Chapter Outline

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Nomenclature of a Ball Bearing

Fig. 11–1
Types of Ball Bearings

(a) Deep groove
(b) Filling notch
(c) Angular contact
(d) Shielded
(e) Sealed

(f) External self-aligning
(g) Double row
(h) Self-aligning
(i) Thrust
(j) Self-aligning thrust

Fig. 11–2
Types of Roller Bearings

(a) Straight Cylindrical
(b) Spherical Roller, thrust
(c) Tapered roller, thrust

(d) Needle
(e) Tapered roller
(f) Steep-angle tapered roller

Fig. 11–3
Bearing Life Definitions

- **Bearing Failure**: Spalling or pitting of an area of 0.01 in$^2$
- **Life**: Number of revolutions (or hours @ given speed) required for failure.
  - For one bearing
- **Rating Life**: Life required for 10% of sample to fail.
  - For a group of bearings
  - Also called *Minimum Life* or $L_{10}$ Life
- **Median Life**: Average life required for 50% of sample to fail.
  - For many groups of bearings
  - Also called *Average Life* or *Average Median Life*
  - *Median Life* is typically 4 or 5 times the $L_{10}$ Life
Load Rating Definitions

- **Catalog Load Rating**, \( C_{10} \): Constant radial load that causes 10% of a group of bearings to fail at the bearing manufacturer’s rating life.
  - Depends on type, geometry, accuracy of fabrication, and material of bearing
  - Also called Basic Dynamic Load Rating, and Basic Dynamic Capacity

- **Basic Load Rating**, \( C \): A catalog load rating based on a rating life of \( 10^6 \) revolutions of the inner ring.
  - The radial load that would be necessary to cause failure at such a low life is unrealistically high.
  - The Basic Load Rating is a reference value, not an actual load.
Load Rating Definitions

- **Static Load Rating, \( C_o \):**
  Static radial load which corresponds to a permanent deformation of rolling element and race at the most heavily stressed contact of 0.0001\(d\).
  - \( d = \) diameter of roller
  - Used to check for permanent deformation
  - Used in combining radial and thrust loads into an equivalent radial load

- **Equivalent Radial Load, \( F_e \):**
  Constant stationary load applied to bearing with rotating inner ring which gives the same life as actual load and rotation conditions.
Load-Life Relationship

- Nominally identical groups of bearings are tested to the life-failure criterion at different loads.
- A plot of load vs. life on log-log scale is approximately linear.
- Using a regression equation to represent the line,

\[ FL^{1/a} = \text{constant} \quad (11-1) \]

- \( a = 3 \) for ball bearings
- \( a = 10/3 \) for roller bearings (cylindrical and tapered roller)

Fig. 11–4
Load-Life Relationship

- Applying Eq. (11–1) to two load-life conditions,
  \[ F_1 L_1^{1/a} = F_2 L_2^{1/a} \]  \hspace{1cm} (11–2)

- Denoting condition 1 with \( R \) for catalog rating conditions, and
  condition 2 with \( D \) for the desired design conditions,
  \[ F_R L_R^{1/a} = F_D L_D^{1/a} \]  \hspace{1cm} (a)

- The units of \( L \) are revolutions. If life \( L \) is given in hours at a given
  speed \( n \) in rev/min, applying a conversion of 60 min/h,
  \[ L = 60 \, \text{Ln} \]  \hspace{1cm} (b)

- Solving Eq. (a) for \( F_R \), which is just another notation for the
  catalog load rating,
  \[ C_{10} = F_R = F_D \left( \frac{L_D}{L_R} \right)^{1/a} = F_D \left( \frac{L_D n_D 60}{L_R n_R 60} \right)^{1/a} \]  \hspace{1cm} (11–3)
Load-Life Relationship

\[ C_{10} = F_R = F_D \left( \frac{L_D}{L_R} \right)^{1/a} = F_D \left( \frac{\mathcal{L}_D n_D 60}{\mathcal{L}_R n_R 60} \right)^{1/a} \] (11–3)

- The desired design load \( F_D \) and life \( L_D \) come from the problem statement.
- The rated life \( L_R \) will be stated by the specific bearing manufacturer. Many catalogs rate at \( L_R = 10^6 \) revolutions.
- The catalog load rating \( C_{10} \) is used to find a suitable bearing in the catalog.
Load-Life Relationship

- It is often convenient to define a dimensionless *multiple of rating life*

\[ x_D = \frac{L_D}{L_R} \]
Reliability vs. Life

- At constant load, the life measure distribution is right skewed.
- The Weibull distribution is a good candidate.
- Defining the life measure in dimensionless form as \( x = \frac{L}{L_{10}} \), the reliability is expressed with a Weibull distribution as

\[
R = \exp \left[ - \left( \frac{x - x_0}{\theta - x_0} \right)^b \right]
\]

(11-4)

where

\( R = \) reliability

\( x = \) life measure dimensionless variate, \( \frac{L}{L_{10}} \)

\( x_0 = \) guaranteed, or “minimum,” value of the variate

\( \theta = \) characteristic parameter corresponding to the 63.2121 percentile value of the variate

\( b = \) shape parameter that controls the skewness
Reliability vs. Life

- An explicit expression for the cumulative distribution function is

\[ F = 1 - R = 1 - \exp \left[ - \left( \frac{x - x_0}{\theta - x_0} \right)^b \right] \]  

(11-5)
Example 11–2

Construct the distributional properties of a 02-30 millimeter deep-groove ball bearing if the Weibull parameters are $x_0 = 0.02$, $(\theta - x_0) = 4.439$, and $b = 1.483$. Find the mean, median, 10th percentile life, standard deviation, and coefficient of variation.

Solution

From Eq. (20–28), p. 991, the mean dimensionless life $\mu_x$ is

$$\mu_x = x_0 + (\theta - x_0) \Gamma \left( 1 + \frac{1}{b} \right) = 0.02 + 4.439 \Gamma \left( 1 + \frac{1}{1.483} \right) = 4.033$$

The median dimensionless life is, from Eq. (20–26) where $R = 0.5$,

$$x_{0.50} = x_0 + (\theta - x_0) \left( \ln \frac{1}{R} \right)^{1/b} = 0.02 + 4.439 \left( \ln \frac{1}{0.5} \right)^{1/1.483}$$

$$= 3.487$$
Example 11–2

The 10th percentile value of the dimensionless life $x$ is

$$x_{0.10} = 0.02 + 4.439 \left( \ln \frac{1}{0.90} \right)^{1/1.483} \doteq 1 \quad \text{(as it should be)}$$

The standard deviation of the dimensionless life is given by Eq. (20–29):

$$\hat{\sigma}_x = (\theta - x_0) \left[ \Gamma \left(1 + \frac{2}{b}\right) - \Gamma^2 \left(1 + \frac{1}{b}\right) \right]^{1/2}$$

$$= 4.439 \left[ \Gamma \left(1 + \frac{2}{1.483}\right) - \Gamma^2 \left(1 + \frac{1}{1.483}\right) \right]^{1/2} = 2.753$$

The coefficient of variation of the dimensionless life is

$$C_x = \frac{\hat{\sigma}_x}{\mu_x} = \frac{2.753}{4.033} = 0.683$$
Relating Load, Life, and Reliability

- Catalog information is at point $A$, at coordinates $C_{10}$ and $x_{10} = L_{10}/L_{10} = 1$, on the 0.90 reliability contour.
- The design information is at point $D$, at coordinates $F_D$ and $x_D$, on the $R = R_D$ reliability contour.
- The designer must move from point $D$ to point $A$ via point $B$.

Fig. 11–5
Relating Load, Life, and Reliability

- Along a constant reliability contour ($BD$), Eq. (11–2) applies:

\[
F_B x_B^{1/a} = F_D x_D^{1/a}
\]

\[
F_B = F_D \left( \frac{x_D}{x_B} \right)^{1/a}
\]

(a)
Relating Load, Life, and Reliability

- Along a constant load line (AB), Eq. (11–4) applies:

\[
R_D = \exp \left[ - \left( \frac{x_B - x_0}{\theta - x_0} \right)^b \right]
\]

- Solving for \(x_B\),

\[
x_B = x_0 + (\theta - x_0) \left( \ln \frac{1}{R_D} \right)^{1/b}
\]
Substituting $x_B$ into Eq. (a),

$$F_B = F_D \left( \frac{x_D}{x_B} \right)^{1/a} = F_D \left[ \frac{x_D}{x_0 + (\theta - x_0)(\ln 1/R_D)^{1/b}} \right]^{1/a}$$

Noting that $F_B = C_{10}$, and including an application factor $a_f$,

$$C_{10} = a_f F_D \left[ \frac{x_D}{x_0 + (\theta - x_0)(\ln 1/R_D)^{1/b}} \right]^{1/a} \quad (11-6)$$

Note that when $R_D = 0.90$, the denominator equals one and the equation reduces to Eq. (11–3).
The Weibull parameters \( x_0, \theta, \) and \( b \) are usually provided by the catalog.

Typical values of Weibull parameters are given on p. 608 at the beginning of the end-of-chapter problems, and shown below.

Manufacturer 1 parameters are common for tapered roller bearings

Manufacturer 2 parameters are common for ball and straight roller bearings

<table>
<thead>
<tr>
<th>Manufacturer</th>
<th>Rating Life, Revolutions</th>
<th>Weibull Parameters</th>
<th>Rating Lives</th>
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<tr>
<td>2</td>
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<td>4.459</td>
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</table>
Relating Load, Life, and Reliability

\[ C_{10} = a_f F_D \left[ \frac{x_D}{x_0 + (\theta - x_0)(\ln 1/R_D)^{1/b}} \right]^{1/a} \quad (11-6) \]

- Eq. (11–6) can be simplified slightly for calculator entry. Note that

\[ \ln \frac{1}{R_D} = \ln \frac{1}{1 - p_f} = \ln(1 + p_f + \cdots) = p_f = 1 - R_D \]

where \( p_f \) is the probability for failure

- Thus Eq. (11–6) can be approximated by

\[ C_{10} = a_f F_D \left[ \frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a} \quad R \geq 0.90 \quad (11-7) \]
Combined Reliability of Multiple Bearings

- If the combined reliability of multiple bearings on a shaft, or in a gearbox, is desired, then the total reliability is equal to the product of the individual reliabilities.

  - For two bearings on a shaft, \( R = R_A R_B \)

- If the bearings are to be identical, each bearing should have a reliability equal to the square root of the total desired reliability.

- If the bearings are not identical, their reliabilities need not be identical, so long as the total reliability is realized.
**Dimension-Series Code**

- ABMA standardized *dimension-series code* represents the relative size of the boundary dimensions of the bearing cross section for metric bearings.

- Two digit series number
- First digit designates the width series
- Second digit designates the diameter series
- Specific dimensions are tabulated in catalogs under a specific series

![Diagram showing ABMA standardized dimension-series code for metric bearings](Fig. 11–7)
### Representative Catalog Data for Ball Bearings (Table 11–2)

Dimensions and Load Ratings for Single-Row 02-Series Deep-Groove and Angular-Contact Ball Bearings

<table>
<thead>
<tr>
<th>Bore, mm</th>
<th>OD, mm</th>
<th>Width, mm</th>
<th>Fillet Radius, mm</th>
<th>Shoulder Diameter, mm</th>
<th>Load Ratings, kN</th>
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### Representative Catalog Data for Cylindrical Roller Bearings  
(Table 11–3)

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<th>Load Rating, kN</th>
<th>OD, mm</th>
<th>Width, mm</th>
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Combined Radial and Thrust Loading

- When ball bearings carry both an axial thrust load $F_a$ and a radial load $F_r$, an equivalent radial load $F_e$ that does the same damage is used.
- A plot of $F_e/V F_r$ vs. $F_a/V F_r$ is obtained experimentally.
- $V$ is a rotation factor to account for the difference in ball rotations for outer ring rotation vs. inner ring rotation.
  - $V = 1$ for inner ring rotation
  - $V = 1.2$ for outer ring rotation

Fig. 11–6
Combined Radial and Thrust Loading

- The data can be approximated by two straight lines

\[
\frac{F_e}{V F_r} = 1 \quad \text{when} \quad \frac{F_a}{V F_r} \leq e
\]

\[
\frac{F_e}{V F_r} = X + Y \frac{F_a}{V F_r} \quad \text{when} \quad \frac{F_a}{V F_r} > e
\]

- \(X\) is the ordinate intercept and \(Y\) is the slope

- Basically indicates that \(F_e\) equals \(F_r\) for smaller ratios of \(F_a/F_r\), then begins to rise when \(F_a/F_r\) exceeds some amount \(e\)

Fig. 11–6
Combined Radial and Thrust Loading

- It is common to express the two equations as a single equation

\[ F_e = X_i V F_r + Y_i F_a \quad (11-9) \]

where

- \( i = 1 \) when \( F_a / V F_r \leq e \)
- \( i = 2 \) when \( F_a / V F_r > e \)

- \( X \) and \( Y \) factors depend on geometry and construction of the specific bearing.

**Fig. 11–6**
Equivalent Radial Load Factors for Ball Bearings

\[ F_e = X_i V F_r + Y_i F_a \]  \hspace{1cm} (11–9)

- \( X \) and \( Y \) for specific bearing obtained from bearing catalog.
- Table 11–1 gives representative values in a manner common to many catalogs.

<table>
<thead>
<tr>
<th>( \frac{F_a}{C_0} )</th>
<th>( e )</th>
<th>( \frac{F_a}{(V F_r)} \leq e )</th>
<th>( \frac{F_a}{(V F_r)} &gt; e )</th>
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<td>( Y_1 )</td>
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</tr>
<tr>
<td>0.17</td>
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</tr>
<tr>
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<td>0.38</td>
<td>1.00</td>
<td>0</td>
</tr>
<tr>
<td>0.42</td>
<td>0.42</td>
<td>1.00</td>
<td>0</td>
</tr>
<tr>
<td>0.56</td>
<td>0.44</td>
<td>1.00</td>
<td>0</td>
</tr>
</tbody>
</table>
Equivalent Radial Load Factors for Ball Bearings

\[ F_e = X_i V F_r + Y_i F_a \]  \hfill (11-9)

Table 11–1

<table>
<thead>
<tr>
<th>( F_a/C_0 )</th>
<th>( e )</th>
<th>( F_a/(VF_r) \leq e )</th>
<th>( F_a/(VF_r) &gt; e )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( X_1 )</td>
<td>( Y_1 )</td>
</tr>
<tr>
<td>0.014*</td>
<td>0.19</td>
<td>1.00</td>
<td>0</td>
</tr>
<tr>
<td>0.021</td>
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<tr>
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<tr>
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<td>1.00</td>
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<td>0</td>
</tr>
<tr>
<td>0.28</td>
<td>0.38</td>
<td>1.00</td>
<td>0</td>
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<tr>
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<td>0.42</td>
<td>1.00</td>
<td>0</td>
</tr>
<tr>
<td>0.56</td>
<td>0.44</td>
<td>1.00</td>
<td>0</td>
</tr>
</tbody>
</table>

- \( X \) and \( Y \) are functions of \( e \), which is a function of \( F_a/C_0 \).
- \( C_0 \) is the basic static load rating, which is tabulated in the catalog.
<table>
<thead>
<tr>
<th>Type of Application</th>
<th>Life, kh</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instruments and apparatus for infrequent use</td>
<td>Up to 0.5</td>
</tr>
<tr>
<td>Aircraft engines</td>
<td>0.5–2</td>
</tr>
<tr>
<td>Machines for short or intermittent operation where service interruption is of minor</td>
<td>4–8</td>
</tr>
<tr>
<td>importance</td>
<td></td>
</tr>
<tr>
<td>Machines for intermittent service where reliable operation is of great importance</td>
<td>8–14</td>
</tr>
<tr>
<td>Machines for 8-h service that are not always fully utilized</td>
<td>14–20</td>
</tr>
<tr>
<td>Machines for 8-h service that are fully utilized</td>
<td>20–30</td>
</tr>
<tr>
<td>Machines for continuous 24-h service</td>
<td>50–60</td>
</tr>
<tr>
<td>Machines for continuous 24-h service where reliability is of extreme importance</td>
<td>100–200</td>
</tr>
</tbody>
</table>
## Recommended Load Application Factors (Table 11–5)

<table>
<thead>
<tr>
<th>Type of Application</th>
<th>Load Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precision gearing</td>
<td>1.0–1.1</td>
</tr>
<tr>
<td>Commercial gearing</td>
<td>1.1–1.3</td>
</tr>
<tr>
<td>Applications with poor bearing seals</td>
<td>1.2</td>
</tr>
<tr>
<td>Machinery with no impact</td>
<td>1.0–1.2</td>
</tr>
<tr>
<td>Machinery with light impact</td>
<td>1.2–1.5</td>
</tr>
<tr>
<td>Machinery with moderate impact</td>
<td>1.5–3.0</td>
</tr>
</tbody>
</table>
Variable Loading

\[ F^a L = \text{constant} = K \]  \( (a) \)

**Figure 11-9**

Plot of \( F^a \) as ordinate and \( L \) as abscissa for \( F^a L = \text{constant} \). The linear damage hypothesis says that in the case of load \( F_1 \), the area under the curve from \( L = 0 \) to \( L = L_A \) is a measure of the damage \( D = F_1^a L_A \). The complete damage to failure is measured by \( C_{10}^a L_B \).
Variable Loading with Piecewise Constant Loading

Figure 11-10

A three-part piecewise-continuous periodic loading cycle involving loads $F_{e1}$, $F_{e2}$, and $F_{e3}$. $F_{eq}$ is the equivalent steady load inflicting the same damage when run for $l_1 + l_2 + l_3$ revolutions, doing the same damage $D$ per period.

$$D = F_{e1}^a l_1 + F_{e2}^a l_2 + F_{e3}^a l_3$$

$$D = F_{eq}^a (l_1 + l_2 + l_3)$$
Variable Loading with Piecewise Constant Loading

\[ D = F_{e1}^{a} l_1 + F_{e2}^{a} l_2 + F_{e3}^{a} l_3 \]  

\[ D = F_{eq}^{a} (l_1 + l_2 + l_3) \]  

\[ F_{eq} = \left[ \frac{F_{e1}^{a} l_1 + F_{e2}^{a} l_2 + F_{e3}^{a} l_3}{l_1 + l_2 + l_3} \right]^{1/a} = \left[ \sum f_i F_{ei}^{a} \right]^{1/a} \]  

\[ F_{eq} = \left[ \frac{\sum n_i t_i F_{ei}^{a}}{\sum n_i t_i} \right]^{1/a} \]  

\[ F_{eq} = \left[ \sum f_i (a_f i F_{ei})^{a} \right]^{1/a} \]  

\[ L_{eq} = \frac{K}{F_{eq}^{a}} \]
Variable Loading with Piecewise Constant Loading

\[ F_{eq} L_{eq} = F_{e1} l_1 + F_{e2} l_2 + F_{e3} l_3 \]

\[ K = F_{e1} L_1 = F_{e2} L_2 = F_{e3} L_3 \]

\[ K = F_{e1} l_1 + F_{e2} l_2 + F_{e3} l_3 = \frac{K}{L_1} l_1 + \frac{K}{L_2} l_2 + \frac{K}{L_3} l_3 = K \sum \frac{l_i}{L_i} \]

\[ \sum \frac{l_i}{L_i} = 1 \]  \hspace{1cm} (11-13)
Variable Loading with Periodic Variation

\[ dD = F^a d\theta \]

\[ D = \int dD = \int_0^\phi F^a d\theta = F_{eq}^a \phi \]

\[ F_{eq} = \left( \frac{1}{\phi} \int_0^\phi F^a d\theta \right)^{1/a} \]

\[ L_{eq} = \frac{K}{F_{eq}^a} \]  

(11–14)
Example 11–6

The operation of a particular rotary pump involves a power demand of \( P = \bar{P} + A' \sin \theta \) where \( \bar{P} \) is the average power. The bearings feel the same variation as \( F = \bar{F} + A \sin \theta \). Develop an application factor \( a_f \) for this application of ball bearings.

Solution

From Eq. (11–14), with \( a = 3 \),

\[
F_{eq} = \left( \frac{1}{2\pi} \int_0^{2\pi} F^a d\theta \right)^{1/a} = \left( \frac{1}{2\pi} \int_0^{2\pi} (\bar{F} + A \sin \theta)^3 d\theta \right)^{1/3}
\]

\[
= \left[ \frac{1}{2\pi} \left( \int_0^{2\pi} \bar{F}^3 d\theta + 3 \bar{F}^2 A \int_0^{2\pi} \sin \theta d\theta + 3 \bar{F} A^2 \int_0^{2\pi} \sin^2 \theta d\theta + A^3 \int_0^{2\pi} \sin^3 \theta d\theta \right) \right]^{1/3}
\]

\[
F_{eq} = \left[ \frac{1}{2\pi} (2\pi \bar{F}^3 + 0 + 3\pi \bar{F} A^2 + 0) \right]^{1/3} = \bar{F} \left[ 1 + \frac{3}{2} \left( \frac{A}{\bar{F}} \right)^2 \right]^{1/3}
\]
In terms of $\bar{F}$, the application factor is

$$a_f = \left[ 1 + \frac{3}{2} \left( \frac{A}{\bar{F}} \right)^2 \right]^{1/3}$$

We can present the result in tabular form:

<table>
<thead>
<tr>
<th>$A/\bar{F}$</th>
<th>$a_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0.2</td>
<td>1.02</td>
</tr>
<tr>
<td>0.4</td>
<td>1.07</td>
</tr>
<tr>
<td>0.6</td>
<td>1.15</td>
</tr>
<tr>
<td>0.8</td>
<td>1.25</td>
</tr>
<tr>
<td>1.0</td>
<td>1.36</td>
</tr>
</tbody>
</table>
Tapered Roller Bearings

- Straight roller bearings can carry large radial loads, but no axial load.
- Ball bearings can carry moderate radial loads, and small axial loads.
- Tapered roller bearings rely on roller tipped at an angle to allow them to carry large radial and large axial loads.
- Tapered roller bearings were popularized by the Timken Company.
Tapered Roller Bearings

- Two separable parts
  - Cone assembly
    - Cone (inner ring)
    - Rollers
    - Cage
  - Cup (outer ring)
- Rollers are tapered so virtual apex is on shaft centerline
- Taper allows for pure rolling of angled rollers
- Distance $a$ locates the effective axial location for force analysis

Fig. 11–13
Mounting Directions of Tapered Roller Bearings

- Mount pairs in opposite directions to counter the axial loads
- Can be mounted in *direct mounting* or *indirect mounting* configurations
- For the same effective spread $a_e$, direct mounting requires greater geometric spread $a_g$
- For the same geometric spread $a_g$, direct mounting provides smaller effect spread $a_e$

Fig. 11–14
### Typical Catalog Data (Fig. 11–15)

<table>
<thead>
<tr>
<th>bore (d)</th>
<th>outside diameter (D)</th>
<th>width (T)</th>
<th>rating at 500 rpm for 3000 hours L₁₀</th>
<th>factor (K)</th>
<th>cone eff. load center</th>
<th>part numbers</th>
<th>cup max shaft fillet radius</th>
<th>width</th>
<th>backing shoulder diameters</th>
<th>cup max housing fillet radius</th>
<th>width</th>
<th>backing shoulder diameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.000</td>
<td>52.000</td>
<td>16.250</td>
<td>8190 1840</td>
<td>1.56</td>
<td>–3.6 –0.14</td>
<td>♦30205</td>
<td>1.0 0.04</td>
<td>15.000 0.5906</td>
<td>30.5 1.20</td>
<td>29.0 1.14</td>
<td>1.0 0.04</td>
<td>13.000 0.5118</td>
</tr>
<tr>
<td>25.000</td>
<td>52.000</td>
<td>19.250</td>
<td>9520 2140</td>
<td>1.00</td>
<td>–3.0 –0.12</td>
<td>♦32205-B</td>
<td>3.0 0.5966</td>
<td>15.000 0.5118</td>
<td>30.5 1.20</td>
<td>29.0 1.14</td>
<td>1.0 0.04</td>
<td>13.000 0.5118</td>
</tr>
<tr>
<td>25.000</td>
<td>62.000</td>
<td>22.000</td>
<td>13200 2980</td>
<td>1.66</td>
<td>–7.6 –0.30</td>
<td>♦33205</td>
<td>3.0 0.5966</td>
<td>15.000 0.5118</td>
<td>30.5 1.20</td>
<td>29.0 1.14</td>
<td>1.0 0.04</td>
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<tr>
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<td>25.250</td>
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<td>–9.7 –0.38</td>
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<td>15.000 0.5118</td>
<td>30.5 1.20</td>
<td>29.0 1.14</td>
<td>1.0 0.04</td>
<td>13.000 0.5118</td>
</tr>
</tbody>
</table>

---

**SINGLE-ROW STRAIGHT BORE**

![Diagram of single-row straight bore](image-url)

- **Parameters:**
  - D: Outside diameter
  - d: Bore diameter
  - T: Width
  - L₁₀: Rating at 500 rpm for 3000 hours
  - K: Factor
  - R: Max shaft fillet radius
  - B: Width
  - d_b, d_a: Backing shoulder diameters
  - C: Housing fillet radius
  - D_b, D_a: Backing shoulder diameters
## Typical Catalog Data (Fig. 11–15 continued)

<table>
<thead>
<tr>
<th>d (in)</th>
<th>D (in)</th>
<th>T (in)</th>
<th>N (lbf) @ 500 rpm</th>
<th>N (lbf)</th>
<th>K</th>
<th>a</th>
<th>R (^\circ)</th>
<th>B</th>
<th>d_b</th>
<th>d_a</th>
<th>r (^\circ)</th>
<th>C (in)</th>
<th>D_b (in)</th>
<th>D_a (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25.400</td>
<td>62.000</td>
<td>19.050</td>
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<td>7280</td>
<td>1.67</td>
<td>-5.8</td>
<td>15102</td>
<td>15245</td>
<td>1.5</td>
<td>20.638</td>
<td>34.0</td>
<td>31.5</td>
<td>1.3</td>
<td>14.288</td>
</tr>
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<td>62.000</td>
<td>20.638</td>
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<td>7280</td>
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<td>-5.8</td>
<td>15101</td>
<td>15101</td>
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<td>13500</td>
<td>1.07</td>
<td>-3.3</td>
<td>M86643</td>
<td>23100</td>
<td>15</td>
<td>23100</td>
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<td>1</td>
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<td>14500</td>
<td>13500</td>
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<td>M86643</td>
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<td>23100</td>
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<td>26.162</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**SINGLE-ROW STRAIGHT BORE**

![Diagram of single-row straight bore](image)
Induced Thrust Load

- A radial load induces a thrust reaction due to the roller angle.
  \[ F_i = \frac{0.47 F_r}{K} \]  \hfill (11–15)

- \( K \) is ratio of radial load rating to thrust load rating
- \( K \) is dependent on specific bearing, and is tabulated in catalog

![Diagram of induced thrust load](Shigley's Mechanical Engineering Design)
Equivalent Radial Load

- The equivalent radial load for tapered roller bearings is found in similar form as before,

\[ F_e = X V F_r + Y F_a \]

- Timken recommends \( X = 0.4 \) and \( Y = K \)

\[ F_e = 0.4F_r + K F_a \]

- \( F_a \) is the net axial load carried by the bearing, including induced thrust load from the other bearing and the external axial load carried by the bearing.

- Only one of the bearings will carry the external axial load
Determining Which Bearing Carries External Axial Load

- Regardless of mounting direction or shaft orientation, visually inspect to determine which bearing is being “squeezed”
- Label this bearing as *Bearing A*

**Fig. 11–17**

Shigley’s Mechanical Engineering Design
Net Axial Load

- Generally, *Bearing A* (the squeezed bearing) carries the net axial load.
- Occasionally the induced thrust from *Bearing A*, \( F_{iA} \), is greater than the combination of the induced thrust from *Bearing B*, \( F_{iB} \), and the external axial load \( F_{ae} \), that is
  \[
  F_{iA} > (F_{iB} + F_{ae})
  \]
- If this happens, then *Bearing B* actually carries the net axial load.
Equivalent Radial Load

• Timken recommends using the full radial load for the bearing that is not carrying the net axial load.

• Equivalent radial load equation:

\[
\begin{align*}
F_{iA} &\leq (F_{iB} + F_{ae}) \\
\begin{cases} 
F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) \\
F_{eB} = F_{rB}
\end{cases} \\
F_{eB} = F_{rB} \\
\end{align*}
\]

\[
\begin{align*}
F_{iA} &> (F_{iB} + F_{ae}) \\
\begin{cases} 
F_{eB} = 0.4F_{rB} + K_B(F_{iA} - F_{ae}) \\
F_{eA} = F_{rA}
\end{cases} \\
\end{align*}
\]

• If the equivalent radial load is less than the original radial load, then use the original radial load.
Example 11–8

The shaft depicted in Fig. 11–18a carries a helical gear with a tangential force of 3980 N, a radial force of 1770 N, and a thrust force of 1690 N at the pitch cylinder with directions shown. The pitch diameter of the gear is 200 mm. The shaft runs at a speed of 800 rev/min, and the span (effective spread) between the direct-mount bearings is 150 mm. The design life is to be 5000 h and an application factor of 1 is appropriate. If the reliability of the bearing set is to be 0.99, select suitable single-row tapered-roller Timken bearings.
Example 11–8

The reactions in the $xz$ plane from Fig. 11–18b are

\[ R_{zA} = \frac{3980(50)}{150} = 1327 \text{ N} \]

\[ R_{zB} = \frac{3980(100)}{150} = 2653 \text{ N} \]

The reactions in the $xy$ plane from Fig. 11–18c are

\[ R_{yA} = \frac{1770(50)}{150} + \frac{169 \ 000}{150} = 1716.7 = 1717 \text{ N} \]

\[ R_{yB} = \frac{1770(100)}{150} - \frac{169 \ 000}{150} = 53.3 \text{ N} \]

Fig. 11–18
Example 11–8

The radial loads $F_{rA}$ and $F_{rB}$ are the vector additions of $R_{yA}$ and $R_{zA}$, and $R_{yB}$ and $R_{zB}$, respectively:

$$F_{rA} = \left( R_{zA}^2 + R_{yA}^2 \right)^{1/2} = (1327^2 + 1717^2)^{1/2} = 2170 \text{ N}$$

$$F_{rB} = \left( R_{zB}^2 + R_{yB}^2 \right)^{1/2} = (2653^2 + 53.3^2)^{1/2} = 2654 \text{ N}$$

**Trial 1:** With direct mounting of the bearings and application of the external thrust to the shaft, the squeezed bearing is bearing $A$ as labeled in Fig. 11–18a. Using $K$ of 1.5 as the initial guess for each bearing, the induced loads from the bearings are

$$F_{iA} = \frac{0.47 F_{rA}}{K_A} = \frac{0.47(2170)}{1.5} = 680 \text{ N}$$

$$F_{iB} = \frac{0.47 F_{rB}}{K_B} = \frac{0.47(2654)}{1.5} = 832 \text{ N}$$
Example 11–8

Since \( F_iA \) is clearly less than \( F_iB + F_{ae} \), bearing A carries the net thrust load, and Eq. (11–16) is applicable. Therefore, the dynamic equivalent loads are

\[
F_{eA} = 0.4F_{rA} + K_A(F_iB + F_{ae}) = 0.4(2170) + 1.5(832 + 1690) = 4651 \, \text{N}
\]

\[
F_{eB} = F_{rB} = 2654 \, \text{N}
\]

The multiple of rating life is

\[
x_D = \frac{L_D}{L_R} = \frac{L_Dn_D60}{L_R} = \frac{(5000)(800)(60)}{90(10^6)} = 2.67
\]

Estimate \( R_D \) as \( \sqrt{0.99} = 0.995 \) for each bearing. For bearing A, from Eq. (11–7) the catalog entry \( C_{10} \) should equal or exceed

\[
C_{10} = (1)(4651) \left[ \frac{2.67}{(4.48)(1 - 0.995)^{2/3}} \right]^{3/10} = 11\,486 \, \text{N}
\]

From Fig. 11–15, tentatively select type TS 15100 cone and 15245 cup, which will work: \( K_A = 1.67, C_{10} = 12\,100 \, \text{N} \).
Example 11–8

For bearing $B$, from Eq. (11–7), the catalog entry $C_{10}$ should equal or exceed

$$
C_{10} = (1)2654 \left[ \frac{2.67}{(4.48)(1 - 0.995)^{2/3}} \right]^{3/10} = 6554 \text{ N}
$$

Tentatively select the bearing identical to bearing $A$, which will work: $K_B = 1.67$, $C_{10} = 12\ 100 \text{ N}$. 
Trial 2: Repeat the process with $K_A = K_B = 1.67$ from tentative bearing selection.

$$F_{iA} = \frac{0.47F_{rA}}{K_A} = \frac{0.47(2170)}{1.67} = 611 \text{ N}$$

$$F_{iB} = \frac{0.47F_{rB}}{K_B} = \frac{0.47(2654)}{1.67} = 747 \text{ N}$$

Since $F_{iA}$ is still less than $F_{iB} + F_{ae}$, Eq. (11–16) is still applicable.

$$F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) = 0.4(2170) + 1.67(747 + 1690) = 4938 \text{ N}$$

$$F_{eB} = F_{rB} = 2654 \text{ N}$$

For bearing $A$, from Eq. (11–7) the corrected catalog entry $C_{10}$ should equal or exceed

$$C_{10} = (1)(4938) \left[ \frac{2.67}{(4.48)(1 - 0.995)^{2/3}} \right]^{3/10} = 12195 \text{ N}$$
Example 11–8

Although this catalog entry exceeds slightly the tentative selection for bearing A, we will keep it since the reliability of bearing B exceeds 0.995. In the next section we will quantitatively show that the combined reliability of bearing A and B will exceed the reliability goal of 0.99.

For bearing B, \( F_{eB} = F_{rB} = 2654 \text{ N} \). From Eq. (11–7),

\[
C_{10} = (1)2654 \left[ \frac{2.67}{(4.48)(1 - 0.995)^{2/3}} \right]^{3/10} = 6554 \text{ N}
\]

Select cone and cup 15100 and 15245, respectively, for both bearing A and B. Note from Fig. 11–14 the effective load center is located at \( a = -5.8 \text{ mm} \), that is, 5.8 mm into the cup from the back. Thus the shoulder-to-shoulder dimension should be \( 150 - 2(5.8) = 138.4 \text{ mm} \). Note that in each iteration of Eq. (11–7) to find the catalog load rating, the bracketed portion of the equation is identical and need not be re-entered on a calculator each time.
Realized Bearing Reliability

- Eq. (11–6) was previously derived to determine a suitable catalog rated load for a given design situation and reliability goal.

\[
C_{10} = a_f F_D \left[ \frac{x_D}{x_0 + (\theta - x_0)(\ln 1/R_D)^{1/b}} \right]^{1/a}
\]  

- An actual bearing is selected from a catalog with a rating greater than \( C_{10} \).
- Sometimes it is desirable to determine the realized reliability from the actual bearing (that was slightly higher capacity than needed).
- Solving Eq. (11–6) for the reliability,

\[
R = \exp \left( - \left\{ \frac{x_D \left( \frac{a_f F_D}{C_{10}} \right)^a - x_0}{\theta - x_0} \right\}^b \right)
\]  

(11–18)
Realized Bearing Reliability

Similarly for the alternate approximate equation, Eq. (11–7),

\[ C_{10} = a_f F_D \left[ \frac{x_D}{x_0 + (\theta - x_0)(1 - R_D)^{1/b}} \right]^{1/a} \quad R \geq 0.90 \]  \hspace{1cm} (11–7)

\[ R = 1 - \left[ \frac{x_D \left( \frac{a_f F_D}{C_{10}} \right)^a - x_0}{\theta - x_0} \right]^b \quad R \geq 0.90 \]  \hspace{1cm} (11–19)
Realized Reliability for Tapered Roller Bearings

Substituting typical Weibull parameters for tapered roller bearings into Eqs. (11–18) and (11–19) give realized reliability equations customized for tapered roller bearings.

\[
R = \exp \left\{ - \left[ \frac{x_D}{\theta[C_{10}/(a_F F_D)]^a} \right]^b \right\}
\]

\[
= \exp \left\{ - \left[ \frac{x_D}{4.48[C_{10}/(a_F F_D)]^{10/3}} \right]^{3/2} \right\}
\] (11–20)

\[
R = 1 - \left\{ \frac{x_D}{\theta[C_{10}/(a_F F_D)]^a} \right\}^b = 1 - \left\{ \frac{x_D}{4.48[C_{10}/(a_F F_D)]^{10/3}} \right\}^{3/2}
\] (11–21)
Example 11–10

In Ex. 11–8 bearings $A$ and $B$ (cone 15100 and cup 15245) have $C_{10} = 12100$ N. What is the reliability of the pair of bearings $A$ and $B$?

Solution

The desired life $x_D$ was $5000(800)60/[90(10^6)] = 2.67$ rating lives. Using Eq. (11–21) for bearing $A$, where from Ex. 11–8, $F_D = F_{e_A} = 4938$ N, and $a_f = 1$, gives

$$R_A = 1 - \left\{ \frac{2.67}{4.48 \left[ \frac{12100}{(1 \times 4938)} \right]^{10/3}} \right\}^{3/2} = 0.994791$$

which is less than 0.995, as expected. Using Eq. (11–21) for bearing $B$ with $F_D = F_{e_B} = 2654$ N gives

$$R_B = 1 - \left\{ \frac{2.67}{4.48 \left[ \frac{12100}{(1 \times 2654)} \right]^{10/3}} \right\}^{3/2} = 0.999766$$
Example 11–10

The reliability of the bearing pair is

\[ R = R_A R_B = 0.994791(0.999766) = 0.994558 \]

which is greater than the overall reliability goal of 0.99. When two bearings are made identical for simplicity, or reducing the number of spares, or other stipulation, and the loading is not the same, both can be made smaller and still meet a reliability goal. If the loading is disparate, then the more heavily loaded bearing can be chosen for a reliability goal just slightly larger than the overall goal.
Example 11–11

Consider a constrained housing as depicted in Fig. 11–19 with two direct-mount tapered roller bearings resisting an external thrust $F_{ae}$ of 8000 N. The shaft speed is 950 rev/min, the desired life is 10,000 h, the expected shaft diameter is approximately 1 in. The reliability goal is 0.95. The application factor is appropriately $a_f = 1$.

(a) Choose a suitable tapered roller bearing for A.
(b) Choose a suitable tapered roller bearing for B.
(c) Find the reliabilities $R_A$, $R_B$, and $R$. 

Fig. 11–19
(a) By inspection, note that the left bearing carries the axial load and is properly labeled as bearing A. The bearing reactions at A are

\[ F_{rA} = F_{rB} = 0 \]
\[ F_{aA} = F_{ae} = 8000 \text{ N} \]

Since bearing B is unloaded, we will start with \( R = R_A = 0.95 \).

With no radial loads, there are no induced thrust loads. Eq. (11–16) is applicable.

\[ F_{eA} = 0.4F_{rA} + K_A(F_{iB} + F_{ae}) = K_A F_{ae} \]

If we set \( K_A = 1 \), we can find \( C_{10} \) in the thrust column and avoid iteration:

\[ F_{eA} = (1)8000 = 8000 \text{ N} \]
\[ F_{eB} = F_{rB} = 0 \]
Example 11–11

The multiple of rating life is

\[ x_D = \frac{L_D}{L_R} = \frac{L_D n_D 60}{L_R} = \frac{(10 \, 000)(950)(60)}{90(10^6)} = 6.333 \]

Then, from Eq. (11–7), for bearing A

\[ C_{10} = a_f F_e A \left[ \frac{x_D}{4.48(1 - R_D)^{2/3}} \right]^{3/10} \]

\[ = (1)8000 \left[ \frac{6.33}{4.48(1 - 0.95)^{2/3}} \right]^{3/10} = 16 159 \text{ N} \]

Figure 11–15 presents one possibility in the 1-in bore (25.4-mm) size: cone, HM88630, cup HM88610 with a thrust rating \((C_{10})_a = 17 200 \text{ N}\).
(b) Bearing B experiences no load, and the cheapest bearing of this bore size will do, including a ball or roller bearing.

(c) The actual reliability of bearing A, from Eq. (11–21), is

\[ R_A = 1 - \left\{ \frac{x_D}{4.48\left[C_{10}/(a_f F_D)\right]^{10/3}} \right\}^{3/2} \]

\[ = 1 - \left\{ \frac{6.333}{4.48 \left[17200/(1 \times 8000)\right]^{10/3}} \right\}^{3/2} = 0.963 \]

which is greater than 0.95, as one would expect. For bearing B,

\[ F_D = F_{eB} = 0 \]

\[ R_B = 1 - \left[ \frac{6.333}{0.85(17200/0)^{10/3}} \right]^{3/2} = 1 - 0 = 1 \]

as one would expect. The combined reliability of bearings A and B as a pair is

\[ R = R_A R_B = 0.953(1) = 0.953 \]

which is greater than the reliability goal of 0.95, as one would expect.
Bearing Lubrication

The purposes of bearing lubrication

- To provide a film of lubricant between the sliding and rolling surfaces
- To help distribute and dissipate heat
- To prevent corrosion of the bearing surfaces
- To protect the parts from the entrance of foreign matter
Bearing Lubrication

- Either oil or grease may be used, with each having advantages in certain situations.

<table>
<thead>
<tr>
<th>Use Grease When</th>
<th>Use Oil When</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The temperature is not over 200°F.</td>
<td>1. Speeds are high.</td>
</tr>
<tr>
<td>2. The speed is low.</td>
<td>2. Temperatures are high.</td>
</tr>
<tr>
<td>3. Unusual protection is required from the entrance</td>
<td>3. Oiltight seals are readily employed.</td>
</tr>
<tr>
<td>of foreign matter.</td>
<td></td>
</tr>
<tr>
<td>4. Simple bearing enclosures are desired.</td>
<td>4. Bearing type is not suitable for grease lubrication.</td>
</tr>
<tr>
<td>5. Operation for long periods without attention is</td>
<td>5. The bearing is lubricated from a central supply</td>
</tr>
<tr>
<td>desired.</td>
<td>which is also used for other machine parts.</td>
</tr>
</tbody>
</table>
Some Common Bearing Mounting Configurations

Fig. 11–20

Fig. 11–21
Some Common Bearing Mounting Configurations

Fig. 11–22
Some Common Bearing Mounting Configurations

Fig. 11–23
Duplexing

- When maximum stiffness and resistance to shaft misalignment is desired, pairs of angular-contact bearings can be used in an arrangement called *duplexing*.
- Duplex bearings have rings ground with an offset.
- When pairs are clamped together, a preload is established.
Duplexing Arrangements

- Three common duplexing arrangements:
  (a) DF mounting – Face to face, good for radial and thrust loads from either direction
  (b) DB mounting – Back to back, same as DF, but with greater alignment stiffness
  (c) DT mounting – Tandem, good for thrust only in one direction

Fig. 11–24
Preferred Fits

- Rotating ring usually requires a press fit
- Stationary ring usually best with a push fit
- Allows stationary ring to creep, bringing new portions into the load-bearing zone to equalize wear
Preloading

- Object of preloading
  - Remove internal clearance
  - Increase fatigue life
  - Decrease shaft slope at bearing

Fig. 11–25
Alignment

- Catalogs will specify alignment requirements for specific bearings.
- Typical maximum ranges for shaft slopes at bearing locations:

<table>
<thead>
<tr>
<th>Bearing Type</th>
<th>Maximum Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tapered roller</td>
<td>0.0005—0.0012 rad</td>
</tr>
<tr>
<td>Cylindrical roller</td>
<td>0.0008—0.0012 rad</td>
</tr>
<tr>
<td>Deep-groove ball</td>
<td>0.001—0.003 rad</td>
</tr>
<tr>
<td>Spherical ball</td>
<td>0.026—0.052 rad</td>
</tr>
<tr>
<td>Self-align ball</td>
<td>0.026—0.052 rad</td>
</tr>
</tbody>
</table>
Enclosures

- Common shaft seals to exclude dirt and retain lubricant

Fig. 11–26