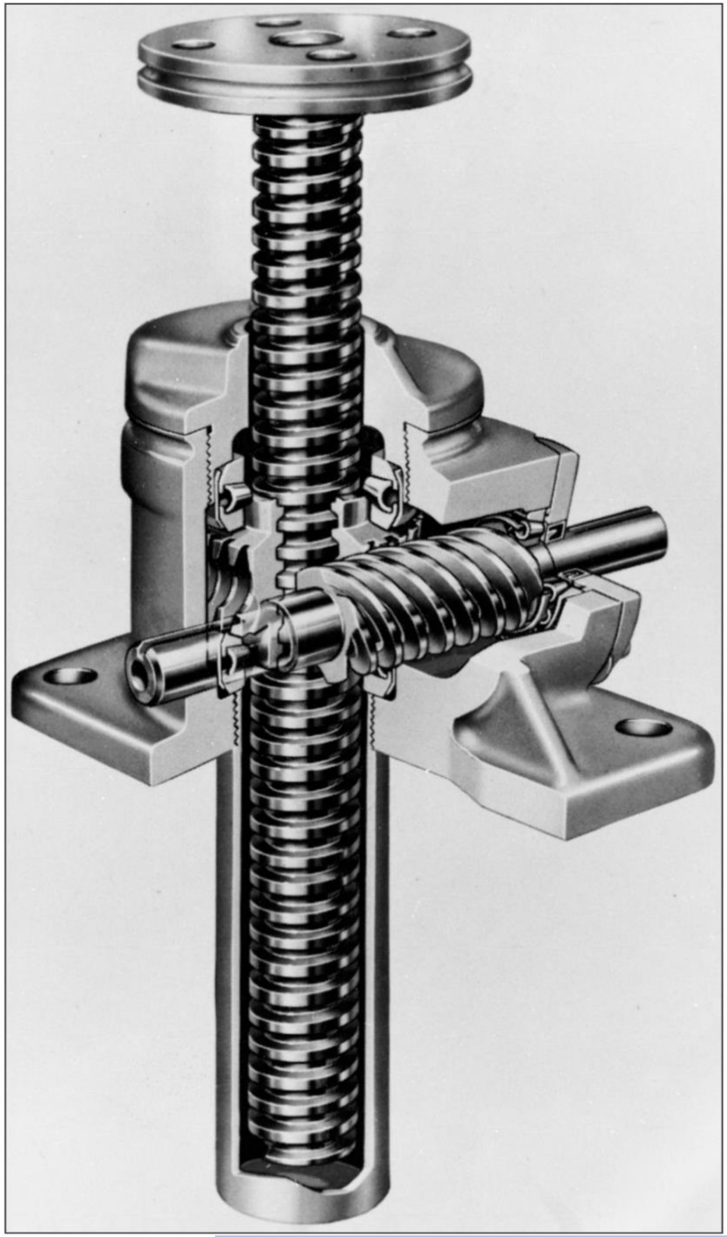


Fig. 8–4

Mechanics of Power Screws

- Find expression for torque required to raise or lower a load
- Unroll one turn of a thread
- Treat thread as inclined plane
- Do force analysis

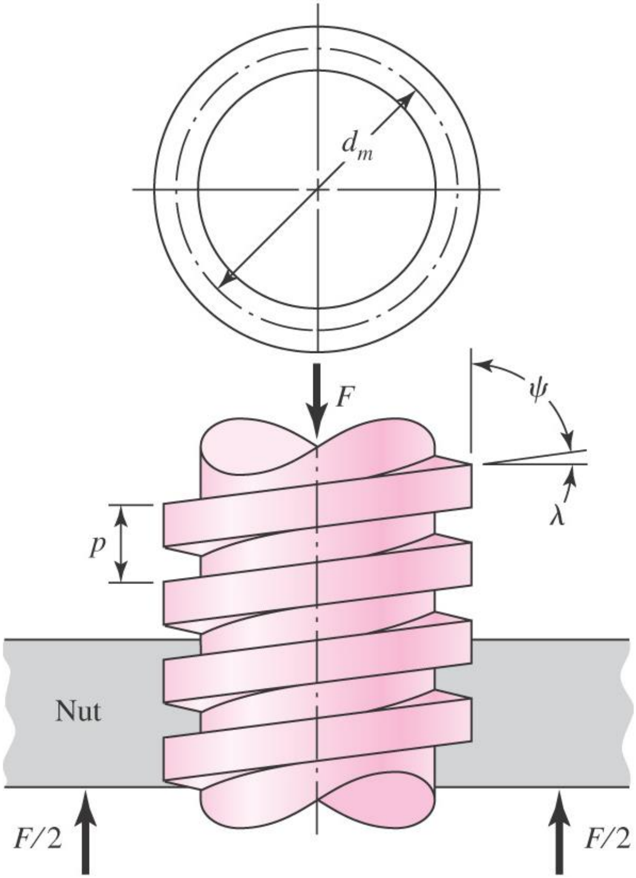


Fig. 8–5

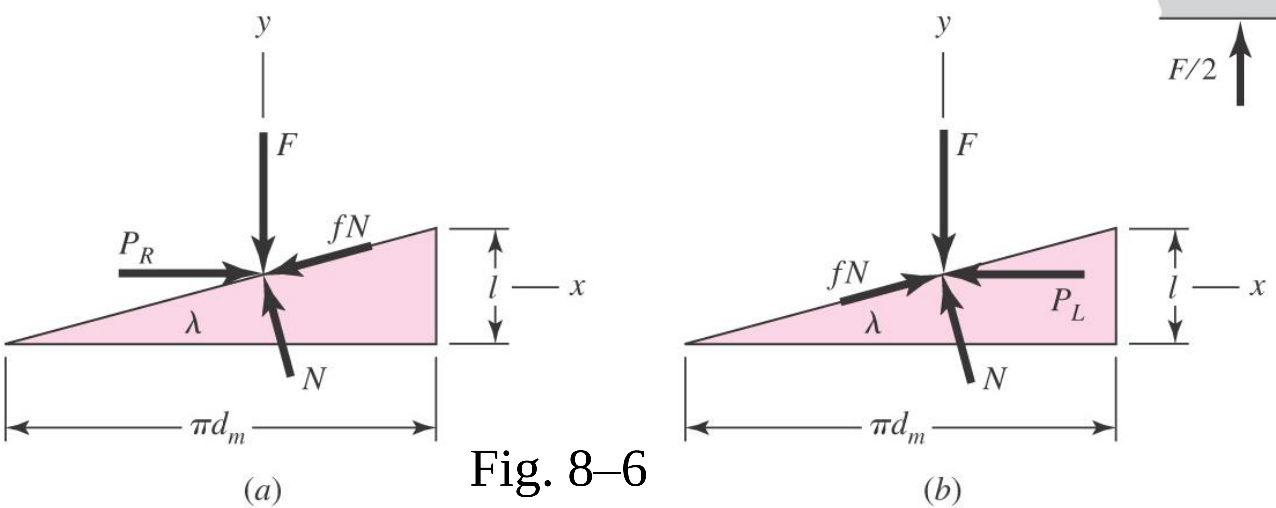


Fig. 8–6

Mechanics of Power Screws

- For raising the load

$$\sum F_x = P_R - N\sin\lambda - fN\cos\lambda = 0$$

$$\sum F_y = -F - fN\sin\lambda + N\cos\lambda = 0$$

(a)

- For lowering the load

$$\sum F_x = -P_L - N\sin\lambda + fN\cos\lambda = 0$$

$$\sum F_y = -F + fN\sin\lambda + N\cos\lambda = 0$$

(b)

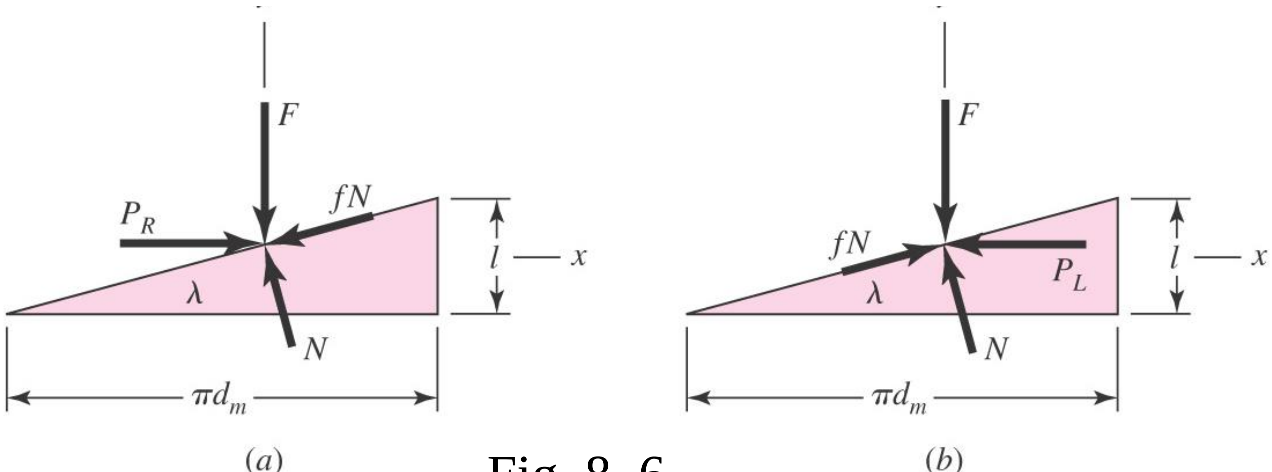


Fig. 8–6

Mechanics of Power Screws

- Eliminate N and solve for P to raise and lower the load

$$P_R = \frac{F(\sin\lambda + f\cos\lambda)}{\cos\lambda - f\sin\lambda} \qquad (c)$$

$$P_L = \frac{F(f\cos\lambda - \sin\lambda)}{\cos\lambda + f\sin\lambda} \qquad (d)$$

- Divide numerator and denominator by $\cos\lambda$ and use relation $\tan\lambda = l/\pi d_m$

$$P_R = \frac{F[(l/\pi d_m) + f]}{1 - (fl/\pi d_m)} \qquad (e)$$

$$P_L = \frac{F[f - (l/\pi d_m)]}{1 + (fl/\pi d_m)} \qquad (f)$$

Raising and Lowering Torque

- Noting that the torque is the product of the force and the mean radius,

$$T_R = \frac{F d_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - f l}\right) \qquad (8\text{–}1)$$

$$T_L = \frac{F d_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + f l}\right) \qquad (8\text{–}2)$$

Self-locking Condition

$$T_L = \frac{F d_m}{2} \left(\frac{\pi f d_m - l}{\pi d_m + f l} \right) \qquad (8\text{–}2)$$

- If the lowering torque is negative, the load will lower itself by causing the screw to spin without any external effort.
- If the lowering torque is positive, the screw is *self-locking*.
- Self-locking condition is $\pi f d_m > l$
- Noting that $l / \pi d_m = \tan \lambda$, the self-locking condition can be seen to only involve the coefficient of friction and the lead angle.

$$f > \tan \lambda \qquad (8\text{–}3)$$

Power Screw Efficiency

- The torque needed to raise the load with no friction losses can be found from Eq. (8–1) with $f = 0$.

$$T_0 = \frac{Fl}{2\pi} \qquad (g)$$

- The efficiency of the power screw is therefore

$$e = \frac{T_0}{T_R} = \frac{Fl}{2\pi T_R} \qquad (8\text{–}4)$$

Power Screws with Acme Threads

- If Acme threads are used instead of square threads, the thread angle creates a wedging action.
- The friction components are increased.
- The torque necessary to raise a load (or tighten a screw) is found by dividing the friction terms in Eq. (8–1) by $\cos\alpha$.

$$T_R = \frac{F\,d_m}{2}\left(\frac{l + \pi f\,d_m \sec\alpha}{\pi d_m - f\,l \sec\alpha}\right) \tag{8–5}$$

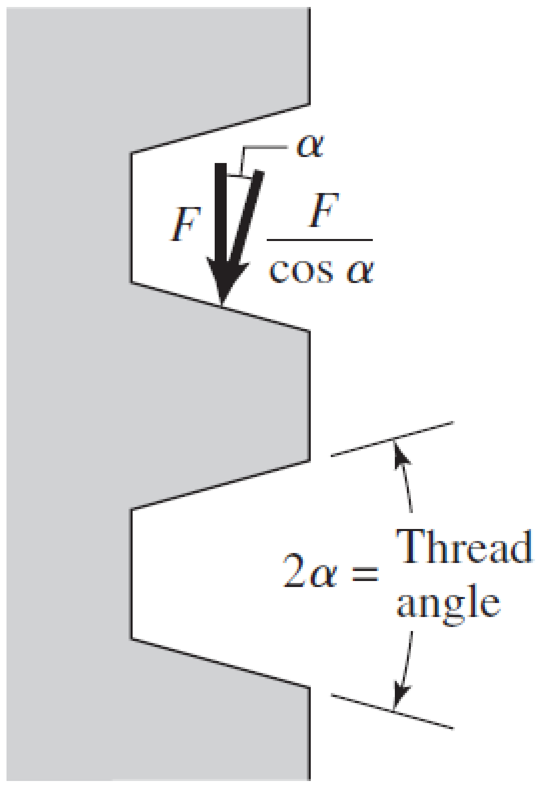


Fig. 8–7 (a)

Collar Friction

- An additional component of torque is often needed to account for the friction between a collar and the load.
- Assuming the load is concentrated at the mean collar diameter d_c

$$T_c = \frac{F f_c d_c}{2} \qquad (8\text{–}6)$$

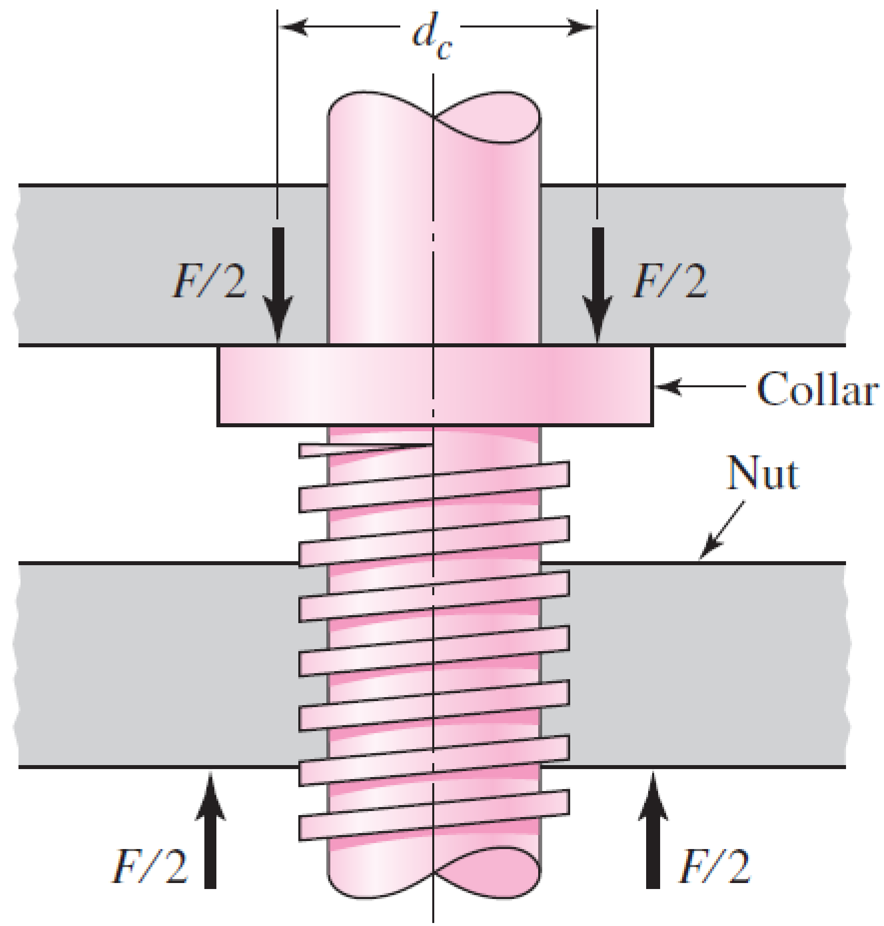


Fig. 8–7 (*b*)

Thread Deformation in Screw-Nut Combination

- Power screw thread is in compression, causing elastic shortening of screw thread pitch.
- Engaging nut is in tension, causing elastic lengthening of the nut thread pitch.
- Consequently, the engaged threads cannot share the load equally.
- Experiments indicate the first thread carries 38% of the load, the second thread 25%, and the third thread 18%. The seventh thread is free of load.
- To find the largest stress in the first thread of a screw-nut combination, use $0.38F$ in place of F, and set $n_t = 1$.

Example 8-1

A square-thread power screw has a major diameter of 32 mm and a pitch of 4 mm with double threads, and it is to be used in an application similar to that in Fig. 8–4. The given data include $f = f_c = 0.08$, $d_c = 40$ mm, and $F = 6.4$ kN per screw.

(*a*) Find the thread depth, thread width, pitch diameter, minor diameter, and lead.

(*b*) Find the torque required to raise and lower the load.

(*c*) Find the efficiency during lifting the load.

(*d*) Find the body stresses, torsional and compressive.

(*e*) Find the bearing stress.

(*f*) Find the thread bending stress at the root of the thread.

(*g*) Determine the von Mises stress at the root of the thread.

(*h*) Determine the maximum shear stress at the root of the thread.

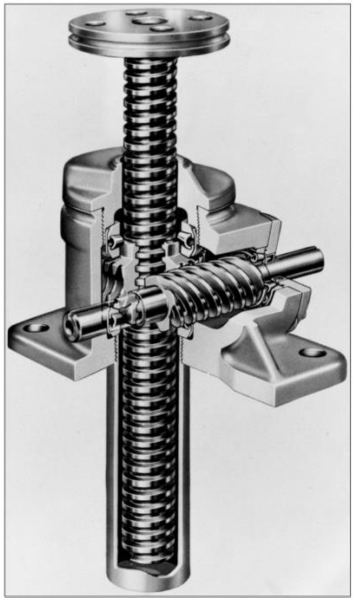


Fig. 8–4

Example 8-1

(*a*) From Fig. 8–3*a* the thread depth and width are the same and equal to half the pitch, or 2 mm. Also

$$d_m = d - p/2 = 32 - 4/2 = 30 \text{ mm}$$

$$d_r = d - p = 32 - 4 = 28 \text{ mm}$$

$$l = np = 2(4) = 8 \text{ mm}$$

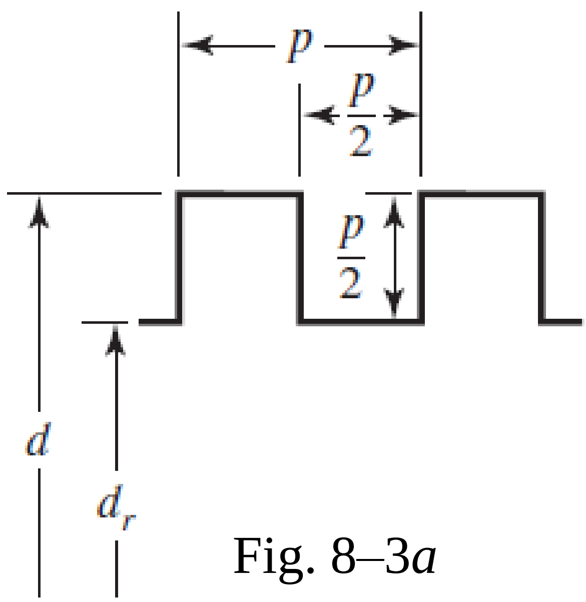


Fig. 8–3*a*

Example 8-1

(*b*) Using Eqs. (8–1) and (8–6), the torque required to turn the screw against the load is

$$T_R = \frac{F d_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - f l}\right) + \frac{F f_c d_c}{2}$$

$$= \frac{6.4(30)}{2}\left[\frac{8 + \pi(0.08)(30)}{\pi(30) - 0.08(8)}\right] + \frac{6.4(0.08)40}{2}$$

$$= 15.94 + 10.24 = 26.18\ \mathrm{N \cdot m}$$

Using Eqs. (8–2) and (8–6), we find the load-lowering torque is

$$T_L = \frac{F d_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + f l}\right) + \frac{F f_c d_c}{2}$$

$$= \frac{6.4(30)}{2}\left[\frac{\pi(0.08)30 - 8}{\pi(30) + 0.08(8)}\right] + \frac{6.4(0.08)(40)}{2}$$

$$= -0.466 + 10.24 = 9.77\ \mathrm{N \cdot m}$$

Example 8-1

(*c*) The overall efficiency in raising the load is

$$e = \frac{Fl}{2\pi T_R} = \frac{6.4(8)}{2\pi(26.18)} = 0.311$$

Example 8-1

(*d*) The body shear stress τ due to torsional moment T_R at the outside of the screw body is

$$\tau = \frac{16T_R}{\pi d_r^3} = \frac{16(26.18)(10^3)}{\pi(28^3)} = 6.07 \text{ MPa}$$

The axial nominal normal stress σ is

$$\sigma = -\frac{4F}{\pi d_r^2} = -\frac{4(6.4)10^3}{\pi(28^2)} = -10.39 \text{ MPa}$$

(*e*) The bearing stress σ_B is, with one thread carrying $0.38F$,

$$\sigma_B = -\frac{2(0.38F)}{\pi d_m(1)p} = -\frac{2(0.38)(6.4)10^3}{\pi(30)(1)(4)} = -12.9 \text{ MPa}$$

(*f*) The thread-root bending stress σ_b with one thread carrying $0.38F$ is

$$\sigma_b = \frac{6(0.38F)}{\pi d_r(1)p} = \frac{6(0.38)(6.4)10^3}{\pi(28)(1)4} = 41.5 \text{ MPa}$$

Example 8-1

(g) The transverse shear at the extreme of the root cross section due to bending is zero. However, there is a circumferential shear stress at the extreme of the root cross section of the thread as shown in part (d) of 6.07 MPa. The three-dimensional stresses, after Fig. 8–8, noting the y coordinate is into the page, are

$$\sigma_x = 41.5 \text{ MPa} \qquad \tau_{xy} = 0$$
$$\sigma_y = -10.39 \text{ MPa} \qquad \tau_{yz} = 6.07 \text{ MPa}$$
$$\sigma_z = 0 \qquad \tau_{zx} = 0$$

For the von Mises stress, Eq. (5–14) of Sec. 5–5 can be written as

$$\sigma' = \frac{1}{\sqrt{2}}\{(41.5 - 0)^2 + [0 - (-10.39)]^2 + (-10.39 - 41.5)^2 + 6(6.07)^2\}^{1/2}$$
$$= 48.7 \text{ MPa}$$

Example 8-1

Alternatively, you can determine the principal stresses and then use Eq. (5–12) to find the von Mises stress. This would prove helpful in evaluating τ_{max} as well. The principal stresses can be found from Eq. (3–15); however, sketch the stress element and note that there are no shear stresses on the x face. This means that σ_x is a principal stress. The remaining stresses can be transformed by using the plane stress equation, Eq. (3–13). Thus, the remaining principal stresses are

$$\frac{-10.39}{2} \pm \sqrt{\left(\frac{-10.39}{2}\right)^2 + 6.07^2} = 2.79, -13.18 \text{ MPa}$$

Ordering the principal stresses gives $\sigma_1, \sigma_2, \sigma_3 = 41.5, 2.79, -13.18$ MPa. Substituting these into Eq. (5–12) yields

$$\sigma' = \left\{\frac{[41.5 - 2.79]^2 + [2.79 - (-13.18)]^2 + [-13.18 - 41.5]^2}{2}\right\}^{1/2}$$

$$= 48.7 \text{ MPa}$$

Example 8-1

(*h*) The maximum shear stress is given by Eq. (3–16), where $\tau_{max} = \tau_{1/3}$, giving

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{41.5 - (-13.18)}{2} = 27.3 \text{ MPa}$$

Power Screw Safe Bearing Pressure

Table 8–4

Screw Bearing Pressure p_b

Source: H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook,* 2nd ed., McGraw-Hill, New York, 2006.

Screw Material	Nut Material	Safe p_b, psi	Notes
Steel	Bronze	2500–3500	Low speed
Steel	Bronze	1600–2500	$\le$10 fpm
	Cast iron	1800–2500	$\le$8 fpm
Steel	Bronze	800–1400	20–40 fpm
	Cast iron	600–1000	20–40 fpm
Steel	Bronze	150–240	$\ge$50 fpm

Power Screw Friction Coefficients

Table 8–5

Coefficients of Friction f for Threaded Pairs

Source: H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook,* 2nd ed., McGraw-Hill, New York, 2006.

Screw Material	Nut Material			
	Steel	Bronze	Brass	Cast Iron
Steel, dry	0.15–0.25	0.15–0.23	0.15–0.19	0.15–0.25
Steel, machine oil	0.11–0.17	0.10–0.16	0.10–0.15	0.11–0.17
Bronze	0.08–0.12	0.04–0.06	—	0.06–0.09

Table 8–6

Thrust-Collar Friction Coefficients

Source: H. A. Rothbart and T. H. Brown, Jr., *Mechanical Design Handbook,* 2nd ed., McGraw-Hill, New York, 2006.

Combination	Running	Starting
Soft steel on cast iron	0.12	0.17
Hard steel on cast iron	0.09	0.15
Soft steel on bronze	0.08	0.10
Hard steel on bronze	0.06	0.08

Head Type of Bolts

- Hexagon head bolt
  - Usually uses nut
  - Heavy duty
- Hexagon head cap screw
  - Thinner head
  - Often used as screw (in threaded hole, without nut)
- Socket head cap screw
  - Usually more precision applications
  - Access from the top
- Machine screws
  - Usually smaller sizes
  - Slot or philips head common
  - Threaded all the way

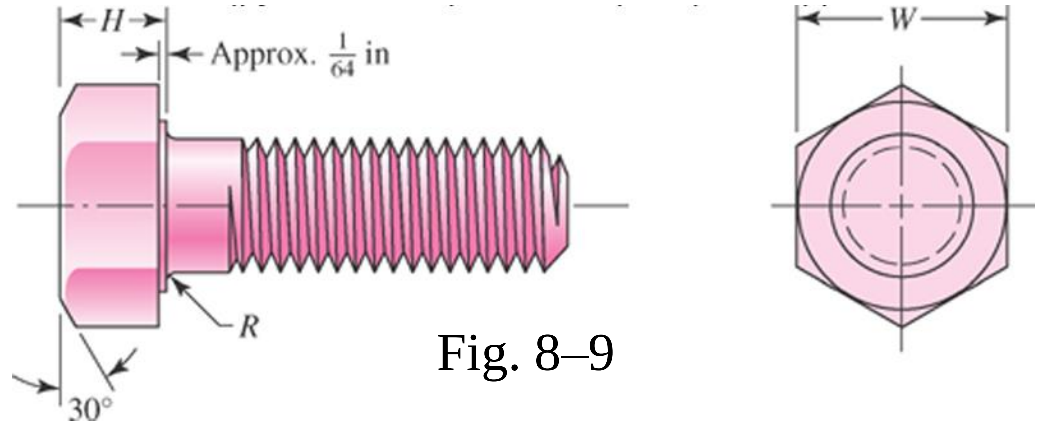


Fig. 8–9

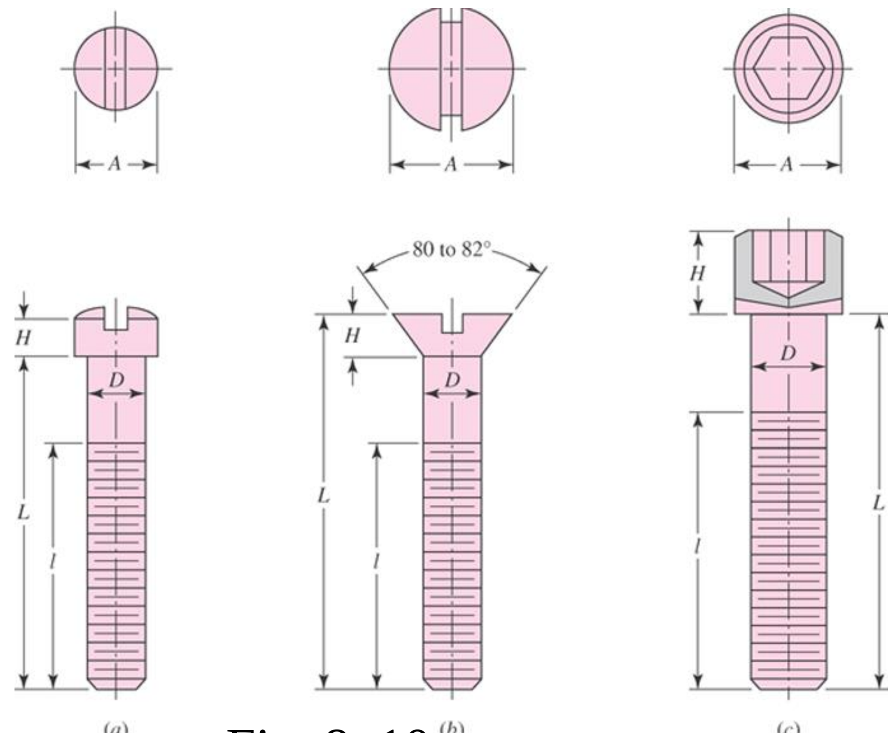
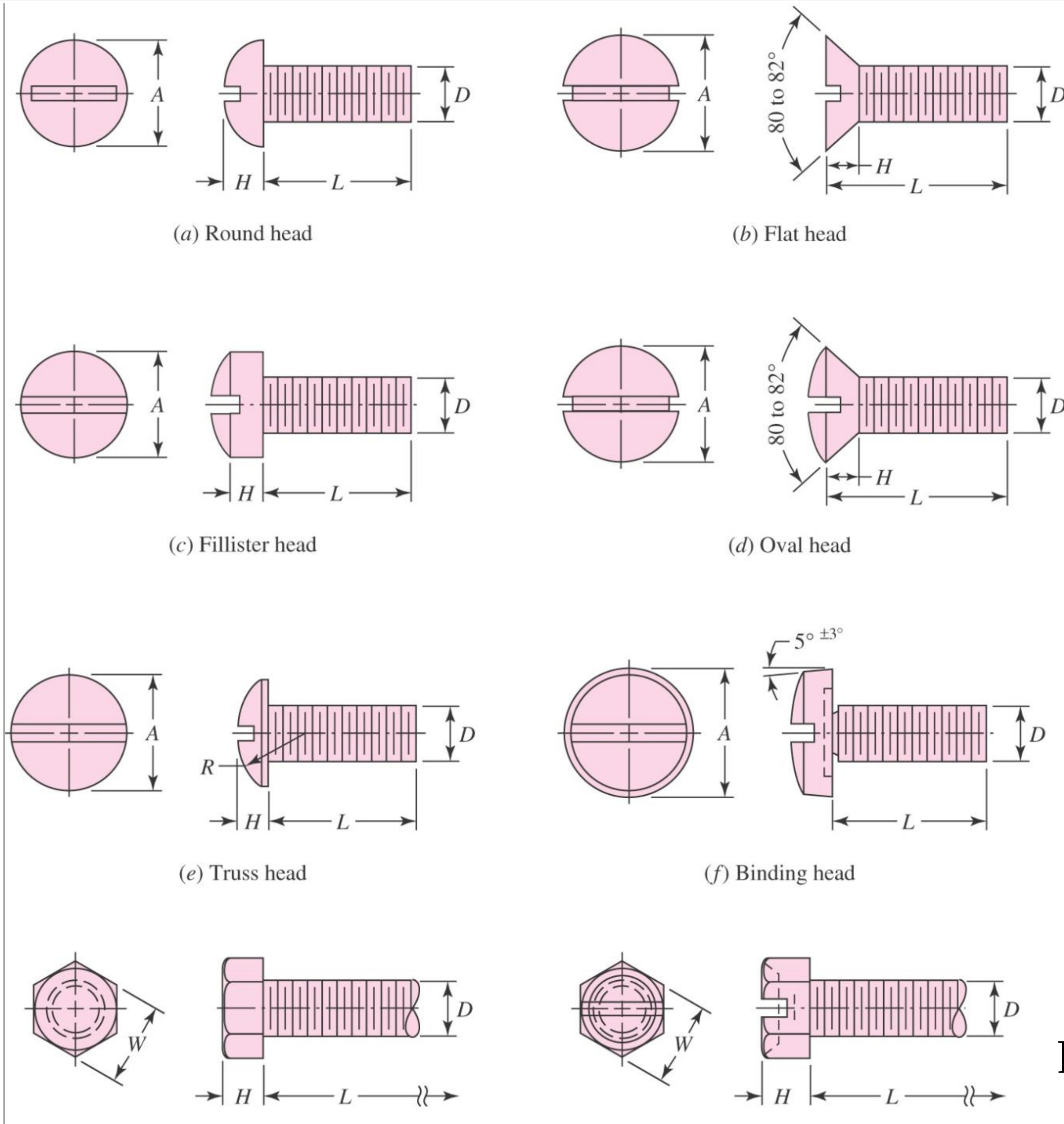


Fig. 8–10

Machine Screws



(*a*) Round head

(*b*) Flat head

(*c*) Fillister head

(*d*) Oval head

(*e*) Truss head

(*f*) Binding head

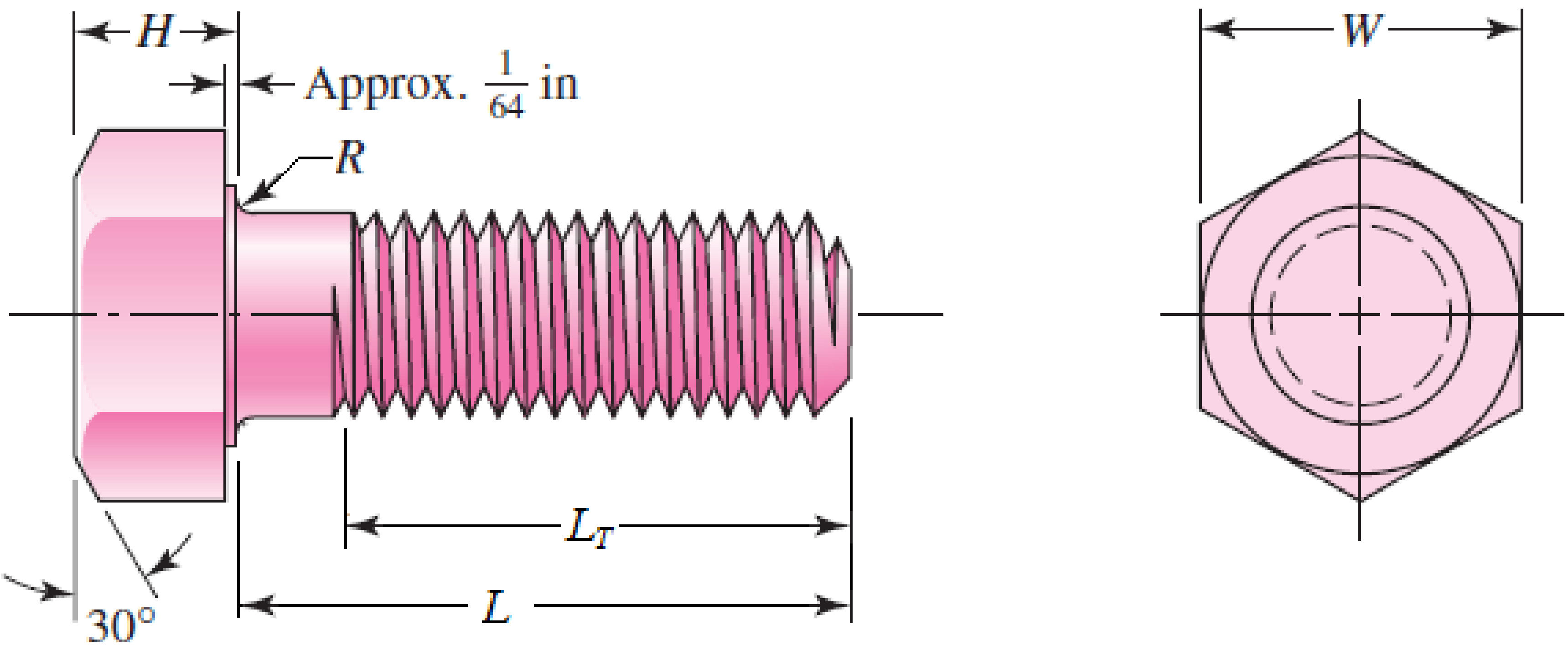
(*g*) Hex head (trimmed)

(*h*) Hex head (upset)

Fig. 8–11

Hexagon-Head Bolt

- Hexagon-head bolts are one of the most common for engineering applications
- Standard dimensions are included in Table A–29
- *W* is usually about 1.5 times nominal diameter
- Bolt length *L* is measured from below the head



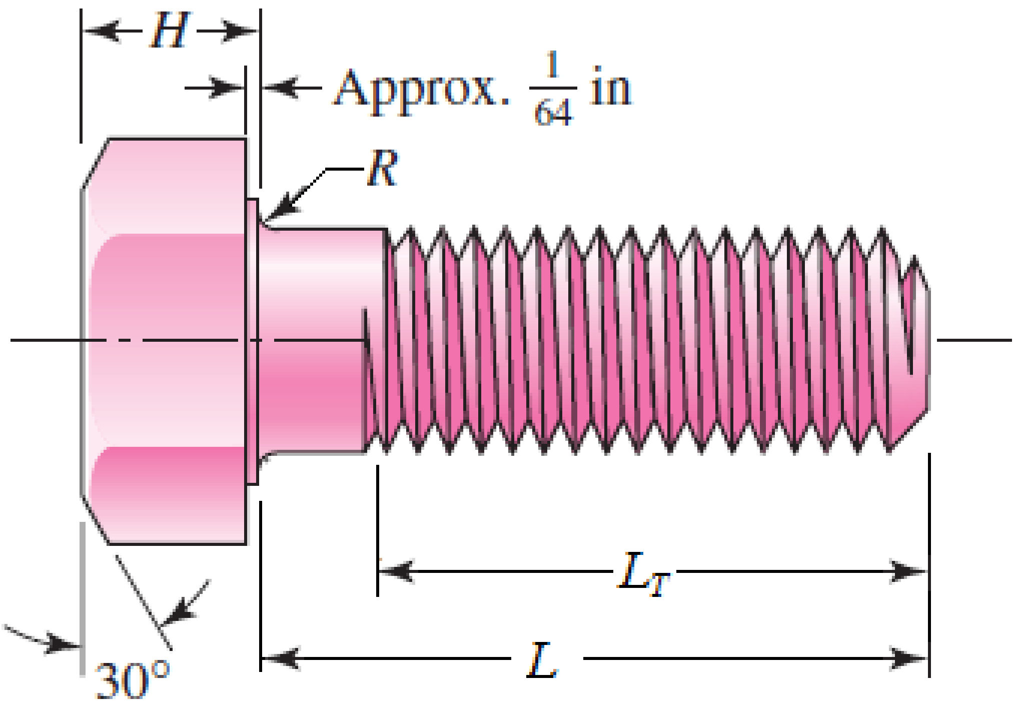
Threaded Lengths

English

$$L_T = \begin{cases} 2d + \frac{1}{4} \text{ in} & L \leq 6 \text{ in} \\ 2d + \frac{1}{2} \text{ in} & L > 6 \text{ in} \end{cases} \quad (8\text{–}13)$$

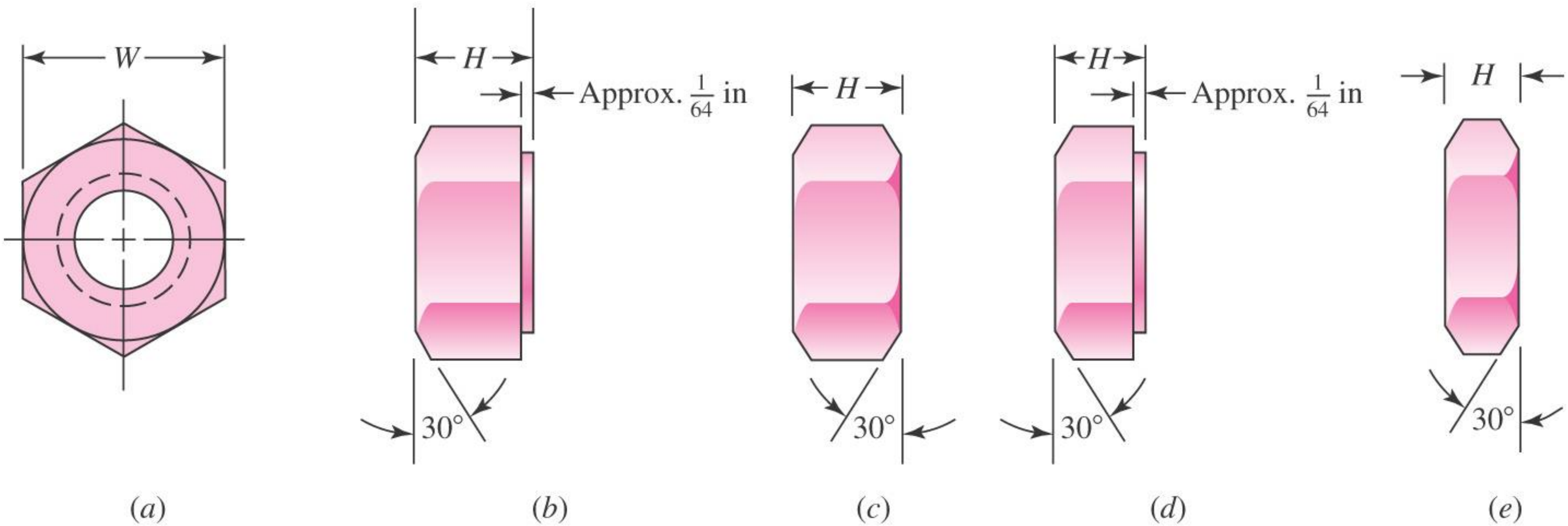
Metric

$$L_T = \begin{cases} 2d + 6 & L \leq 125 \quad d \leq 48 \\ 2d + 12 & 125 < L \leq 200 \\ 2d + 25 & L > 200 \end{cases} \quad (8\text{–}14)$$



Nuts

- See Appendix A–31 for typical specifications
- First three threads of nut carry majority of load
- Localized plastic strain in the first thread is likely, so nuts should not be re-used in critical applications.



End view | Washer-faced, regular | Chamfered both sides, regular | Washer-faced, jam nut | Chamfered both sides, jam nut

Fig. 8–12

Tension Loaded Bolted Joint

- Grip length *l* includes everything being compressed by bolt preload, including washers
- Washer under head prevents burrs at the hole from gouging into the fillet under the bolt head

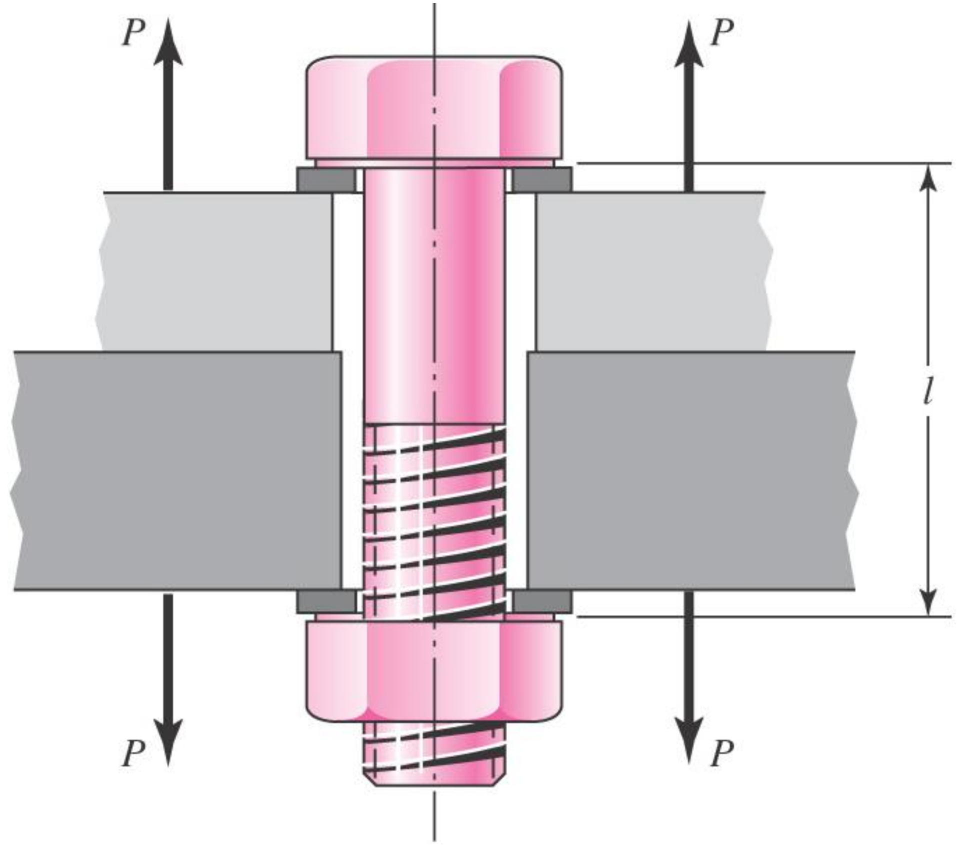


Fig. 8–13

Pressure Vessel Head

- Hex-head cap screw in tapped hole used to fasten cylinder head to cylinder body
- Note O-ring seal, not affecting the stiffness of the members within the grip
- Only part of the threaded length of the bolt contributes to the effective grip *l*

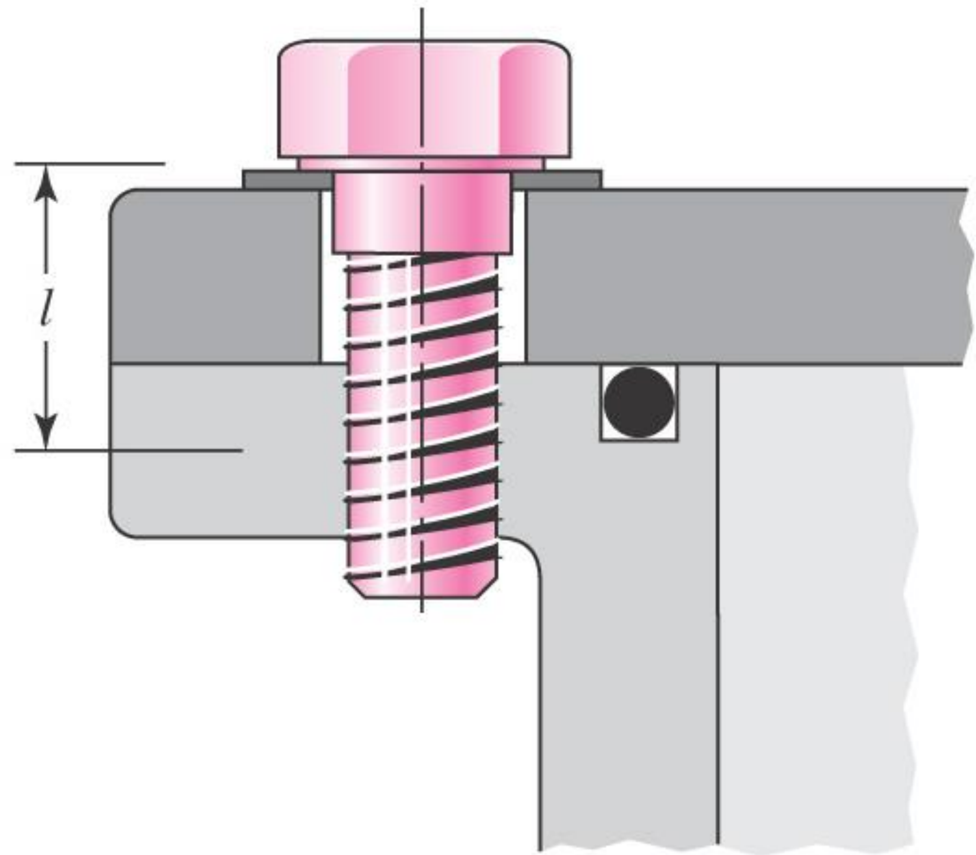
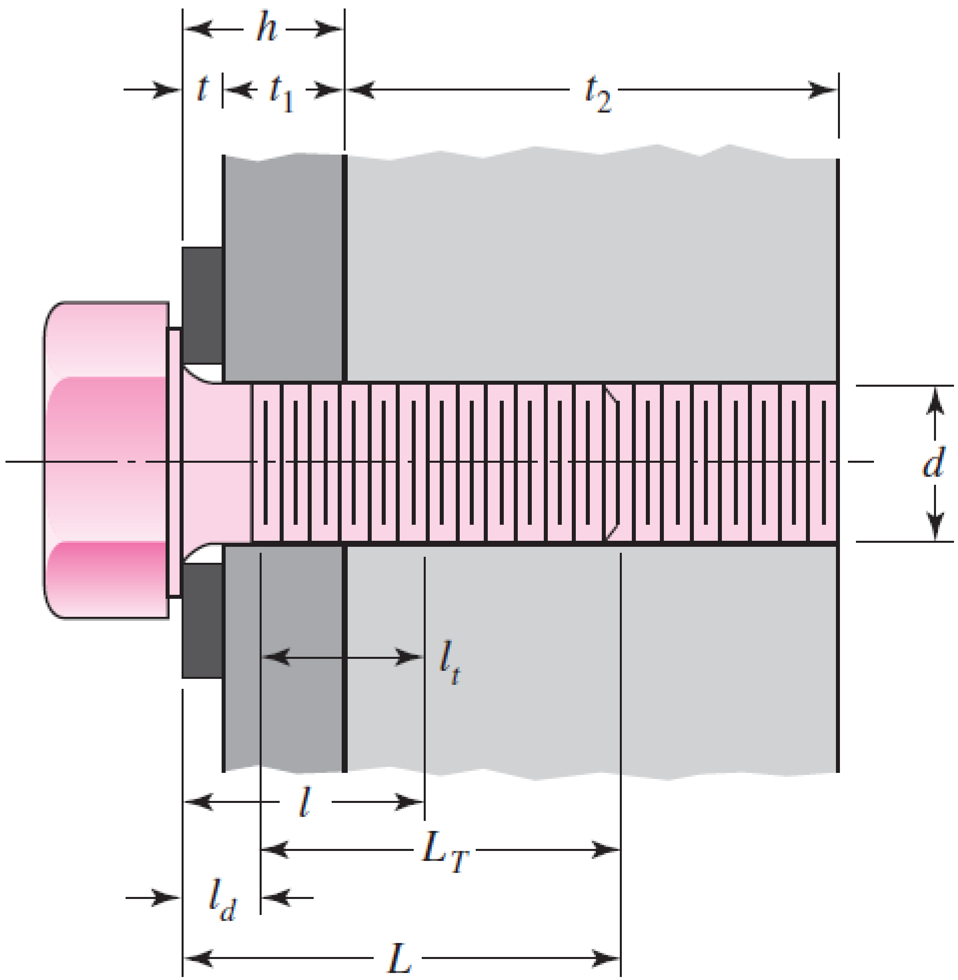


Fig. 8–14

Effective Grip Length for Tapped Holes

- For screw in tapped hole, effective grip length is

$$l = \begin{cases} h + t_2/2, & t_2 < d \\ h + d/2, & t_2 \geq d \end{cases}$$



Bolted Joint Stiffnesses

- During bolt preload
  - bolt is stretched
  - members in grip are compressed
- When external load *P* is applied
  - Bolt stretches further
  - Members in grip uncompress some
- Joint can be modeled as a soft bolt spring in parallel with a stiff member spring

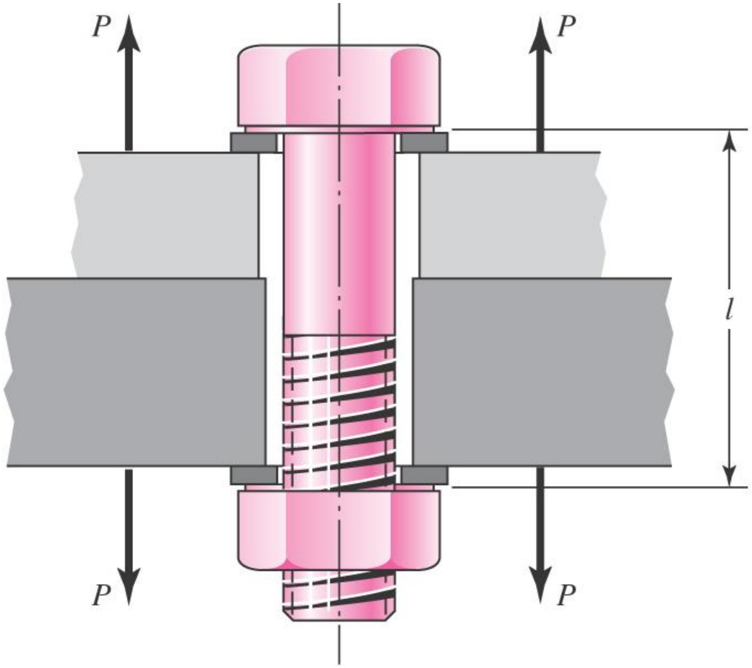
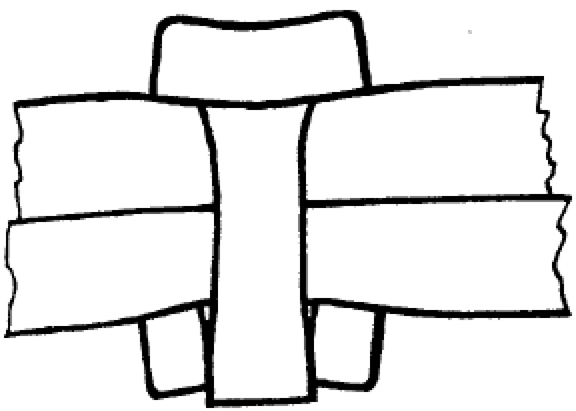
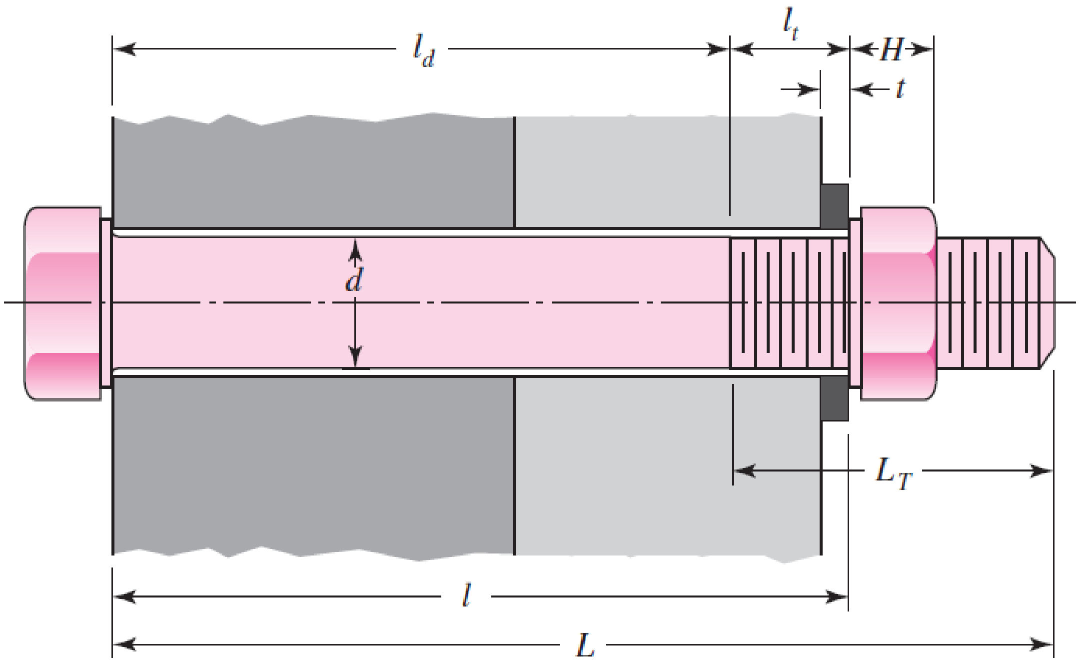


Fig. 8–13



Bolt Stiffness

- Axially loaded rod, partly threaded and partly unthreaded
- Consider each portion as a spring
- Combine as two springs in series



$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \qquad \text{or} \qquad k = \frac{k_1 k_2}{k_1 + k_2} \tag{8–15}$$

$$k_t = \frac{A_t E}{l_t} \qquad k_d = \frac{A_d E}{l_d} \tag{8–16}$$

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} \tag{8–17}$$

Procedure to Find Bolt Stiffness

Given fastener diameter d and pitch p in mm or number of threads per inch

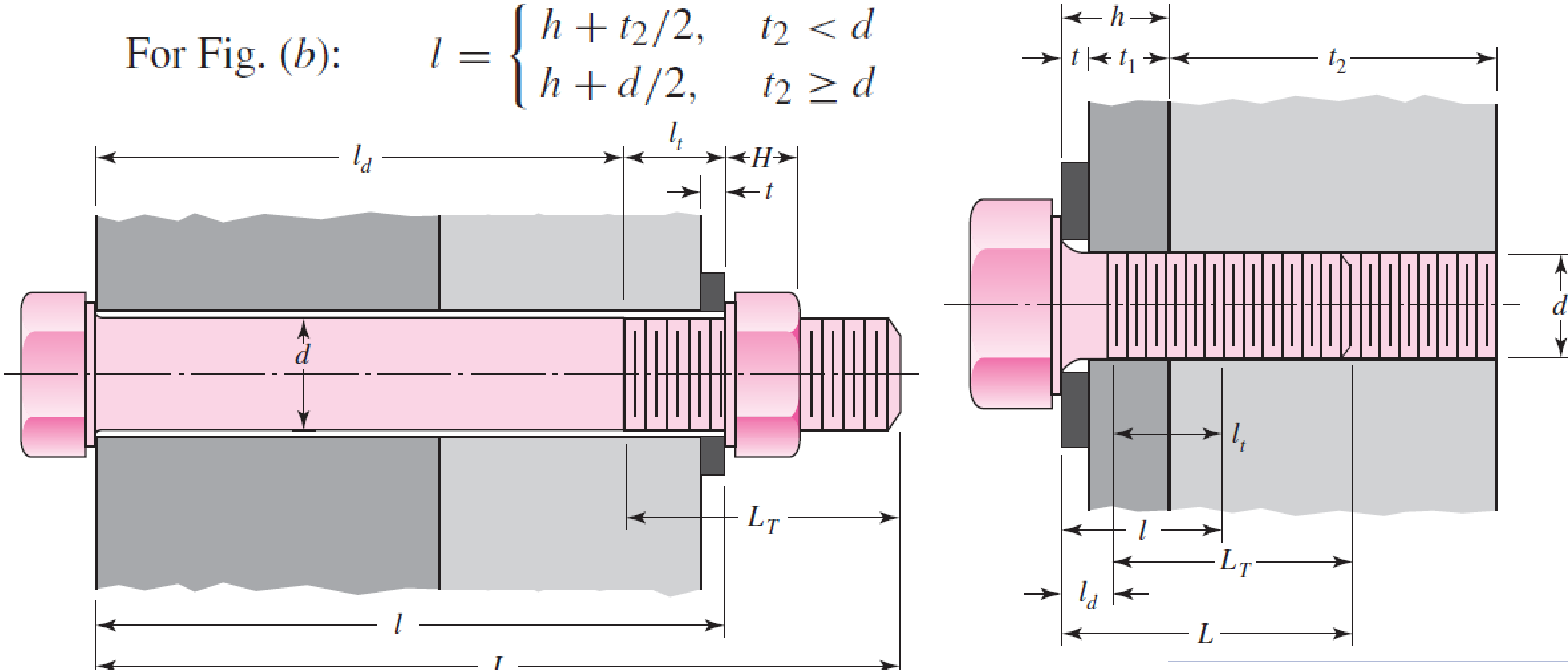
Washer thickness: t from Table A–32 or A–33

Nut thickness [Fig. (a) only]: H from Table A–31

Grip length:

For Fig. (a): l = thickness of all material squeezed between face of bolt and face of nut

For Fig. (b): $$l = \begin{cases} h + t_2/2, & t_2 < d \\ h + d/2, & t_2 \geq d \end{cases}$$



Procedure to Find Bolt Stiffness

Fastener length (round up using Table A–17*):

For Fig. (*a*): $L > l + H$

For Fig. (*b*): $L > h + 1.5d$

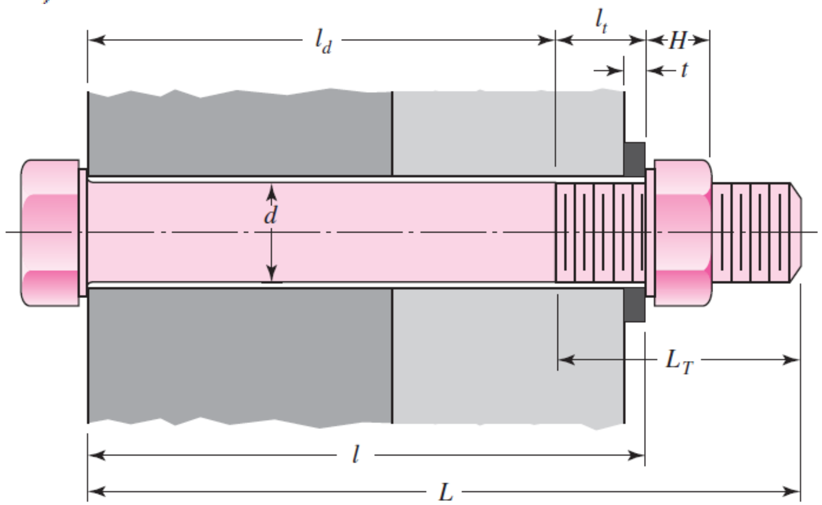
Threaded length L_T:

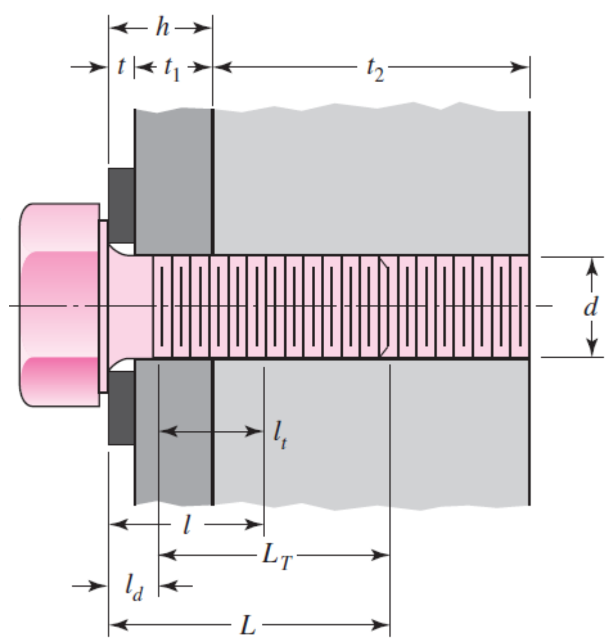
Inch series:

$$L_T = \begin{cases} 2d + \frac{1}{4} \text{ in}, & L \le 6 \text{ in} \\ 2d + \frac{1}{2} \text{ in}, & L > 6 \text{ in} \end{cases}$$

Metric series:

$$L_T = \begin{cases} 2d + 6 \text{ mm}, & L \le 125 \text{ mm}, d \le 48 \text{ mm} \\ 2d + 12 \text{ mm}, & 125 < L \le 200 \text{ mm} \\ 2d + 25 \text{ mm}, & L > 200 \text{ mm} \end{cases}$$





Procedure to Find Bolt Stiffness

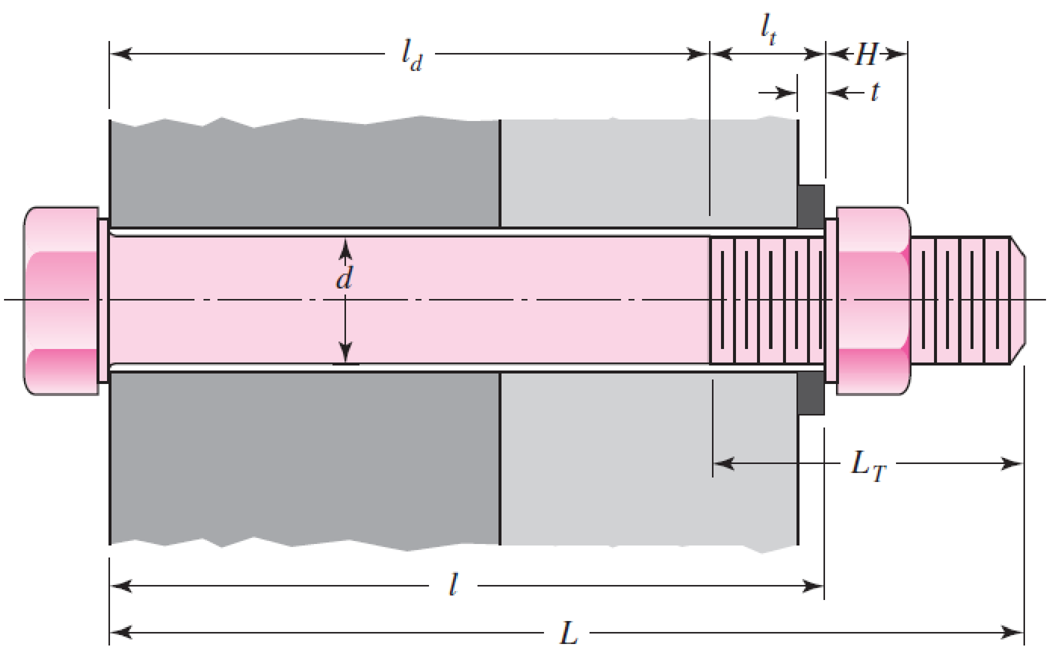
Length of unthreaded portion in grip: $l_d = L - L_T$

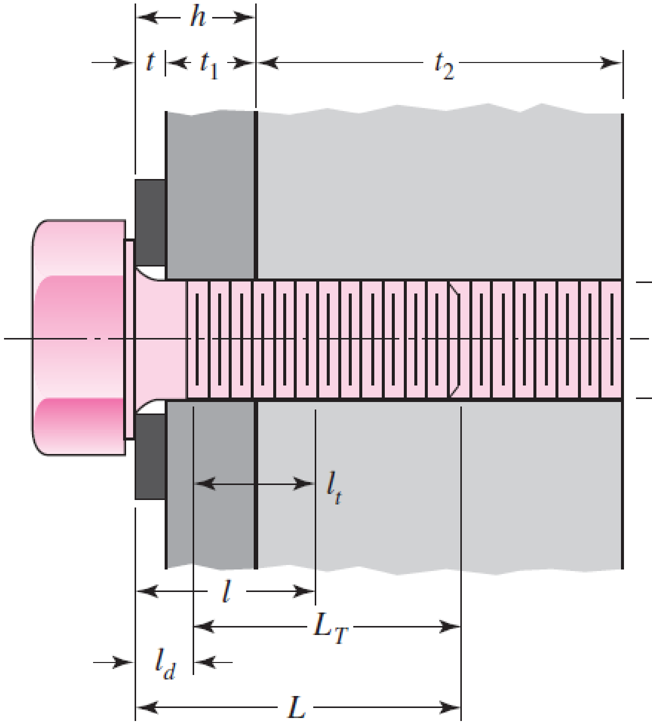
Length of threaded portion in grip: $l_t = l - l_d$

Area of unthreaded portion: $A_d = \pi d^2/4$

Area of threaded portion: A_t from Table 8–1 or 8–2

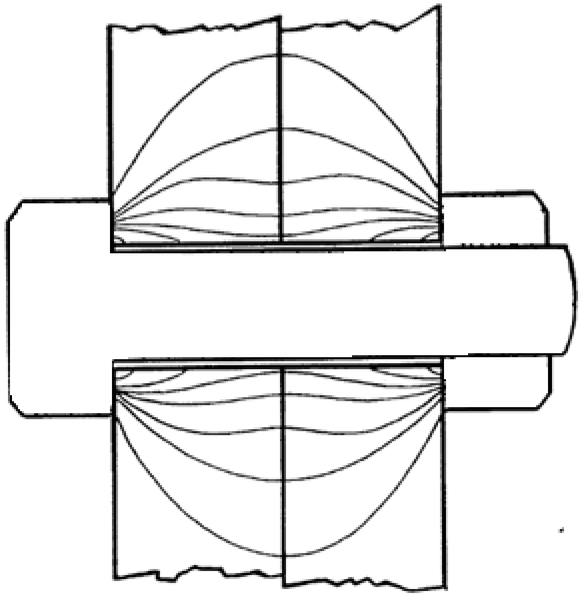
Fastener stiffness: $$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$





Member Stiffness

- Stress distribution spreads from face of bolt head and nut
- Model as a cone with top cut off
- Called a *frustum*



Member Stiffness

- Model compressed members as if they are frusta spreading from the bolt head and nut to the midpoint of the grip
- Each frustum has a half-apex angle of α
- Find stiffness for frustum in compression

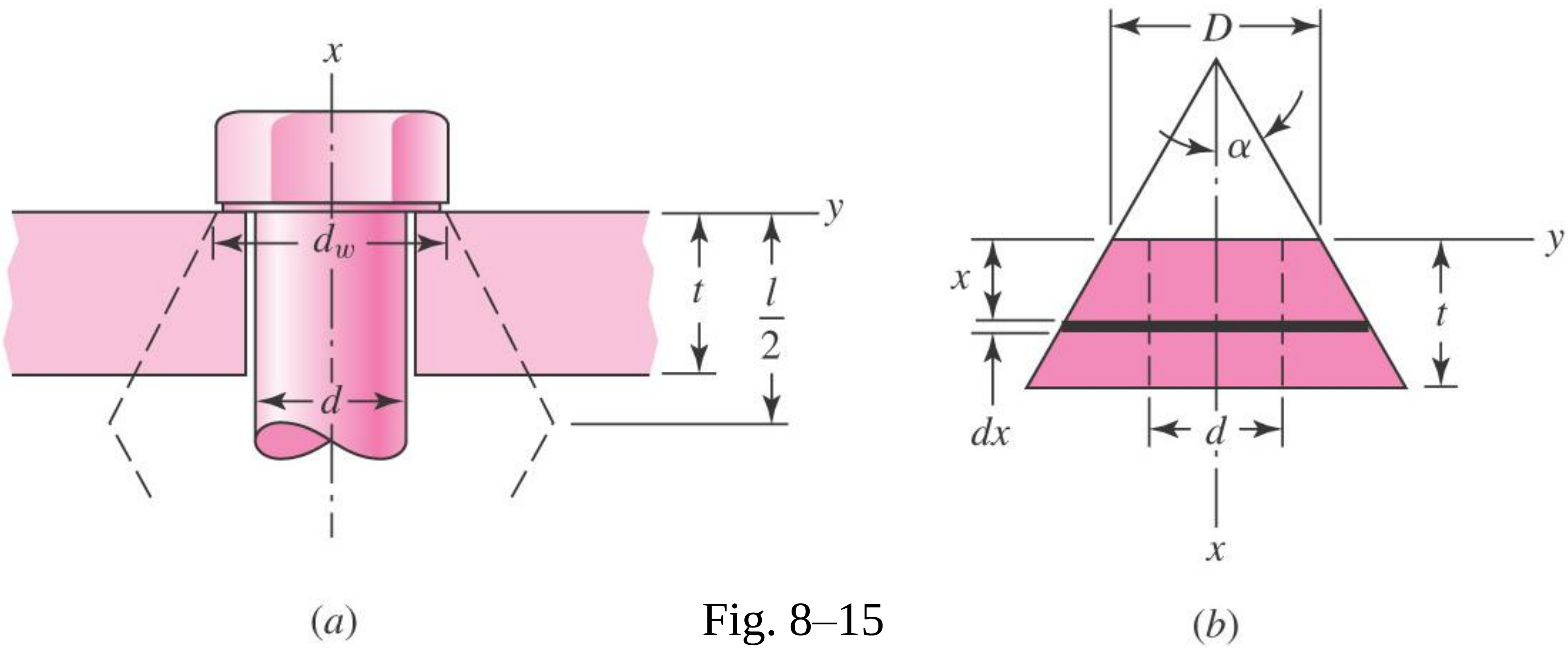


Fig. 8–15

Member Stiffness

$$d\delta = \frac{P\,dx}{EA} \qquad (a)$$

$$A = \pi\left(r_o^2 - r_i^2\right) = \pi\left[\left(x\tan\alpha + \frac{D}{2}\right)^2 - \left(\frac{d}{2}\right)^2\right]$$

$$= \pi\left(x\tan\alpha + \frac{D+d}{2}\right)\left(x\tan\alpha + \frac{D-d}{2}\right) \qquad (b)$$

$$\delta = \frac{P}{\pi E}\int_0^t \frac{dx}{[x\tan\alpha + (D+d)/2][x\tan\alpha + (D-d)/2]} \qquad (c)$$

$$\delta = \frac{P}{\pi E d\tan\alpha}\ln\frac{(2t\tan\alpha + D - d)(D+d)}{(2t\tan\alpha + D + d)(D-d)} \qquad (d)$$

$$k = \frac{P}{\delta} = \frac{\pi E d\tan\alpha}{\ln\dfrac{(2t\tan\alpha + D - d)(D+d)}{(2t\tan\alpha + D + d)(D-d)}} \qquad (8\text{–}19)$$

Member Stiffness

- With typical value of $\alpha = 30^{\circ}$,

$$k = \frac{0.5774\pi E d}{\ln \dfrac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}} \qquad (8\text{–}20)$$

- Use Eq. (8–20) to find stiffness for each frustum
- Combine all frusta as springs in series

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \cdots + \frac{1}{k_i} \qquad (8\text{–}18)$$

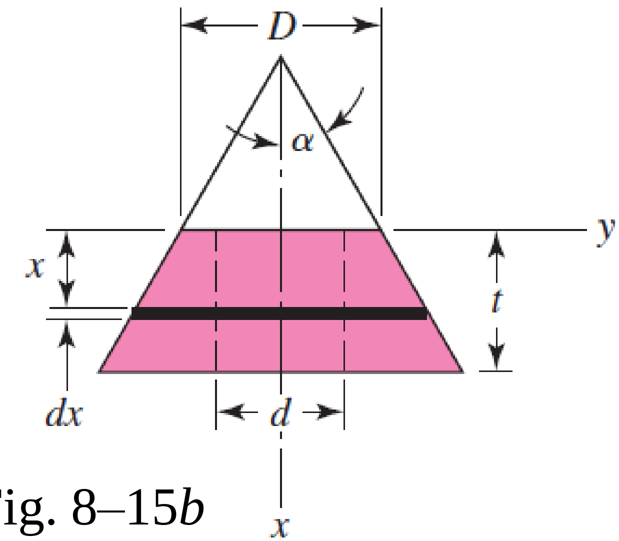


Fig. 8–15*b*

Member Stiffness for Common Material in Grip

- If the grip consists of any number of members all of the same material, two identical frusta can be added in series. The entire joint can be handled with one equation,

$$k_m = \frac{\pi E d \tan\alpha}{2 \ln \dfrac{(l \tan\alpha + d_w - d)(d_w + d)}{(l \tan\alpha + d_w + d)(d_w - d)}} \tag{8–21}$$

- d_w is the washer face diameter
- Using standard washer face diameter of 1.5d, and with $\alpha = 30^\circ$,

$$k_m = \frac{0.5774\pi E d}{2 \ln\left(5\dfrac{0.5774l + 0.5d}{0.5774l + 2.5d}\right)} \tag{8–22}$$

Finite Element Approach to Member Stiffness

- For the special case of common material within the grip, a finite element model agrees with the frustum model

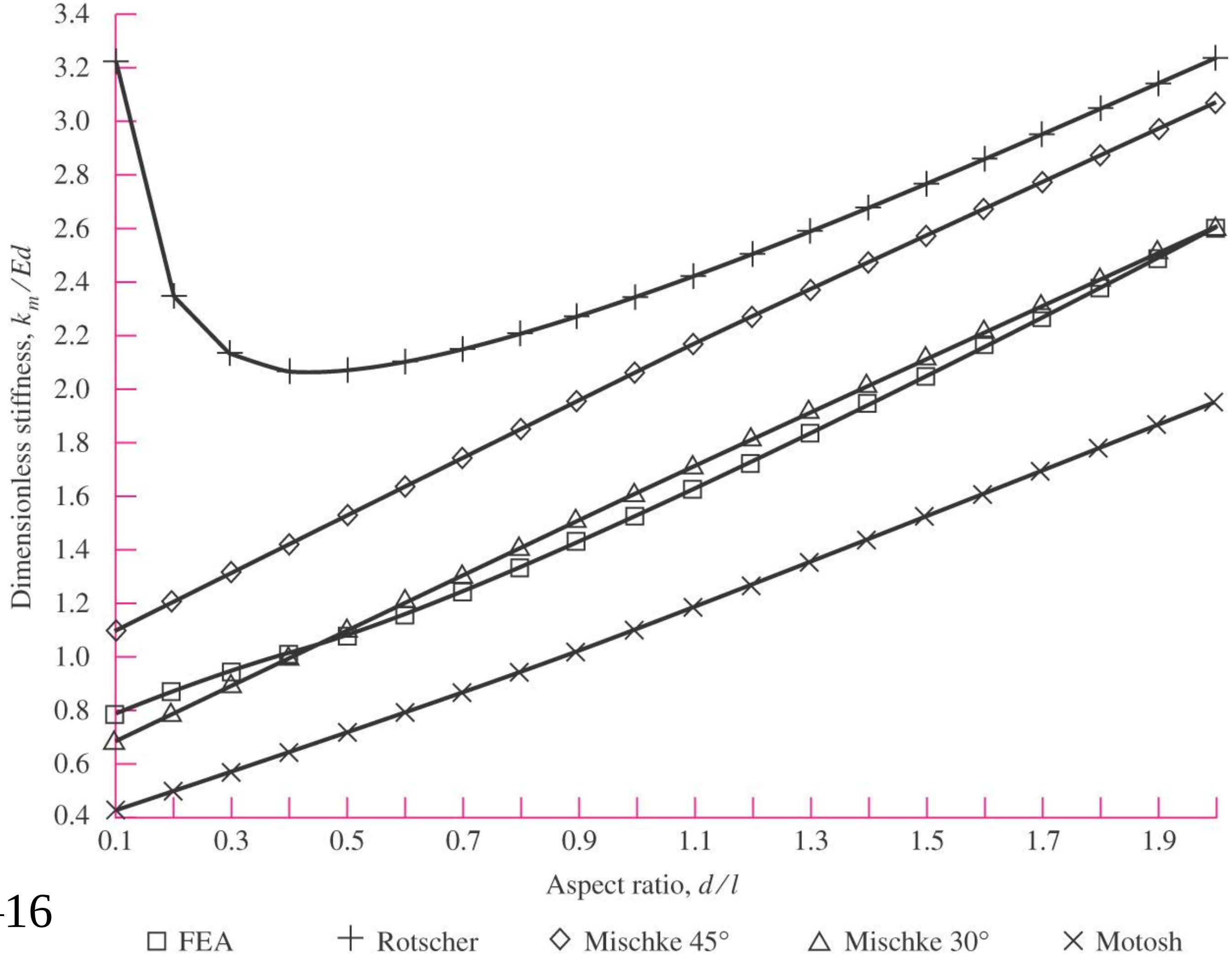


Fig. 8–16

Finite Element Approach to Member Stiffness

- Exponential curve-fit of finite element results can be used for case of common material within the grip

$$\frac{k_m}{Ed} = A \exp(Bd/l) \tag{8–23}$$

Table 8–8

Stiffness Parameters of Various Member Materials†

†*Source:* J. Wileman, M. Choudury, and I. Green, "Computation of Member Stiffness in Bolted Connections," *Trans. ASME, J. Mech. Design,* vol. 113, December 1991, pp. 432–437.

Material Used	Poisson Ratio	Elastic GPa	Modulus Mpsi	*A*	*B*
Steel	0.291	207	30.0	0.787 15	0.628 73
Aluminum	0.334	71	10.3	0.796 70	0.638 16
Copper	0.326	119	17.3	0.795 68	0.635 53
Gray cast iron	0.211	100	14.5	0.778 71	0.616 16
General expression				0.789 52	0.629 14

Bolt Materials

- Grades specify material, heat treatment, strengths
  - Table 8–9 for SAE grades
  - Table 8–10 for ASTM designations
  - Table 8–11 for metric property class
- Grades should be marked on head of bolt

Bolt Materials

- *Proof load* is the maximum load that a bolt can withstand without acquiring a permanent set
- *Proof strength* is the quotient of proof load and tensile-stress area
  - Corresponds to proportional limit
  - Slightly lower than yield strength
  - Typically used for static strength of bolt
- Good bolt materials have stress-strain curve that continues to rise to fracture

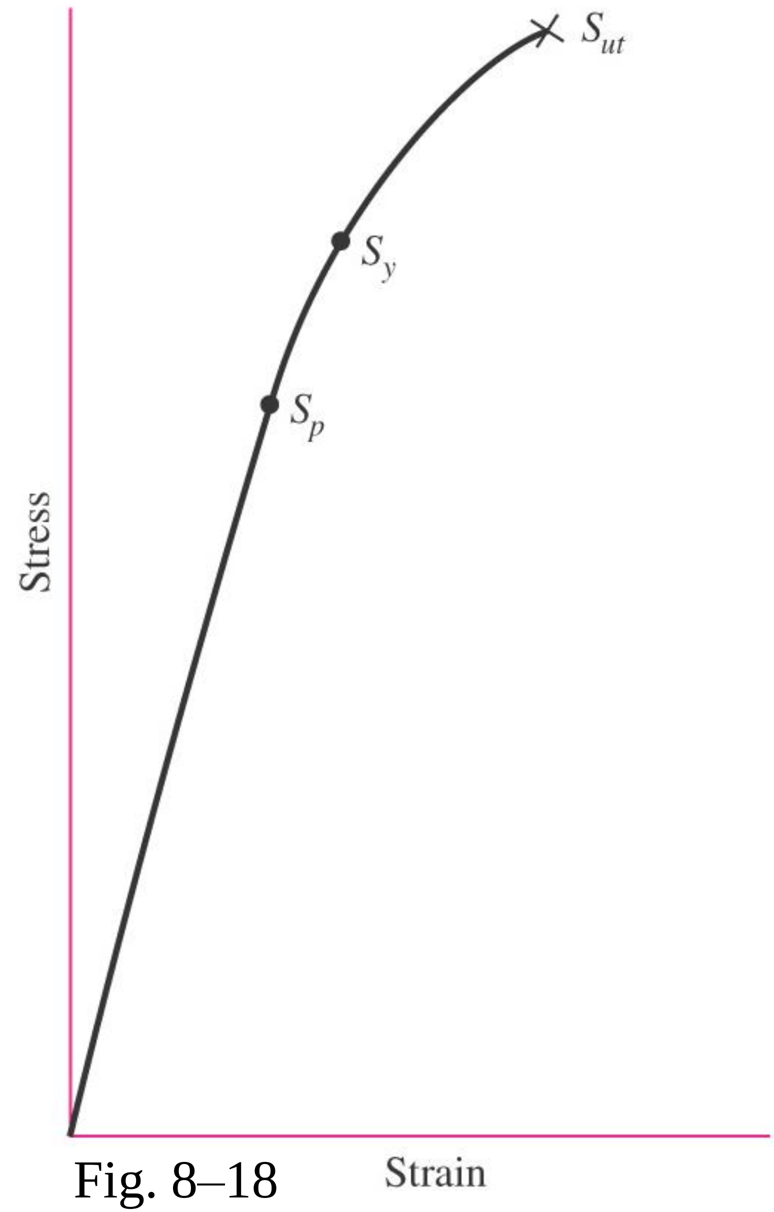
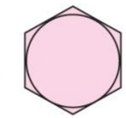
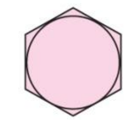
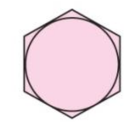
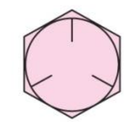
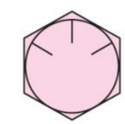
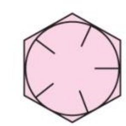
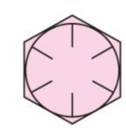
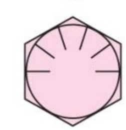


Fig. 8–18

SAE Specifications for Steel Bolts

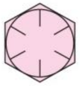
Table 8–9

SAE Grade No.	Size Range Inclusive, in	Minimum Proof Strength,* kpsi	Minimum Tensile Strength,* kpsi	Minimum Yield Strength,* kpsi	Material	Head Marking
1	$\frac{1}{4}$–$1\frac{1}{2}$	33	60	36	Low or medium carbon	
2	$\frac{1}{4}$–$\frac{3}{4}$	55	74	57	Low or medium carbon	
	$\frac{7}{8}$–$1\frac{1}{2}$	33	60	36		
4	$\frac{1}{4}$–$1\frac{1}{2}$	65	115	100	Medium carbon, cold-drawn	
5	$\frac{1}{4}$–1	85	120	92	Medium carbon, Q&T	
	$1\frac{1}{8}$–$1\frac{1}{2}$	74	105	81		
5.2	$\frac{1}{4}$–1	85	120	92	Low-carbon martensite, Q&T	
7	$\frac{1}{4}$–$1\frac{1}{2}$	105	133	115	Medium-carbon alloy, Q&T	
8	$\frac{1}{4}$–$1\frac{1}{2}$	120	150	130	Medium-carbon alloy, Q&T	
8.2	$\frac{1}{4}$–1	120	150	130	Low-carbon martensite, Q&T	

*Minimum strengths are strengths exceeded by 99 percent of fasteners.

ASTM Specification for Steel Bolts

Table 8–10

ASTM Designation No.	Size Range, Inclusive, in	Minimum Proof Strength,* kpsi	Minimum Tensile Strength,* kpsi	Minimum Yield Strength,* kpsi	Material	Head Marking
A307	$\frac{1}{4}$–$1\frac{1}{2}$	33	60	36	Low carbon	
A325, type 1	$\frac{1}{2}$–1	85	120	92	Medium carbon, Q&T	
	$1\frac{1}{8}$–$1\frac{1}{2}$	74	105	81		
A325, type 2	$\frac{1}{2}$–1	85	120	92	Low-carbon, martensite, Q&T	
	$1\frac{1}{8}$–$1\frac{1}{2}$	74	105	81		
A325, type 3	$\frac{1}{2}$–1	85	120	92	Weathering steel, Q&T	
	$1\frac{1}{8}$–$1\frac{1}{2}$	74	105	81		
A354, grade BC	$\frac{1}{4}$–$2\frac{1}{2}$	105	125	109	Alloy steel, Q&T	
	$2\frac{3}{4}$–4	95	115	99		
A354, grade BD	$\frac{1}{4}$–4	120	150	130	Alloy steel, Q&T	
A449	$\frac{1}{4}$–1	85	120	92	Medium-carbon, Q&T	
	$1\frac{1}{8}$–$1\frac{1}{2}$	74	105	81		
	$1\frac{3}{4}$–3	55	90	58		
A490, type 1	$\frac{1}{2}$–$1\frac{1}{2}$	120	150	130	Alloy steel, Q&T	
A490, type 3	$\frac{1}{2}$–$1\frac{1}{2}$	120	150	130	Weathering steel, Q&T	

*Minimum strengths are strengths exceeded by 99 percent of fasteners.

Metric Mechanical-Property Classes for Steel Bolts

Property Class	Size Range, Inclusive	Minimum Proof Strength,† MPa	Minimum Tensile Strength,† MPa	Minimum Yield Strength,† MPa	Material	Head Marking
4.6	M5–M36	225	400	240	Low or medium carbon	4.6
4.8	M1.6–M16	310	420	340	Low or medium carbon	4.8
5.8	M5–M24	380	520	420	Low or medium carbon	5.8
8.8	M16–M36	600	830	660	Medium carbon, Q&T	8.8
9.8	M1.6–M16	650	900	720	Medium carbon, Q&T	9.8
10.9	M5–M36	830	1040	940	Low-carbon martensite, Q&T	10.9
12.9	M1.6–M36	970	1220	1100	Alloy, Q&T	12.9

Table 8–11

*The thread length for bolts and cap screws is

Bolt Specification

Threads per inch

Thread series

Material grade

¼-20 x ¾ in UNC-2 Grade 5 Hex head bolt

Nominal diameter

length

Class fit

Head type

Metric

Pitch

M12 x 1.75 ISO 4.8 Hex head bolt

Nominal diameter

Material class

Tension Loaded Bolted Joints

$$F_i = \text{preload}$$

$$P_{\text{total}} = \text{Total external tensile load applied to the joint}$$

$$P = \text{external tensile load per bolt}$$

$$P_b = \text{portion of } P \text{ taken by bolt}$$

$$P_m = \text{portion of } P \text{ taken by members}$$

$$F_b = P_b + F_i = \text{resultant bolt load}$$

$$F_m = P_m - F_i = \text{resultant load on members}$$

$$C = \text{fraction of external load } P \text{ carried by bolt}$$

$$1 - C = \text{fraction of external load } P \text{ carried by members}$$

$$N = \text{Number of bolts in the joint}$$

Tension Loaded Bolted Joints

- During bolt preload
  - bolt is stretched
  - members in grip are compressed
- When external load P is applied
  - Bolt stretches an additional amount δ
  - Members in grip uncompress same amount δ

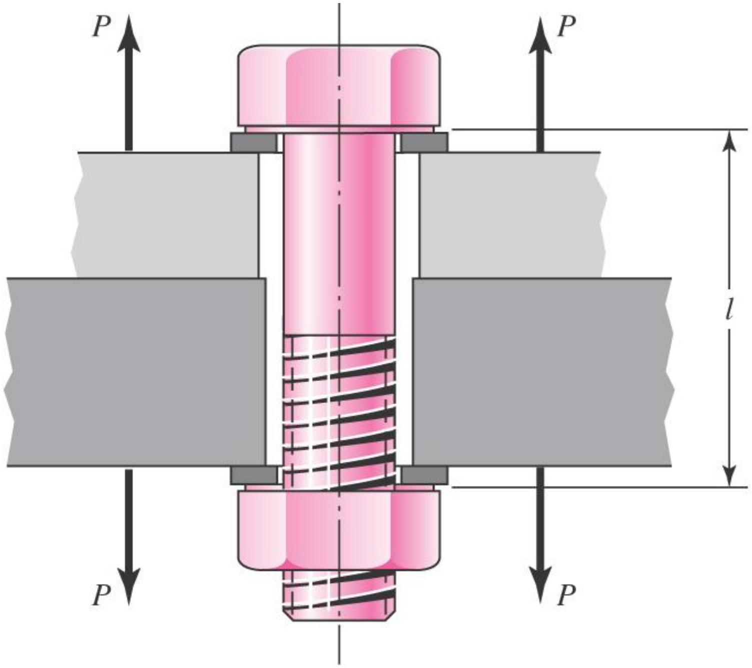


Fig. 8–13

$$\delta = \frac{P_b}{k_b} \qquad \text{and} \qquad \delta = \frac{P_m}{k_m} \tag{b}$$

$$P_m = P_b \frac{k_m}{k_b} \tag{c}$$

Stiffness Constant

- Since $P = P_b + P_m$,

$$P_b = \frac{k_b P}{k_b + k_m} = C P \qquad (d)$$

$$P_m = P - P_b = (1 - C) P \qquad (e)$$

- C is defined as the *stiffness constant* of the joint

$$C = \frac{k_b}{k_b + k_m} \qquad (f)$$

- C indicates the proportion of external load P that the bolt will carry. A good design target is around 0.2.

Table 8–12

Computation of Bolt and Member Stiffnesses. Steel members clamped using a $\frac{1}{2}$ in-13 NC steel bolt. $C = \frac{k_b}{k_b + k_m}$

	Stiffnesses, M lbf/in			
Bolt Grip, in	k_b	k_m	C	$1 - C$
2	2.57	12.69	0.168	0.832
3	1.79	11.33	0.136	0.864
4	1.37	10.63	0.114	0.886

Bolt and Member Loads

- The resultant bolt load is

$$F_b = P_b + F_i = CP + F_i \qquad F_m < 0 \tag{8–24}$$

- The resultant load on the members is

$$F_m = P_m - F_i = (1 - C)P - F_i \qquad F_m < 0 \tag{8–25}$$

- These results are only valid if the load on the members remains negative, indicating the members stay in compression.

Relating Bolt Torque to Bolt Tension

- Best way to measure bolt preload is by relating measured bolt elongation and calculated stiffness
- Usually, measuring bolt elongation is not practical
- Measuring applied torque is common, using a torque wrench
- Need to find relation between applied torque and bolt preload

Relating Bolt Torque to Bolt Tension

- From the power screw equations, Eqs. (8–5) and (8–6), we get

$$T = \frac{F_i d_m}{2}\left(\frac{l + \pi f d_m \sec\alpha}{\pi d_m - f l \sec\alpha}\right) + \frac{F_i f_c d_c}{2} \qquad (a)$$

- Applying $\tan\lambda = l/\pi d_m$,

$$T = \frac{F_i d_m}{2}\left(\frac{\tan\lambda + f \sec\alpha}{1 - f \tan\lambda \sec\alpha}\right) + \frac{F_i f_c d_c}{2} \qquad (b)$$

- Assuming a washer face diameter of 1.5d, the collar diameter is $d_c = (d + 1.5d)/2 = 1.25d$, giving

$$T = \left[\left(\frac{d_m}{2d}\right)\left(\frac{\tan\lambda + f \sec\alpha}{1 - f \tan\lambda \sec\alpha}\right) + 0.625 f_c\right] F_i d \qquad (c)$$

Relating Bolt Torque to Bolt Tension

$$T = \left[\left(\frac{d_m}{2d} \right) \left(\frac{\tan\lambda + f \sec\alpha}{1 - f \tan\lambda \sec\alpha} \right) + 0.625 f_c \right] F_i d \qquad \text{(c)}$$

- Define term in brackets as *torque coefficient K*

$$K = \left(\frac{d_m}{2d} \right) \left(\frac{\tan\lambda + f \sec\alpha}{1 - f \tan\lambda \sec\alpha} \right) + 0.625 f_c \qquad \text{(8–26)}$$

$$T = K F_i d \qquad \text{(8–27)}$$

Typical Values for Torque Coefficient *K*

$$T = K F_i d \qquad (8\text{–}27)$$

- Some recommended values for *K* for various bolt finishes is given in Table 8–15
- Use *K* = 0.2 for other cases

Table 8–15

Torque Factors *K* for Use with Eq. (8–27)

Bolt Condition	*K*
Nonplated, black finish	0.30
Zinc-plated	0.20
Lubricated	0.18
Cadmium-plated	0.16
With Bowman Anti-Seize	0.12
With Bowman-Grip nuts	0.09

Distribution of Preload vs Torque

- Measured preloads for 20 tests at same torque have considerable variation
  - Mean value of 34.3 kN
  - Standard deviation of 4.91

Table 8–13

23.6,	27.6,	28.0,	29.4,	30.3,	30.7,	32.9,	33.8,	33.8,	33.8,
34.7,	35.6,	35.6,	37.4,	37.8,	37.8,	39.2,	40.0,	40.5,	42.7

Mean value $\bar{F}_i = 34.3$ kN. Standard deviation, $\hat{\sigma} = 4.91$ kN.

Distribution of Preload vs Torque

- Same test with *lubricated* bolts
  - Mean value of 34.18 kN (unlubricated 34.3 kN)
  - Standard deviation of 2.88 kN (unlubricated 4.91 kN)

Table 8–14

30.3,	32.5,	32.5,	32.9,	32.9,	33.8,	34.3,	34.7,	37.4,	40.5

Mean value, $\bar{F}_i = 34.18$ kN. Standard deviation, $\hat{\sigma} = 2.88$ kN.

- Lubrication made little change to average preload vs torque
- Lubrication significantly reduces the standard deviation of preload vs torque

Tension Loaded Bolted Joints: Static Factors of Safety

Axial Stress:

$$\sigma_b = \frac{F_b}{A_t} = \frac{CP + F_i}{A_t}$$

Yielding Factor of Safety:

$$n_p = \frac{S_p}{\sigma_b} = \frac{S_p}{(CP + F_i)/A_t} = \frac{S_p A_t}{CP + F_i} \qquad (8\text{–}28)$$

Load Factor:

$$\frac{C n_L P + F_i}{A_t} = S_p \qquad n_L = \frac{S_p A_t - F_i}{CP} \qquad (8\text{–}29)$$

Joint Separation Factor:

$$n_0 = \frac{F_i}{P(1 - C)} \qquad (8\text{–}30)$$

Recommended Preload

$$F_i = \begin{cases} 0.75F_p & \text{for nonpermanent connections, reused fasteners} \\ 0.90F_p & \text{for permanent connections} \end{cases} \qquad (8\text{–}31)$$

$$F_p = A_t S_p \qquad (8\text{–}32)$$

Fatigue Loading of Tension Joints

- Fatigue methods of Ch. 6 are directly applicable
- Distribution of typical bolt failures is
  - 15% under the head
  - 20% at the end of the thread
  - 65% in the thread at the nut face
- Fatigue stress-concentration factors for threads and fillet are given in Table 8–16

Table 8–16

Fatigue Stress-Concentration Factors K_f for Threaded Elements

SAE Grade	Metric Grade	Rolled Threads	Cut Threads	Fillet
0 to 2	3.6 to 5.8	2.2	2.8	2.1
4 to 8	6.6 to 10.9	3.0	3.8	2.3

Endurance Strength for Bolts

- Bolts are standardized, so endurance strengths are known by experimentation, including all modifiers. See Table 8–17.
- Fatigue stress-concentration factor K_f is also included as a reducer of the endurance strength, so it should not be applied to the bolt stresses.
- Ch. 6 methods can be used for cut threads.

Table 8–17

Fully Corrected Endurance Strengths for Bolts and Screws with Rolled Threads*

Grade or Class	Size Range	Endurance Strength
SAE 5	$\frac{1}{4}$–1 in	18.6 kpsi
	$1\frac{1}{8}$–$1\frac{1}{2}$ in	16.3 kpsi
SAE 7	$\frac{1}{4}$–$1\frac{1}{2}$ in	20.6 kpsi
SAE 8	$\frac{1}{4}$–$1\frac{1}{2}$ in	23.2 kpsi
ISO 8.8	M16–M36	129 MPa
ISO 9.8	M1.6–M16	140 MPa
ISO 10.9	M5–M36	162 MPa
ISO 12.9	M1.6–M36	190 MPa

*Repeatedly applied, axial loading, fully corrected.

Fatigue Stresses

- With an external load on a per bolt basis fluctuating between $P_{\min}$ and $P_{\max}$,

$$F_{b\min} = CP_{\min} + F_i \qquad (a)$$

$$F_{b\max} = CP_{\max} + F_i \qquad (b)$$

$$\sigma_a = \frac{(F_{b\max} - F_{b\min})/2}{A_t} = \frac{(CP_{\max} + F_i) - (CP_{\min} + F_i)}{2A_t}$$

$$\sigma_a = \frac{C(P_{\max} - P_{\min})}{2A_t} \qquad (8\text{–}35)$$

$$\sigma_m = \frac{(F_{b\max} + F_{b\min})/2}{A_t} = \frac{(CP_{\max} + F_i) + (CP_{\min} + F_i)}{2A_t}$$

$$\sigma_m = \frac{C(P_{\max} + P_{\min})}{2A_t} + \frac{F_i}{A_t} \qquad (8\text{–}36)$$

Typical Fatigue Load Line for Bolts

- Typical load line starts from constant preload, then increases with a constant slope

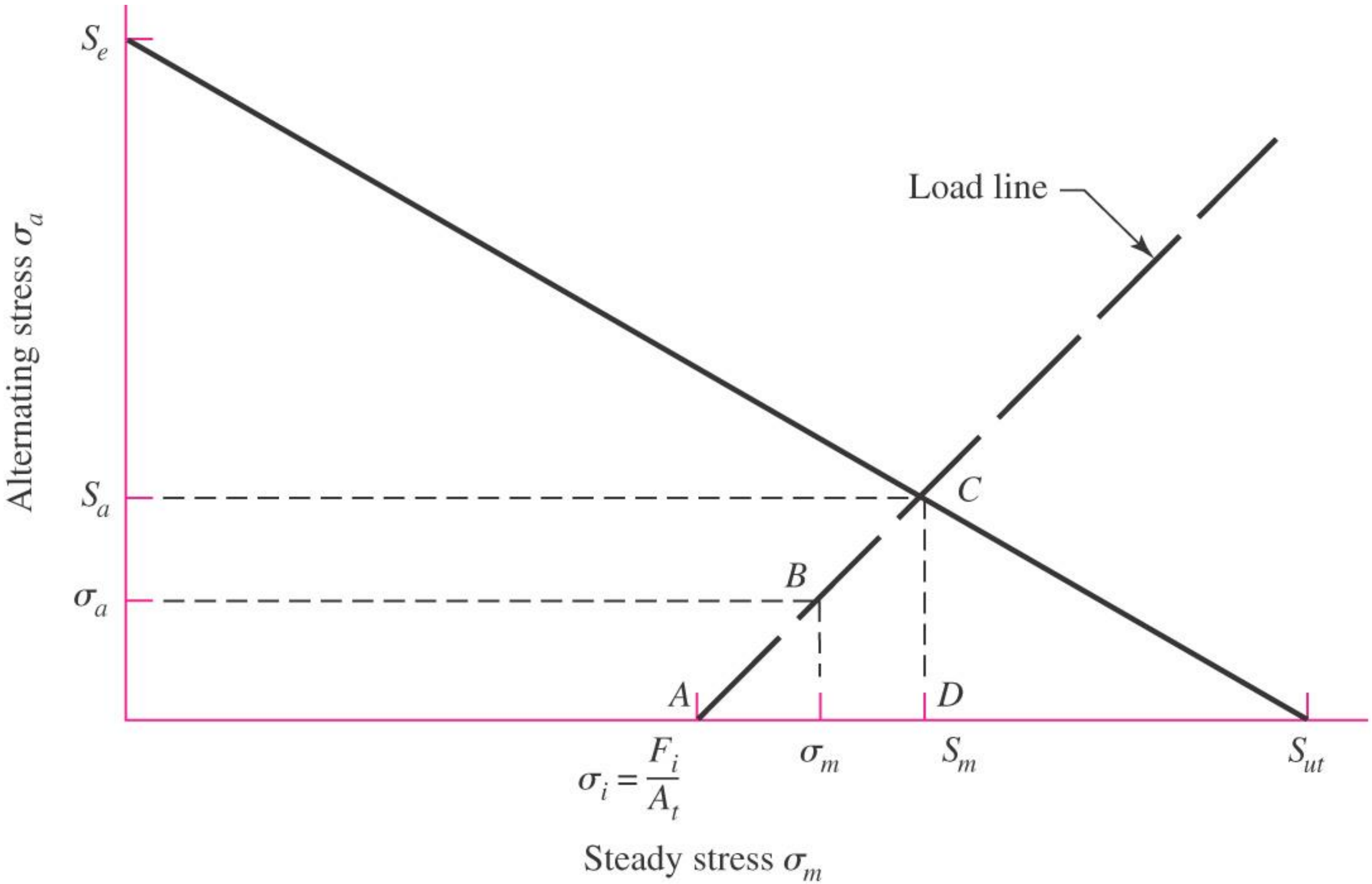


Fig. 8–20

Typical Fatigue Load Line for Bolts

- Equation of load line:

$$S_a = \frac{\sigma_a}{\sigma_m - \sigma_i}(S_m - \sigma_i) \qquad (a)$$

- Equation of Goodman line:

$$S_a = S_e - \frac{S_e}{S_{ut}} S_m \qquad (b)$$

- Solving (*a*) and (*b*) for intersection point,

$$S_a = \frac{S_e \sigma_a (S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m - \sigma_i)} \qquad (c)$$

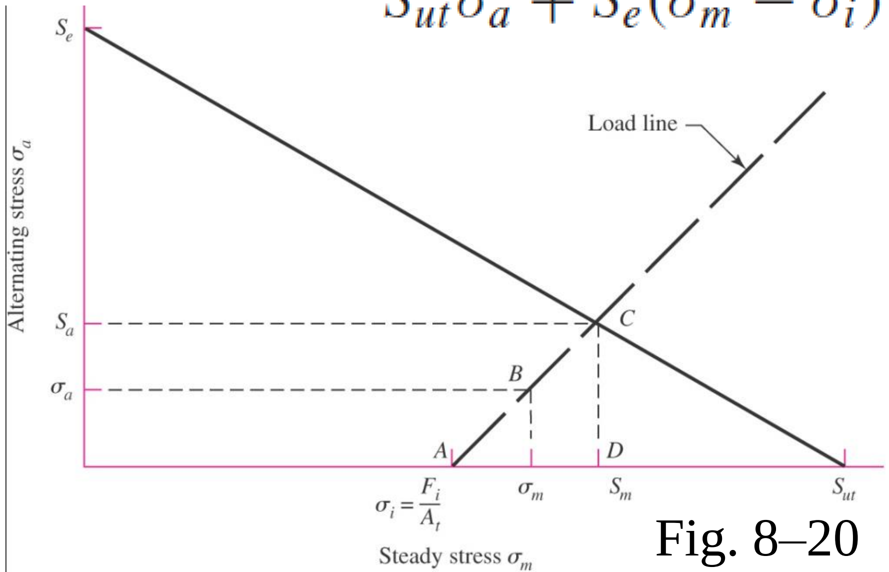


Fig. 8–20

Fatigue Factor of Safety

- Fatigue factor of safety based on Goodman line and constant preload load line,

$$n_f = \frac{S_a}{\sigma_a} \tag{8–37}$$

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m - \sigma_i)} \tag{8–38}$$

- Other failure curves can be used, following the same approach.

Repeated Load Special Case

- Bolted joints often experience *repeated load*, where external load fluctuates between 0 and $P_{\max}$
- Setting $P_{\min} = 0$ in Eqs. (8-35) and (8-36),

$$\sigma_a = \frac{CP}{2A_t} \tag{8–39}$$

$$\sigma_m = \frac{CP}{2A_t} + \frac{F_i}{A_t} \tag{8–40}$$

- With constant preload load line,

$$\sigma_m = \sigma_a + \sigma_i \tag{8–41}$$

- Load line has slope of unity for repeated load case

Repeated Load Special Case

- Intersect load line equation with failure curves to get intersection coordinate S_a
- Divide S_a by σ_a to get fatigue factor of safety for repeated load case for each failure curve.

Load line:

$$\sigma_m = \sigma_a + \sigma_i \qquad (8\text{–}41)$$

Goodman:

$$\frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1 \qquad (8\text{–}42)$$

Gerber:

$$\frac{S_a}{S_e} + \left(\frac{S_m}{S_{ut}}\right)^2 = 1 \qquad (8\text{–}43)$$

ASME-elliptic:

$$\left(\frac{S_a}{S_e}\right)^2 + \left(\frac{S_m}{S_p}\right)^2 = 1 \qquad (8\text{–}44)$$

Repeated Load Special Case

- Fatigue factor of safety equations for repeated loading, constant preload load line, with various failure curves:

Goodman:

$$n_f = \frac{S_e(S_{ut} - \sigma_i)}{\sigma_a(S_{ut} + S_e)} \quad (8\text{–}45)$$

Gerber·

$$n_f = \frac{1}{2\sigma_a S_e}\left[S_{ut}\sqrt{S_{ut}^2 + 4S_e(S_e + \sigma_i)} - S_{ut}^2 - 2\sigma_i S_e\right] \quad (8\text{–}46)$$

ASME-elliptic:

$$n_f = \frac{S_e}{\sigma_a(S_p^2 + S_e^2)}\left(S_p\sqrt{S_p^2 + S_e^2 - \sigma_i^2} - \sigma_i S_e\right) \quad (8\text{–}47)$$

Further Reductions for Goodman

- For convenience, σ_a and σ_i can be substituted into any of the fatigue factor of safety equations.
- Doing so for the Goodman criteria in Eq. (8–45),

$$n_f = \frac{2S_e(S_{ut}A_t - F_i)}{CP(S_{ut} + S_e)} \tag{8–48}$$

- If there is no preload, $C = 1$ and $F_i = 0$, resulting in

$$n_{f0} = \frac{2S_e S_{ut} A_t}{P(S_{ut} + S_e)} \tag{8–49}$$

- Preload is beneficial for resisting fatigue when n_f / n_{f0} is greater than unity. This puts an upper bound on the preload,

$$F_i \le (1 - C)S_{ut}A_t \tag{8–50}$$

Yield Check with Fatigue Stresses

- As always, static yielding must be checked.
- In fatigue loading situations, since σ_a and σ_m are already calculated, it may be convenient to check yielding with

$$n_p = \frac{S_p}{\sigma_m + \sigma_a} \tag{8–51}$$

- This is equivalent to the yielding factor of safety from Eq. (8–28).

$$n_p = \frac{S_p}{\sigma_b} = \frac{S_p}{(CP + F_i)/A_t} = \frac{S_p A_t}{CP + F_i} \tag{8–28}$$

Bolted and Riveted Joints Loaded in Shear

- Shear loaded joints are handled the same for rivets, bolts, and pins
- Several failure modes are possible
  - (a) Joint loaded in shear
  - (b) Bending of bolt or members
  - (c) Shear of bolt
  - (d) Tensile failure of members
  - (e) Bearing stress on bolt or members
  - (f) Shear tear-out
  - (g) Tensile tear-out

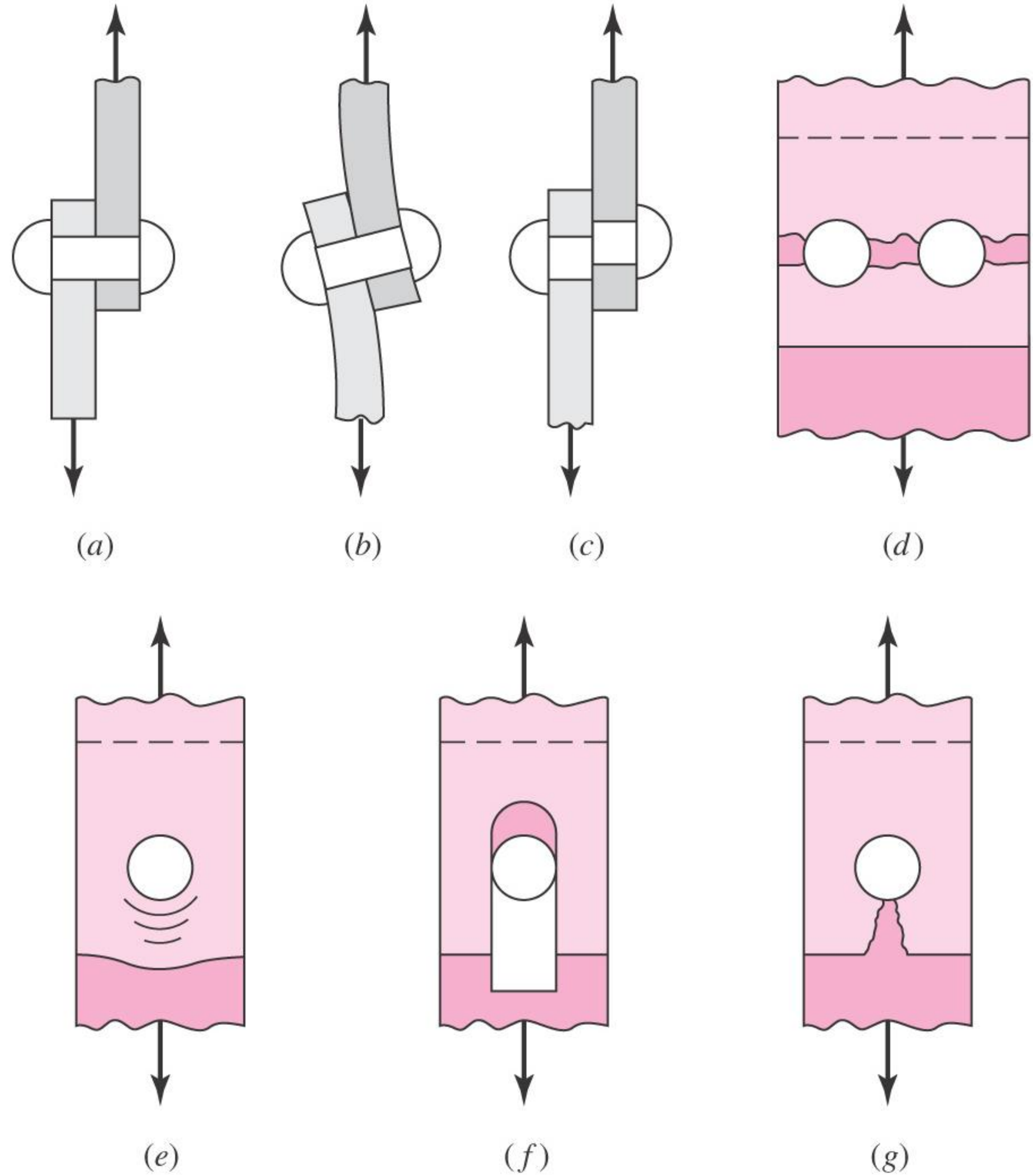
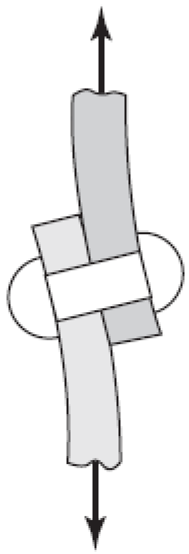


Fig. 8–23

Failure by Bending

- Bending moment is approximately $M = Ft / 2$, where t is the grip length, i.e. the total thickness of the connected parts.
- Bending stress is determined by regular mechanics of materials approach, where I/c is for the weakest member or for the bolt(s).

$$\sigma = \frac{M}{I/c} \tag{8–52}$$

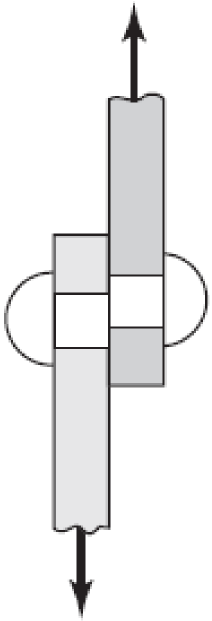


Failure by Shear of Bolt

- Simple direct shear

$$\tau = \frac{F}{A} \tag{8–53}$$

- Use the total cross sectional area of bolts that are carrying the load.
- For bolts, determine whether the shear is across the nominal area or across threaded area. Use area based on nominal diameter or minor diameter, as appropriate.

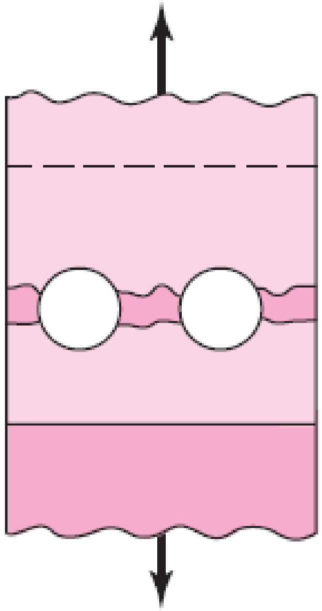


Failure by Tensile Rupture of Member

- Simple tensile failure

$$\sigma = \frac{F}{A} \qquad (8\text{–}54)$$

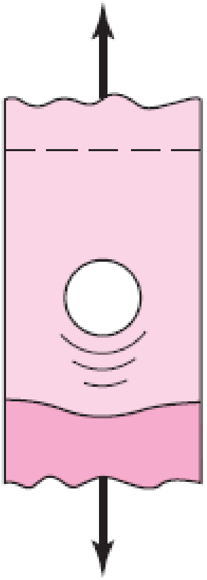
- Use the smallest net area of the member, with holes removed



Failure by Bearing Stress

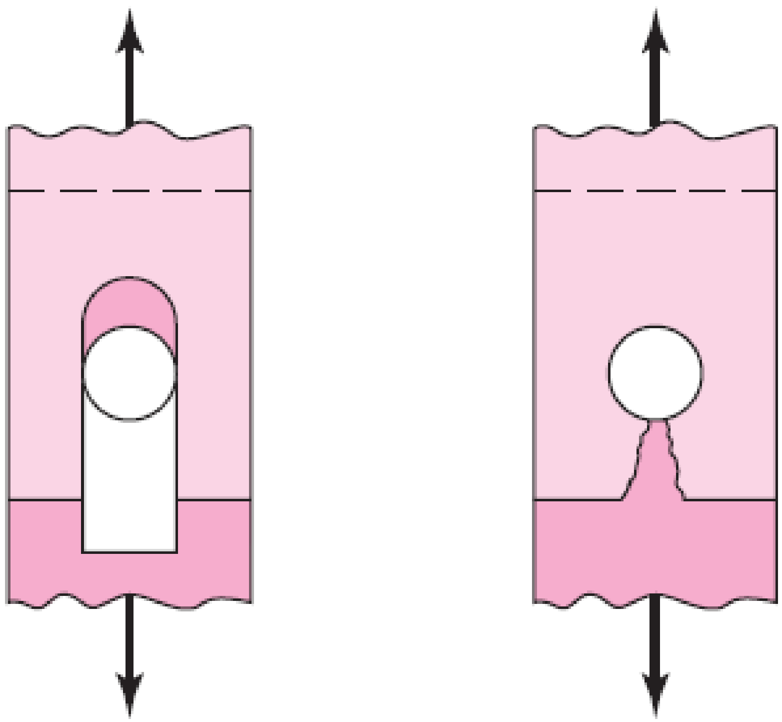
- Failure by crushing known as *bearing stress*
- Bolt or member with lowest strength will crush first
- Load distribution on cylindrical surface is non-trivial
- Customary to assume uniform distribution over projected contact area, $A = td$
- t is the thickness of the thinnest plate and d is the bolt diameter

$$\sigma = -\frac{F}{A} \qquad (8\text{–}55)$$



Failure by Shear-out or Tear-out

- Edge shear-out or tear-out is avoided by spacing bolts at least 1.5 diameters away from the edge



Shear Joints with Eccentric Loading

- *Eccentric* loading is when the load does not pass along a line of symmetry of the fasteners.
- Requires finding moment about centroid of bolt pattern
- Centroid location

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4 + A_5x_5}{A_1 + A_2 + A_3 + A_4 + A_5} = \frac{\sum_1^n A_i x_i}{\sum_1^n A_i} \quad (8\text{–}56)$$

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3 + A_4y_4 + A_5y_5}{A_1 + A_2 + A_3 + A_4 + A_5} = \frac{\sum_1^n A_i y_i}{\sum_1^n A_i}$$

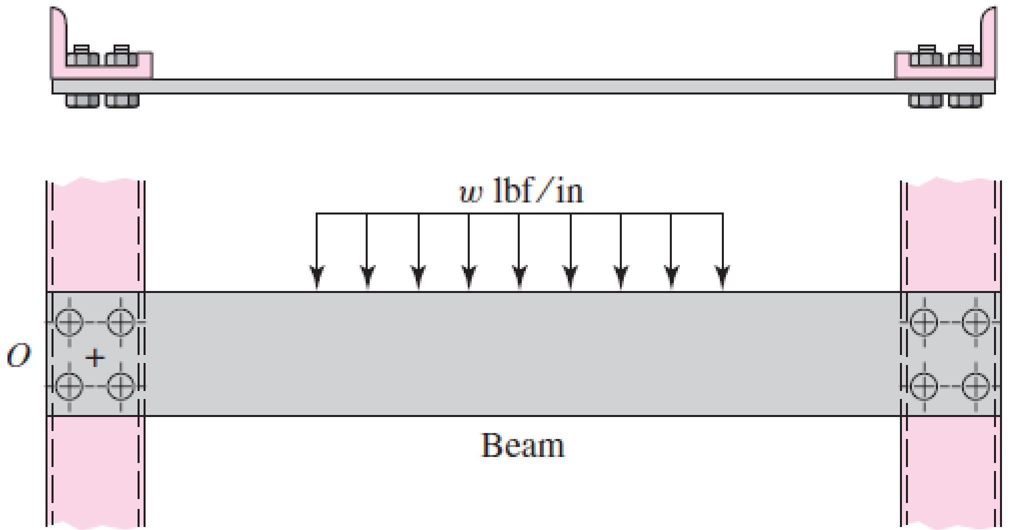
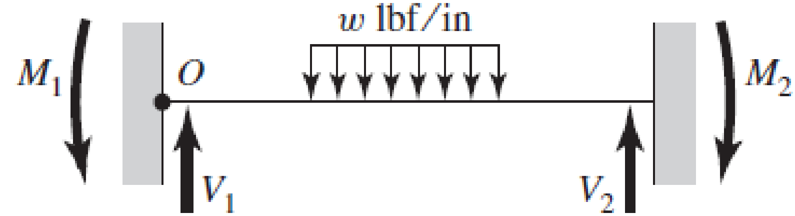


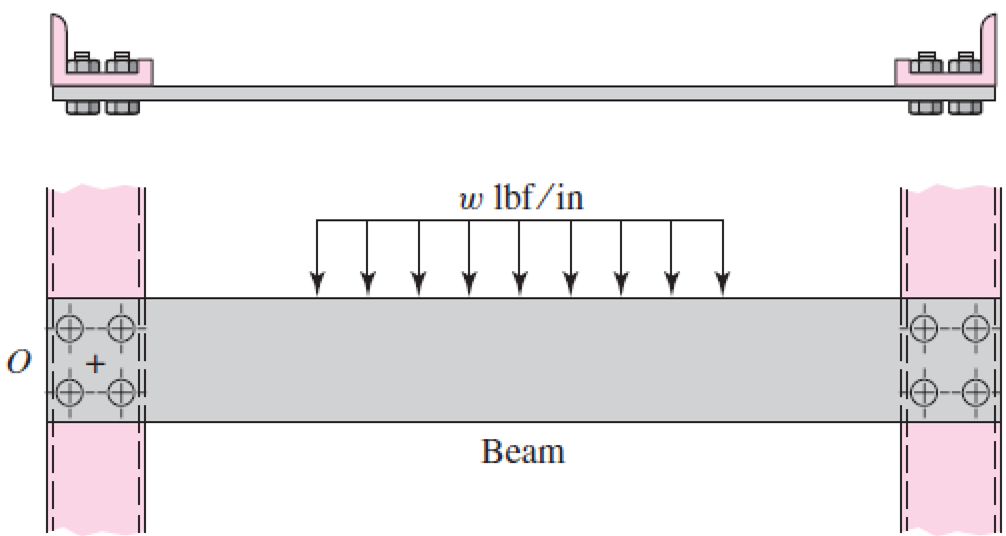
Fig. 8–27*a*

Shear Joints with Eccentric Loading

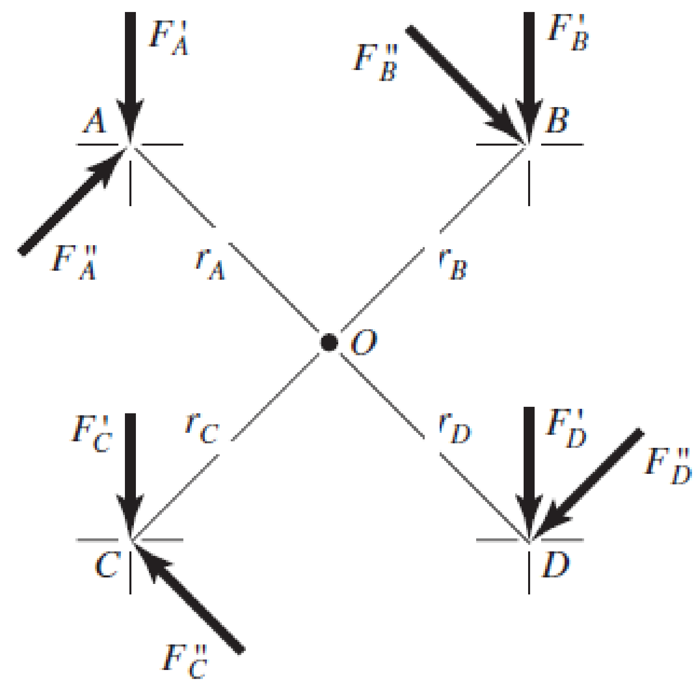
(a) Example of eccentric loading
(b) Free body diagram
(c) Close up of bolt pattern



(b)



(a)



(c)

Fig. 8–27

Shear Joints with Eccentric Loading

- *Primary Shear*

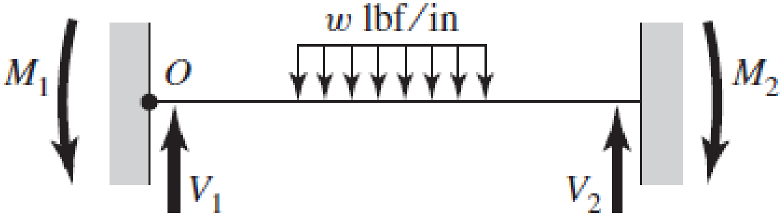
$$F' = V_1/n$$

- *Secondary Shear*, due to moment load around centroid

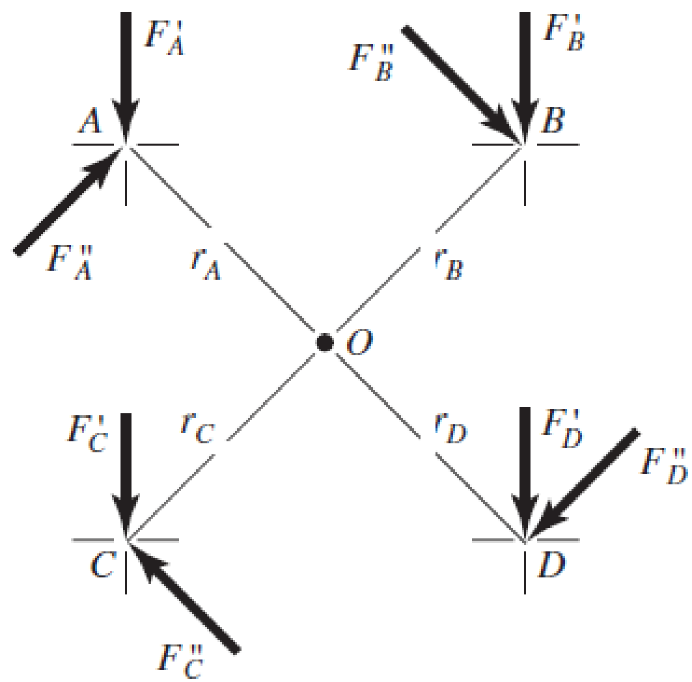
$$M_1 = F''_A r_A + F''_B r_B + F''_C r_C + \cdots$$

$$\frac{F''_A}{r_A} = \frac{F''_B}{r_B} = \frac{F''_C}{r_C}$$

$$F''_n = \frac{M_1 r_n}{r_A^2 + r_B^2 + r_C^2 + \cdots} \qquad (8\text{–}57)$$



(b)



(c)

Example 8-7

Shown in Fig. 8–28 is a 15- by 200-mm rectangular steel bar cantilevered to a 250-mm steel channel using four tightly fitted bolts located at A, B, C, and D.

For a $F = 16$ kN load find

(a) The resultant load on each bolt

(b) The maximum shear stress in each bolt

(c) The maximum bearing stress

(d) The critical bending stress in the bar

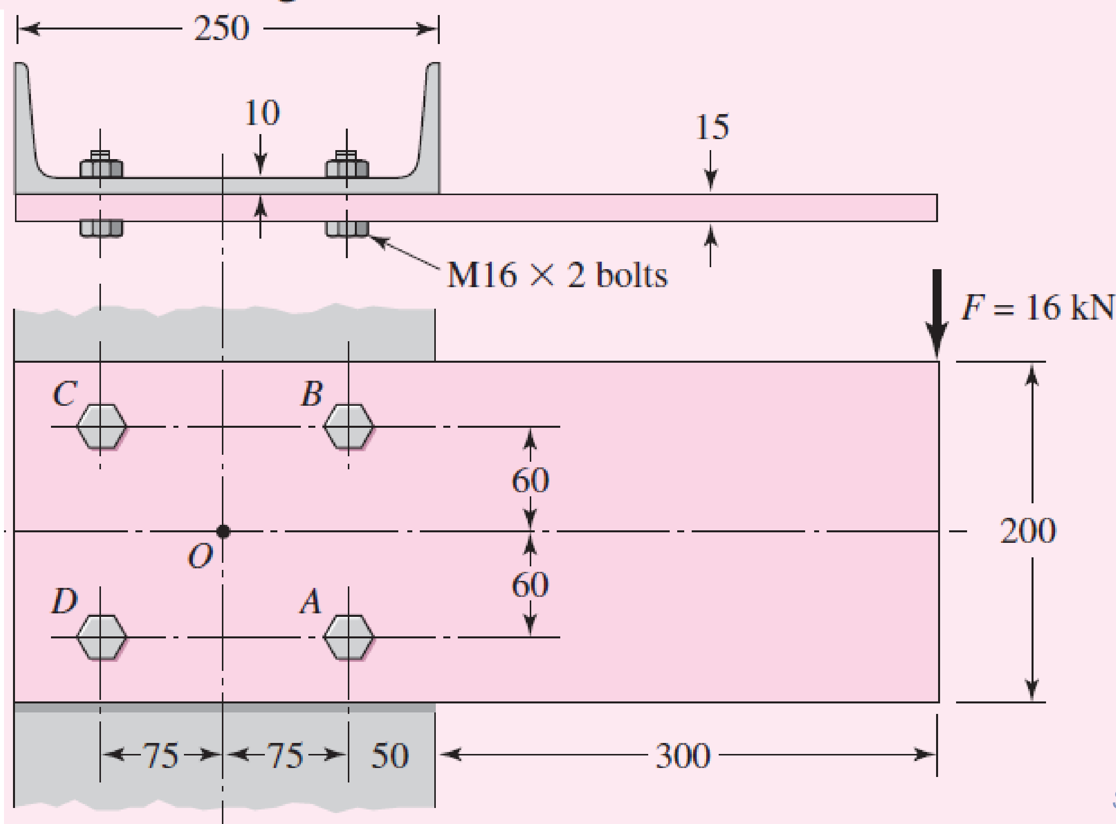


Fig. 8–28

Example 8-7

(*a*) Point *O*, the centroid of the bolt group in Fig. 8–28, is found by symmetry. If a free-body diagram of the beam were constructed, the shear reaction *V* would pass through *O* and the moment reactions *M* would be about *O*. These reactions are

$$V = 16\text{ kN} \qquad M = 16(425) = 6800\text{ N}\cdot\text{m}$$

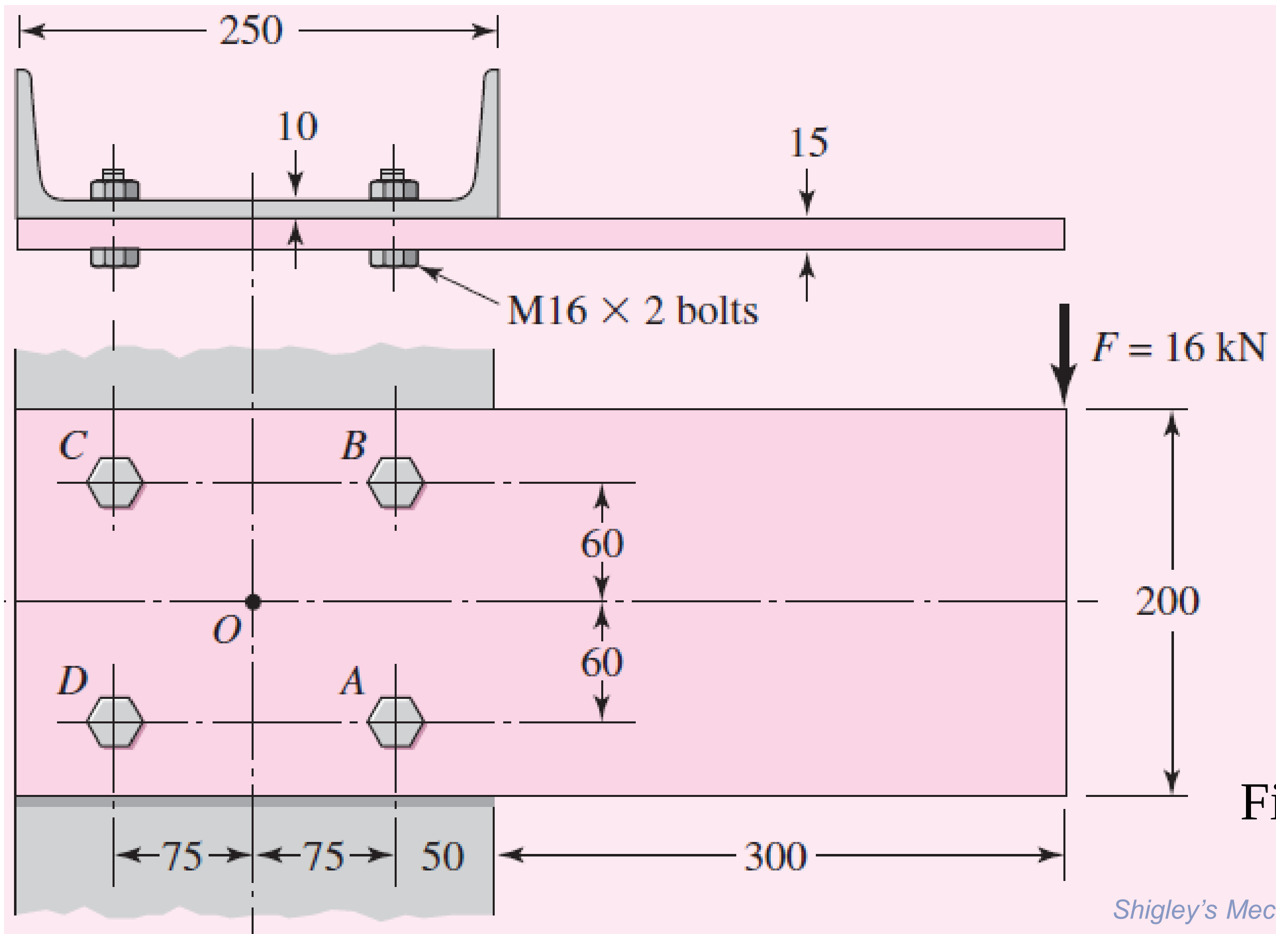


Fig. 8–28

Example 8-7

In Fig. 8–29, the bolt group has been drawn to a larger scale and the reactions are shown. The distance from the centroid to the center of each bolt is

$$r = \sqrt{(60)^2 + (75)^2} = 96.0 \text{ mm}$$

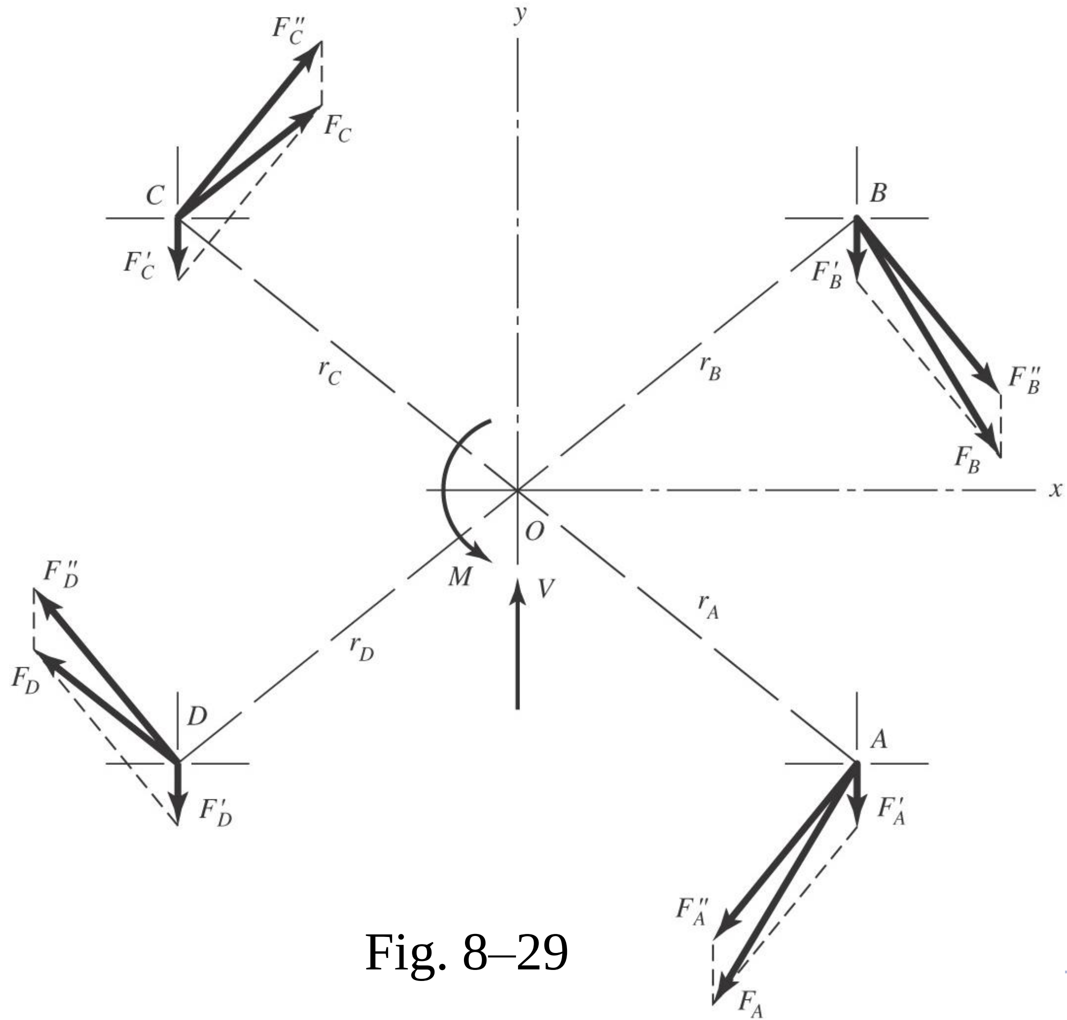


Fig. 8–29

Example 8-7

The primary shear load per bolt is

$$F' = \frac{V}{n} = \frac{16}{4} = 4 \text{ kN}$$

Since the secondary shear forces are equal, Eq. (8–57) becomes

$$F'' = \frac{Mr}{4r^2} = \frac{M}{4r} = \frac{6800}{4(96.0)} = 17.7 \text{ kN}$$

The primary and secondary shear forces are plotted to scale in Fig. 8–29 and the resultants obtained by using the parallelogram rule. The magnitudes are found by measurement (or analysis) to be

$$F_A = F_B = 21.0 \text{ kN}$$

$$F_C = F_D = 14.8 \text{ kN}$$

Example 8-7

(b) Bolts A and B are critical because they carry the largest shear load. Does this shear act on the threaded portion of the bolt, or on the unthreaded portion? The bolt length will be 25 mm plus the height of the nut plus about 2 mm for a washer. Table A–31 gives the nut height as 14.8 mm. Including two threads beyond the nut, this adds up to a length of 43.8 mm, and so a bolt 46 mm long will be needed. From Eq. (8–14) we compute the thread length as $L_T = 38$ mm. Thus the unthreaded portion of the bolt is $46 - 38 = 8$ mm long. This is less than the 15 mm for the plate in Fig. 8–28, and so the bolt will tend to shear across its minor diameter. Therefore the shear-stress area is $A_s = 144\ \text{mm}^2$, and so the shear stress is

$$\tau = \frac{F}{A_s} = -\frac{21.0(10)^3}{144} = 146\ \text{MPa}$$

Example 8-7

(c) The channel is thinner than the bar, and so the largest bearing stress is due to the pressing of the bolt against the channel web. The bearing area is $A_b = td = 10(16) = 160\ \text{mm}^2$. Thus the bearing stress is

$$\sigma = -\frac{F}{A_b} = -\frac{21.0(10)^3}{160} = -131\ \text{MPa}$$

Example 8-7

(*d*) The critical bending stress in the bar is assumed to occur in a section parallel to the y axis and through bolts A and B. At this section the bending moment is

$$M = 16(300 + 50) = 5600 \text{ N} \cdot \text{m}$$

The second moment of area through this section is obtained by the use of the transfer formula, as follows:

$$\begin{aligned} I &= I_{\text{bar}} - 2(I_{\text{holes}} + \bar{d}^2 A) \\ &= \frac{15(200)^3}{12} - 2\left[\frac{15(16)^3}{12} + (60)^2(15)(16)\right] = 8.26(10)^6 \text{ mm}^4 \end{aligned}$$

Then

$$\sigma = \frac{Mc}{I} = \frac{5600(100)}{8.26(10)^6}(10)^3 = 67.8 \text{ MPa}$$