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Abstract Book
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EFFECTIVE LENGTHS OF IRREGULAR FRAME COLUMNS

Gunay OZMEN¹, Konuralp GIRGIN² and Engin ORAKDOGEN³

SUMMARY

In several design codes and specifications, simplified formulae and diagrams are given for determining the effective lengths of frame columns. It is shown that these formulae may yield rather erroneous results especially for irregular frames. This is due to the fact that, the code formulae utilise only local stiffness distributions. In this paper, a simplified procedure for determining approximate values for the buckling loads of both regular and irregular frames is developed. The procedure utilises lateral load analysis of frames and yields errors in the order of 5%, which may be considered suitable for design purposes. The proposed procedure is applied to several numerical examples and it is shown that all the errors are in the acceptable range and on the safe side.

Key words: buckling load, effective length, sway mode, unbraced frames, irregular frames, isolated subassembly, design codes.

1. INTRODUCTION

Determination of the effective lengths of frame columns is one of the significant phases of frame design. Theoretically, effective length of an individual column is determined by calculating the system-buckling load of the frame. Since a full system instability analysis, may be quite involved for frames met in practical applications, simplified formulae and diagrams are given for determining the effective lengths of frame columns in most of the design codes and specifications, [AISC, 1988], [ACI 318-89, 1989], [Eurocode 3, 2002], [DIN 18800, 1990]. The so-called “isolated subassembly approach” used in codes and specifications, was originally developed by [Galambos, 1968]. A major limitation of the isolated subassembly approach is that it does not consider the interaction effects of structural elements other than those in the immediate neighbourhood of the joints. Erroneous results corresponding to this fact have been recognised by several authors and numerous publications have been made to improve the applicability of the subassembly approach. Most of these publications use the so-called “storey-buckling approach” which accounts for the horizontal interaction between the columns in a story of the unbraced frame. Among the papers, which use storey-buckling approach, the publications of [Lui, 1992], [Aristizabal-Ochoa, 1997], [Hellesland and Bjarhovde, 1997] and [Cheong-Siat-
Moy, 1999] may be highlighted. A reasonably comprehensive list of these improvement studies is given by [Ozmen and Girgin, 2005].

Apart from the above mentioned improvement studies, certain independent methods for determining an approximate value for the overall buckling load of plane frames are also developed, whereby the displacements due to a fictitious lateral loading is utilised [Cakiroglu, 1962], [Stevens, 1967] and [Horne, 1975].

In this paper, a practical method, which is applicable to both regular and irregular frames will be explained and applied to numerical examples. The method, which is developed by using the procedure given by [Cakiroglu, 1962] is performed by applying a simple quotient based on the results of a fictitious lateral load analysis.

**2. IRREGULAR FRAMES**

A plane frame may be considered as being regular when all the beams are continuous along the width of the frame at all levels as shown in Fig. 1a.

![Fig. 1: Regular and irregular frames](image)

The frame becomes “Irregular” when the beams of at least one level are curtailed as shown in Fig. 1b. In other words, it is not possible to define a “storey” for certain levels of irregular frames. In practice several frames of this nature exist as shown in the examples in Fig. 2.

![Fig. 2: Irregular frame examples](image)
In case of irregular frames, the error orders of code procedures are far greater, mainly because the isolated subassembly assumptions are hardly satisfied. Moreover, almost all of the improvement studies mentioned above hardly offer any remedy, since most of them use the storey buckling approach and it is not possible to define a storey at certain (or all) levels of an irregular frame. On the other hand, the method presented in this paper offers an approximate but simple solution for both regular and irregular frames.

3. SYSTEM BUCKLING LOAD OF UNBRACED MULTI-STOREY FRAMES

A multi-storey frame which is composed of beams and columns made of linear elastic material is under the effect of vertical loads as shown in Fig. 3a.

![Multi storey frame and buckling mode](image)

Fig. 3: Multi storey frame and buckling mode

The axial load $N_j$ of the $j^{th}$ column may be expressed as

$$N_j = n_j P$$  \(1\)

where $n_j$ is a dimensionless coefficient and $P$ is an arbitrarily chosen load parameter. The frame is in the state of “Stable Equilibrium” and, if the axial deformations are neglected, all the displacements and deformations are zero. Internal forces of the frame columns consist of only axial forces $N_j$ while all the internal forces of beams are zero. However, when the load parameter reaches to a critical $P_{cr}$ value, another state of “Unstable Equilibrium” may exist. The lateral displacement diagram corresponding to this new state, which is shown schematically in Fig. 3b, is called the “Buckling Mode” of the structure, [Horne and Merchant, 1965]. Once the buckling load parameter $P_{cr}$ is determined, the effective length $s_j$ of an individual column can be computed by

$$s_j = \pi \sqrt{\frac{EI_j}{n_j P_{cr}}}$$  \(2\)

where $EI_j$ is the bending stiffness of the $j^{th}$ column.

In certain simple cases, buckling load parameter may be determined by using the so-called stability functions, [Horne and Merchant, 1965]. For general cases however, it is necessary to utilise specially prepared software.

In this study, a practical method is developed for determining the effective lengths of columns in unbraced frames. The method is based on computing an approximate value for system buckling load by using the results of a fictitious lateral loading.
4. EFFECTIVE LENGTHS ACCORDING TO DESIGN CODES

In several design codes and specifications, simplified formulae and diagrams are given for calculating the effective lengths of individual columns. These simple formulae have the advantage of enabling the designer to obtain the effective lengths, without applying the tedious computations (or special software) which are necessary for the calculation of the overall-buckling load. Application of code formulae on several numerical examples have shown that erroneous results may be encountered for both sway and non-sway modes. This is mainly because, only local stiffness distributions are considered in these formulae, while the general behaviour of the frame is not taken into account. Discussion of effective lengths of non-sway frames is left out of the scope of this study for the sake of brevity. The erroneous results encountered for sway mode will presently be demonstrated on several numerical examples.

4.1 Numerical Example

Dimensions and loading of an irregular frame is shown in the schematic elevation in Fig. 4.

The exact value of the buckling load for this frame is found to be

\[
P_{cr} = 2.004 \frac{EI}{h^2}.
\]

The exact values for effective length multipliers \(\beta\) are found by utilising Eq. (2), which are shown in the 4th column of Table 1. Effective length multipliers \(\beta\) according to Eurocode 3 are also shown in the Table 1 together with the corresponding relative errors.

<table>
<thead>
<tr>
<th>Column</th>
<th>n</th>
<th>Length</th>
<th>(\beta) (Exact)</th>
<th>(\beta) (Eurocode 3)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1.00</td>
<td>2h</td>
<td>0.906</td>
<td>1.225</td>
<td>35.2</td>
</tr>
<tr>
<td>C2</td>
<td>2.00</td>
<td>h</td>
<td>1.812</td>
<td>1.985</td>
<td>9.5</td>
</tr>
<tr>
<td>C3</td>
<td>1.00</td>
<td>h</td>
<td>1.404</td>
<td>1.458</td>
<td>3.8</td>
</tr>
<tr>
<td>C4</td>
<td>1.00</td>
<td>h</td>
<td>2.219</td>
<td>1.301</td>
<td>-41.4</td>
</tr>
</tbody>
</table>

Relative errors on \(\beta\) values vary between -41.4% and 35.1%, which may be considered as excessive, i.e. not acceptable for design purposes. In other numerical examples, even larger errors may be encountered as explained below.
4.2 Other examples and other design codes

Similar formulae and diagrams for calculating effective length multipliers are given in other design codes. The calculations for the above example have been carried out using [AISC, 1988] charts and [ACI, 1989] formulae and similar results are obtained. The ranges of errors for the codes under consideration are shown together in Table 2.

Table 2: Relative error ranges for several examples and different codes (%)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Min</td>
<td>Max</td>
<td>Min</td>
</tr>
<tr>
<td>a</td>
<td>-41.4</td>
<td>35.2</td>
<td>-44.9</td>
</tr>
<tr>
<td>b</td>
<td>-48.9</td>
<td>31.2</td>
<td>-40.3</td>
</tr>
<tr>
<td>c</td>
<td>-37.1</td>
<td>38.1</td>
<td>-32.6</td>
</tr>
<tr>
<td>d</td>
<td>-52.5</td>
<td>18.2</td>
<td>-41.5</td>
</tr>
<tr>
<td>e</td>
<td>-57.4</td>
<td>58.5</td>
<td>-47.8</td>
</tr>
<tr>
<td>f</td>
<td>-45.6</td>
<td>43.1</td>
<td>-37.1</td>
</tr>
<tr>
<td>g</td>
<td>-46.8</td>
<td>2.4</td>
<td>-39.2</td>
</tr>
<tr>
<td>h</td>
<td>-60.1</td>
<td>13.9</td>
<td>-55.4</td>
</tr>
</tbody>
</table>

Types a ~ g in the first column of the Table 2, refer to the designations shown in Fig. 2, while Type h represents the irregular frame shown in Fig. 1b. The characteristics of Type a are as shown in Fig. 4 (Numerical Example). Detailed characteristics of other examples are not included for the sake of brevity.

It is clearly seen that all the considered codes yield errors, which are almost of the same order. This is due to the fact that all codes use similar formulae, which consider only the local (isolated) stiffness distributions. It is concluded that, the effective length multipliers should be determined by taking into account all the attributes of the frame i.e. considering not only the local stiffness distributions, but also the overall characteristics of the structure.

5. A SIMPLIFIED PROCEDURE FOR DETERMINING THE BUCKLING LOAD

In the following, a practical method will be explained and applied to the numerical examples. The method, which is developed by using the procedure given by [Cakiroglu, 1962], is applied by using a simple quotient based on the results obtained by standard frame analysis software.

Consider the fictitious lateral loading shown in Fig. 5 applied to the frame shown in Fig.3. It is assumed that this loading provides displacements identical to (or proportional with) those corresponding to the buckling mode.

![Fig. 5: Multi storey frame and fictitious lateral loading](image-url)
The buckling load parameter can be determined by using Betti’s Reciprocal Theorem applied to the states shown in Figs. 3 and 5. According to this theorem, it can be written that

\[ W_1 = W_2 \]  

(4)

where \( W_1 \) is the virtual work of the force system in Fig. 3a in conjunction with the displacements in Fig. 5b, and \( W_2 \) is the virtual work of the force system in Fig. 5a in conjunction with the displacements in Fig. 3b, [Neal, 1964]. Since the displacements of Figs. 3b and 5b are assumed as being the same, the displacements and deformations corresponding to the lateral fictitious loading will be used in the following.

5.1 Determination of \( W_1 \)

According to the Principle of Virtual Works, \( W_1 \) can be computed as the work done by the internal forces of the loading shown in Fig. 3, in conjunction with the deformations induced by the fictitious lateral loading. The displacement diagram of an infinitely small portion of one of the columns together with the internal forces is shown in Fig 6.

If the axial deformations are neglected, the total virtual work can be expressed as

\[ W_1 = P \sum_{\text{Columns}} n_j \left( \frac{dv}{dx} \right)^2 dx . \]  

(5)

Here \( P \) and \( n_j \) are the load parameter and the dimensionless coefficient defined in Eq. (1), respectively. \( h_{ij} \) represents the height of \( j^{\text{th}} \) column. After certain mathematical operations

\[ W_1 = 1.20P \sum_{\text{Columns}} n_j \frac{\delta_j^2}{h_{ij}} , \]  

(6)

is obtained, [Ozmen and Girgin, 2005]. Here \( \delta_j \) represents the relative displacement of the column.

5.2 Determination of \( W_2 \)

The virtual work of the force system in Fig. 5a in conjunction with the displacements in Fig. 3b (Fig. 5b) can simply be written as

\[ W_2 = \sum_{\text{Jo int s}} H_i d_i \]  

(7)

where \( H_i \) and \( d_i \) represent the lateral joint loads and joint displacements at joint \( i \), respectively. Eq. (7) may more conveniently be expressed in terms of column shear force \( Q_j \) and relative column displacement \( \delta_j \) as
\[ W_2 = \sum_{\text{Columns}} Q_j \delta_j. \] (8)

### 5.3 Simplified Buckling Load Formula

Substituting the expressions for \( W_1 \) and \( W_2 \) given respectively by Eqs. (6) and (8) into Eq. (4) and solving for \( P \) (\( P_{cr} \)), the buckling load is obtained as

\[ P_{cr} = \frac{\sum Q \delta}{1.20 \sum \frac{n \delta^2}{h_c}}. \] (9)

Both summations in the above formula will be carried out on columns. Column indices are omitted for the sake of simplicity. It must be noted that, this formula is approximate since the lateral loading corresponding to the buckling load displacements, are not known initially. However, application on several numerical examples has shown that, the value of \( P_{cr} \) is not strongly dependent to the initial choice of lateral loads. It may be recommended that, lateral load at each joint should be selected as proportional to the vertical load \( P_i \) existing at the joint.

### 5.4 Analysis procedure

Effective lengths of frame columns can be determined as follows:

- Apply lateral forces proportional to the vertical loads at each joint,
- Compute relative storey displacements using any existing software,
- Compute the critical load \( P_{cr} \) by using Eq. (9),
- Determine the effective lengths of columns by using Eq. (2).

This procedure is applicable to both regular and irregular frames easily.

6. **NUMERICAL APPLICATIONS**

In the following, the procedure outlined above will be applied to examples and the results will be discussed.

6.1 **Numerical Example**

Dimensions and loading of the numerical example is the same as shown in the schematic elevation in Fig. 4 of Section 4.1. The fictitious lateral loading is shown in Fig. 7, where loads are chosen as being proportional to vertical loads at the joints.

![Fig. 7: Fictitious lateral loading for Numerical Example](image)

Lateral load analysis yields relative displacements \( \delta \) and column shear forces \( Q \). Using these values, the terms used for the application of Eq.(9) are calculated as shown in Table 3.
Table 3: Buckling load calculations for Numerical Example

<table>
<thead>
<tr>
<th>Column</th>
<th>Q</th>
<th>$h_c$</th>
<th>$\frac{EI}{h^3\delta}$</th>
<th>$\frac{EI}{h^3}Q\delta$</th>
<th>n</th>
<th>$\frac{(EI)^2}{h^5}n\frac{\delta^2}{h_c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.974</td>
<td>2.00</td>
<td>0.8821</td>
<td>0.8592</td>
<td>1.50</td>
<td>0.5836</td>
</tr>
<tr>
<td>C2</td>
<td>2.026</td>
<td>1.00</td>
<td>0.5633</td>
<td>1.1400</td>
<td>1.50</td>
<td>0.4760</td>
</tr>
<tr>
<td>C3</td>
<td>1.446</td>
<td>1.00</td>
<td>0.3188</td>
<td>0.4600</td>
<td>2.50</td>
<td>0.2541</td>
</tr>
<tr>
<td>C4</td>
<td>2.580</td>
<td>1.00</td>
<td>0.3188</td>
<td>0.8200</td>
<td>1.00</td>
<td>0.1016</td>
</tr>
<tr>
<td>Sum</td>
<td></td>
<td></td>
<td>3.2792</td>
<td>1.4153</td>
<td></td>
<td>1.4153</td>
</tr>
</tbody>
</table>

Eq. (9) yields

$$P_{cr} = \frac{3.2792}{1.20 \times 1.4153} \frac{EI}{h^2} = 1.931 \frac{EI}{h^2}$$

which has an error of –3.5%. Effective length multipliers $\beta$, which are calculated by means of Eq. (2), are shown and compared with the exact values in Table 4.

Table 4: Effective length multipliers for Numerical Example

<table>
<thead>
<tr>
<th>Column</th>
<th>$\beta$ (Exact)</th>
<th>$\beta$ (Prop. Method)</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.906</td>
<td>0.923</td>
<td>1.8</td>
</tr>
<tr>
<td>C2</td>
<td>1.812</td>
<td>1.846</td>
<td>1.8</td>
</tr>
<tr>
<td>C3</td>
<td>1.404</td>
<td>1.430</td>
<td>1.8</td>
</tr>
<tr>
<td>C4</td>
<td>2.219</td>
<td>2.261</td>
<td>1.8</td>
</tr>
</tbody>
</table>

All the effective length parameters are on the safe side. It is interesting to note that the effective lengths (and errors) of all the columns are the same due to the fact that they are computed by using the same equation used for exact calculations, namely Eq. (2).

Buckling load calculations are repeated by using wind and earthquake loadings for the same frame, and the errors on effective lengths are found as 2.8% and 3.0%, respectively.

6.2 Other examples

Using the proposed procedure, the irregular examples of Section 4.2 are solved and the errors encountered in effective length multipliers are shown in Table 5.

Table 5: Errors on effective length multipliers

<table>
<thead>
<tr>
<th>Type</th>
<th>Relative error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>1.8</td>
</tr>
<tr>
<td>b</td>
<td>3.0</td>
</tr>
<tr>
<td>c</td>
<td>1.8</td>
</tr>
<tr>
<td>d</td>
<td>1.6</td>
</tr>
<tr>
<td>e</td>
<td>0.7</td>
</tr>
<tr>
<td>f</td>
<td>2.5</td>
</tr>
<tr>
<td>g</td>
<td>2.9</td>
</tr>
<tr>
<td>h</td>
<td>1.6</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.7</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.0</td>
</tr>
<tr>
<td>Average</td>
<td>2.0</td>
</tr>
</tbody>
</table>
It is seen that all the errors are on the safe side and in the acceptable range. All the irregular examples are solved again by using wind and earthquake loadings as well and the corresponding error orders are shown in Table 6.

Table 6: Errors for wind and earthquake loadings (%)

<table>
<thead>
<tr>
<th>Type</th>
<th>Wind</th>
<th>Earthquake</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>2.8</td>
<td>3.0</td>
</tr>
<tr>
<td>b</td>
<td>3.0</td>
<td>2.2</td>
</tr>
<tr>
<td>c</td>
<td>2.2</td>
<td>2.7</td>
</tr>
<tr>
<td>d</td>
<td>0.7</td>
<td>1.7</td>
</tr>
<tr>
<td>e</td>
<td>-1.0</td>
<td>0.1</td>
</tr>
<tr>
<td>f</td>
<td>0.3</td>
<td>-1.9</td>
</tr>
<tr>
<td>g</td>
<td>3.0</td>
<td>2.0</td>
</tr>
<tr>
<td>h</td>
<td>1.9</td>
<td>0.4</td>
</tr>
<tr>
<td>Minimum</td>
<td>-1.0</td>
<td>-1.9</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Average</td>
<td>1.9</td>
<td>1.8</td>
</tr>
</tbody>
</table>

It is seen that, the error orders do not vary considerably due to the kind of lateral loading, and they are on the safe side for the great majority of all the cases.

6.3 Other approximate methods

All the improvement methods mentioned in Section 1 are developed for regular frames and most of them use “Storey buckling approach”. Since it is not possible to define a “Storey” at least for certain levels of irregular frames, these methods are not applicable to them. The only improvement method, which considers all the frame columns, is the [Hellesland and Bjorhovde, 1997] approach. The methods using the “Fictitious loading approach” of [Cakiroglu, 1962] and [Horne, 1975] are also applicable to irregular frames by applying slight modifications. The irregular examples of Section 4.2, have once again solved by using these three methods and the errors encountered are shown in Table 7.

Table 7: Errors on effective length multipliers for approximate methods

<table>
<thead>
<tr>
<th>Type</th>
<th>Cakiroglu</th>
<th>Horne</th>
<th>Hellesland-Bjorhovde</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>13.1</td>
<td>-0.8</td>
<td>-3.0</td>
</tr>
<tr>
<td>b</td>
<td>8.8</td>
<td>-7.5</td>
<td>3.5</td>
</tr>
<tr>
<td>c</td>
<td>8.6</td>
<td>-4.2</td>
<td>3.8</td>
</tr>
<tr>
<td>d</td>
<td>16.6</td>
<td>8.2</td>
<td>0.1</td>
</tr>
<tr>
<td>e</td>
<td>12.8</td>
<td>-2.4</td>
<td>11.7</td>
</tr>
<tr>
<td>f</td>
<td>10.2</td>
<td>7.9</td>
<td>3.5</td>
</tr>
<tr>
<td>g</td>
<td>2.0</td>
<td>9.4</td>
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<tr>
<td>h</td>
<td>-1.9</td>
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</tr>
<tr>
<td>Minimum</td>
<td>-1.9</td>
<td>-7.5</td>
<td>-14.7</td>
</tr>
<tr>
<td>Maximum</td>
<td>16.6</td>
<td>9.4</td>
<td>11.7</td>
</tr>
<tr>
<td>Average</td>
<td>9.3</td>
<td>5.7</td>
<td>5.9</td>
</tr>
</tbody>
</table>

The errors for all the columns of a certain frame are the same for these methods also. It is seen that all the errors encountered are less than the ones found by using code formulae. However, they are generally greater than the errors found by applying the method presented in this paper.

7. CONCLUSIONS

In this paper, determination of effective lengths of multi-storey frame columns is investigated. The main conclusions derived, may be summarised as follows:
1. It is shown that, simplified formulae and diagrams, which are given in several design codes and specifications, may yield rather erroneous results for effective lengths of the columns, especially for irregular frames. This is due to the fact that the code formulae refer only to local stiffness distributions, instead of the overall behaviour of the structure.
2. A simplified procedure for determining an approximate value for system buckling load is developed. The procedure utilises a simple quotient based on the results of a fictitious lateral load analysis. Effective lengths of columns may then be calculated by means of a simple formula.
3. The proposed procedure is applicable to both regular and irregular cases equally easily.
4. The proposed procedure yields errors, which are less than 5% for all the considered examples. This order may be regarded acceptable from the designer’s point of view.
5. The buckling load value is not strongly dependent on the choice of lateral loading. Hence, any existing lateral loading on the frame under consideration may be used without losing a significant amount of accuracy.
6. The proposed procedure is applied to several numerical examples and it is seen that all the errors are in the acceptable range and on the safe side for most of the cases.

8. REFERENCES

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