Non-linear behavior and seismic safety of reinforced concrete structures

K. Girgin & E. Ozer
Istanbul Technical University, Faculty of Civil Engineering, Istanbul, Turkey

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ABSTRACT: A method of load increments is presented for the non-linear static (pushover) analysis and determination of collapse loads of reinforced concrete space frame structures subjected to gravity loads combined with earthquake forces. The method accounts for the non-linearities caused by both the elastic-plastic behavior of reinforced concrete and the effect of geometrical changes on equilibrium equations. The elastic-plastic behavior of reinforced concrete members subjected to combined internal forces is idealized through the concept of plastic section with limited plastic deformation capability. At each step of the load increments method, the yield conditions and the effect of geometrical changes are linearized. The numerical illustration of the method is carried out on a twelve-story reinforced concrete building structure.

1 INTRODUCTION

The use of elastic-plastic analysis and design methods, which consider the non-linear behavior of reinforced concrete as well as the non-linearity caused by geometrical changes, may result in more realistic and economical solutions especially for structures built in earthquake zones. Furthermore, the seismic safety of existing reinforced concrete structures can be determined accurately through these methods.

In Figure 1, typical load-deflection diagrams corresponding to the materially non-linear theory are given. As the loads are increased from the initial state, plastic deformations develop in the vicinity of critical sections where internal forces exceed the linear-elastic limit. For reinforced concrete structures which demonstrate sufficient ductility, the plastic deformations can be assumed to accumulate at certain sections, defined as “plastic sections”, while other portions of the structure remain linear-elastic.

Figure 1. Typical load-deflection diagrams.
The first-order load-deflection diagram of a ductile structure, for which the plastic section concept is adopted, is given by curve (I). In the first-order theory, where the effect of geometrical changes on equilibrium equations is neglected, the structure collapses at the “first-order limit load” through the formation of a complete or partial mechanism. This load is denoted by $P_{L1}$.

The typical load-deflection diagram obtained by the second-order theory is represented by curve (II). In this theory, the equilibrium equations are formulated for the deformed shape of structure. When the external loads reach the ultimate value of $P_{L2}$, which is referred to as the “second-order limit load”, the structure collapses through the loss of stability.

In some cases, the structure may be considered as being collapsed at the load level of either $P_{U1}$ or $P_{U2}$, due to excessive deflections and plastic deformations.

The non-linear methods of analysis have two main practical applications, such as

1. to develop design methods which are based on the collapse load criterion (Ziemian et al. 1992) (Orakdogen & Ozer 1996),
2. to determine the factor of safety of existing structures using the collapse load as the basis for comparison (Lu et al. 1977) (Ozer et al. 1993).

2 LOAD INCREMENTS METHOD

In this study, a method of load increments is developed for the non-linear static (pushover) analysis and determination of collapse load of reinforced concrete space frame structures subjected to constant gravity loads and increasing lateral loads. The method accounts for the non-linearities caused by both the elastic-plastic behavior of reinforced concrete and the effect of geometrical changes on equilibrium equations, (Ozer at al. 1996).

2.1 Assumptions

The following assumptions and idealizations are made throughout the study.

1. Ideal elastic-plastic behavior is adopted for reinforced concrete members which demonstrate sufficient ductility.
2. The non-linear axial, flexural and torsional deformations are assumed to accumulate at certain sections, defined as plastic sections, while the remaining part of the structure behaves linearly elastic. The plastic flexural deformations are limited to the plastic rotation capacity. The plastic section concept introduced herein is the extension of conventional plastic hinge hypothesis which is limited to planar members subject to simple bending.
3. The yield conditions are expressed in terms of axial force, bending moments and torsional moment. The effect of shear forces on yield conditions is neglected. Furthermore, by considering that the total longitudinal reinforcement may be divided into two parts, such as torsional and flexural reinforcement, the yield conditions for torsion and biaxial flexure with axial force can be expressed independently. The yield surface for biaxial flexure with axial force can be approximated by planar regions.
4. The plastic deformation (yield) vector is assumed to be normal to the yield curve, for the case of flexure with axial force.
5. Second-order theory is applied to the analysis of slender structures. In this theory, equilibrium equations are formulated for the deformed configuration while the effect of geometrical changes on compatibility equations is ignored.

2.2 Principles of the method

In the load increments method developed in this study, the structural system is analyzed under factored constant gravity loads and proportionally increasing lateral loads. Thus, at the end of the analysis, the factor of safety against lateral earthquake or wind loads is determined under the anticipated safety factor for gravity loads.
When the gravity loads acting on the structure are known, then the member axial forces can be easily estimated by using the equilibrium equations. Thus, the second-order effects (P-Δ effects) are taken into account by calculating the elements of stiffness and loading matrices through the use of stability functions for the estimated axial forces. If the member axial forces obtained at the end of the analysis vary significantly from the estimated values, the analysis is repeated. However, since the axial forces are dependent to the equilibrium equations rather than the compatibility conditions, such a repetition is not required in most cases, especially for multistory building structures.

In the proposed method, the structure is analyzed for successive lateral load increments. A given load increment is terminated when the state of internal forces at a certain critical section reaches the limit state defined by the yield condition, i.e. when a plastic section forms. Since the yield vector is assumed to be normal to the yield surface, the plastic deformation components for a plastic section can be expressed by a single “plastic deformation parameter” which is introduced as a new unknown for the next lateral load increment. Besides, an equation is added to the system of equations to express the incremental yield condition. This equation is also linear, because the yield surface is approximated by planar regions.

Since the system of equations corresponding to the previous load increment has already been solved, the solution for the current load increment is obtained simply by the elimination of the new unknown.

A successive approximations procedure may be applied to structures which have substantial torsional moments. In the first step of this procedure, the analysis is performed by omitting the torsional plastic sections. If any critical sections at which the torsional moment exceed the torsional plastic moment are detected, the analysis is repeated by considering these sections as potential plastic sections. When the torsional plastic sections found at the end of the analysis are same as the estimated ones, the calculation is terminated.

2.3 Idealization of non-linear behavior of reinforced concrete members

In the course of this study, the internal force-deformation relationships and the yield conditions of reinforced concrete members under combined internal forces are idealized as explained in the following.

The theoretical and idealized bending moment-curvature (M-χ) relationships for reinforced concrete sections with flexure and axial force are shown in Figure 2. The idealized bending moment-curvature relationship is assumed to be composed of two straight lines, such as

\[ OA : M = EI\chi \quad \text{for} \quad 0 \leq \chi \leq \chi_{L1} \]

\[ AB : M = M_p \quad \text{for} \quad \chi_{L1} < \chi \leq \chi_{L2} \] (1)

where \(M_p\) is the ultimate bending moment capacity for a given axial load level of \(N = N_o\), \(\chi_{L1}\) and \(\chi_{L2}\) are the curvatures corresponding to the first yielding and the ultimate state, respectively.

For the case of biaxial flexure with axial load, the above idealization may similarly be applied in both directions.

As illustrated in Figure 2, the flexural rigidity of the cross-section is defined as

\[ EI = \frac{M_p}{\chi_{L1}} \] (2)

In the OA portion of the idealized M-χ curve, the section behaves linearly elastic. When the bending moment at a critical section reaches the ultimate value of \(M_p\), plastic deformations occur. For reinforced concrete members which demonstrate sufficient ductility, the plastic deformations are assumed to be concentrated in the vicinity of the critical section. The plastic rotation which is defined as the sum of these plastic flexural deformations is limited to the plastic rotation capacity.
The plastic rotation capacity for reinforced concrete members is expressed by

$$\theta_{p,\text{max}} = (\chi - \chi_{L1}) l_p$$

where $l_p$ is the equivalent length of plastic section over which the plastic curvature is considered to be constant.

As it can be seen from Equation 3, the plastic rotation capacity is mainly dependent to strain-stress relationships of reinforced concrete and reinforcing steel, maximum concrete strain of $\varepsilon_{cu}$, amount and configuration of transversal confining reinforcement, cross-sectional dimensions and shape of bending moment diagram.

Various empirical expressions have been proposed by several investigators for the equivalent length of the plastic section $l_p$ and the maximum concrete strain $\varepsilon_{cu}$ at ultimate curvature (Park and Paulay 1975).

In this study, the torsional moment-twisting angle $(M_y - \phi)$ relationship is also idealized by two straight lines, such as

$$OA : M_y = GJ \phi \quad \text{for} \quad 0 \leq \phi \leq \phi_1$$

$$AB : M_y = M_{yp} \quad \text{for} \quad \phi_1 < \phi \leq \phi_{\text{max}}$$

where $M_{yp}$ is the ultimate torsional moment capacity and $GJ$ is the torsional rigidity of the uncracked cross-section, Figure 3.

In order to prevent excessive torsional deformations, it is recommended that the maximum twisting angle $\phi_{\text{max}}$ should not exceed the limiting value of 0.001 radian (Ersoy 1987).

The generalized yield condition for a ductile reinforced concrete frame element may be written as

$$0, \ldots, (\mathbf{M}, \mathbf{N}, \mathbf{T}, K) = 0$$

where $\mathbf{M}$ and $\mathbf{N}$ are bending moments, $\mathbf{M}_t$ is torsional moment, $\mathbf{N}$ is axial force, $\mathbf{T}$, $\mathbf{T}$ are shear forces and $K$ is a non-linear function of internal forces.

Neglecting the effect of shear forces, this condition becomes

$$K(M_x, M_z, M_y, N, T_y, T_z) = 0$$

Considering that the total longitudinal reinforcement may be divided into two parts, such as flexural and torsional reinforcement, the interaction between flexure and torsion can be ignored. Thus the yield conditions become
Figure 3. Idealized torsional moment-twisting angle relationship.

\[ K_1(M_x, M_z, N) = 0 \] (7)

and

\[ K_2(M_y) = M_y - M_{yp} = 0 \] (8)

where \( K_1 \) is a non-linear function of bending moments and axial force and \( M_{yp} \) is ultimate torsional moment capacity, which is also called as “torsional plastic moment”.

Under increasing loads, when internal forces at a critical section reach the ultimate state defined by the yield conditions, a plastic section develops and finite plastic deformations occur. The state of internal forces is not allowed to violate the yield conditions.

In the case of biaxial flexure with axial force, this property is stated as

\[ \frac{\partial K_1}{\partial M_x} dM_x + \frac{\partial K_1}{\partial M_z} dM_z + \frac{\partial K_1}{\partial N} dN = 0 \] (9)

where

\[ \frac{\partial K_1}{\partial M_x}, \frac{\partial K_1}{\partial M_z}, \frac{\partial K_1}{\partial N} \]

denote the partial derivatives of function \( K_1(M_x, M_z, N) \) with respect to \( M_x, M_z \) and \( N \).

The plastic deformations which develop at plastic sections are represented by the “yield vector” \( d(\phi_x, \phi_z, \Delta) \) in which \( \phi_x, \phi_z \) and \( \Delta \) are the plastic deformation components in the directions of \( M_x, M_z \) and \( N \), respectively.

It is known from the plastic theory that, for a stable elastic-plastic material, the yield vector \( d \) is normal to the yield surface (Hodge 1959). This property is approximately valid for reinforced concrete sections under uniaxial and biaxial flexure combined with axial force.

If the yield vector is assumed to be normal to the yield surface, the plastic deformation components may be expressed in terms of a single parameter, as in the following.

\[ \phi_x = \phi \frac{\partial K_1}{\partial M_x}, \quad \phi_z = \phi \frac{\partial K_1}{\partial M_z}, \quad \Delta = \phi \frac{\partial K_1}{\partial N} \] (10)

The parameter \( \phi \) is called as “plastic deformation parameter”.

In order to linearize the structural behavior, the yield surface is approximated by

\[ K_1(M_x, M_z, N) \equiv A_1 M_x + A_2 M_z + A_3 N + B = 0 \] (11)
where $A_1$, $A_2$, $A_3$ and $B$ are constants which depend on the material and cross-sectional characteristics and the amount and configuration of reinforcement.

According to this approximation, Equation 9 yields

$$
\Delta K_i = A_i \Delta M_1 + A_2 \Delta M_2 + A_3 \Delta N = 0
$$

(12)

where $\Delta M_1, \Delta M_2$ and $\Delta N$ denote the incremental bending moments and the axial force caused by a given load increment.

Finally, combining Equations 10 and 11, the plastic deformation components may be expressed by

$$
\phi_x = \phi_1 \frac{\partial K_1}{\partial M_1} = \phi A_1, \quad \phi_z = \phi_2 \frac{\partial K_2}{\partial M_2} = \phi A_2, \quad \Delta = \phi \frac{\partial K_3}{\partial N} = \phi A_3
$$

(13)

In this study, a linearized yield condition is proposed for reinforced concrete members subject to biaxial flexure and axial force. This yield surface, which is composed of 24 planar regions, is shown in Figures 4, 5 (Girgin 1996).

![Figure 4. Perspective views of idealized yield surface.](image)

![Figure 5. Projections of idealized yield surface on $M_x$-$M_z$ plane.](image)
2.4 Mathematical Formulation

At each step of the proposed method, a structural system with several plastic sections is analyzed for a lateral load increment.

Utilizing the conventional Matrix Displacement Method and making necessary modifications to account for the plastic sections, the unknowns are considered to be composed of two groups, such as

1. Nodal displacement components,
2. Plastic deformation parameters which represent the finite plastic deformations developed in plastic sections.

The equations are also considered in two groups.

1. Equilibrium equations of nodes in the directions of nodal displacement components.
2. Incremental yield conditions for plastic sections, as given by Equation 12.

It is clearly seen that the number of unknowns is equal to the number of equations.

Referring to system coordinates, the equilibrium equations of nodes may be written in matrix form, as

\[
[S_{dd}] [d] + [S_{dp}] [\phi] + [P_o] = [q]
\]  

(14)

The matrices given in Equation 14 are as follows:

- \([d]\) : unknown nodal displacements vector,
- \([q]\) : vector of nodal external loads,
- \([P_o]\) : vector of fixed-end forces due to member external loads.

If the number of nodes is designated by \(n\), the number of elements of vectors \([d]\), \([q]\) and \([P_o]\) is \(6n\) for space frames.

\([S_{dd}]\) : system stiffness matrix obtained by omitting the plastic sections. This matrix is composed of individual member stiffness matrices and is in the order of \(6n\) for space structures.

The above matrices are determined through the conventional Matrix Displacement Method. In the second-order analysis however, the elements of matrices \([S_{dd}]\) and \([P_o]\) should be calculated by means of the stability functions.

The matrices which are introduced to account for the effect of plastic deformations on equilibrium equations are defined below:

- \([\phi]\) : unknown plastic deformation parameters vector. If the number of plastic sections is designated by \(m\), this vector has \(m\) elements.
- \([S_{dp}]\) : a \(6n \times m\) matrix which represents the effect of imposed unit plastic deformations on the equilibrium equations. The elements of this matrix can be obtained through superposition (Ozer 1987) (Girgin 1996).

The incremental yield conditions express that the states of internal forces at the plastic sections satisfy the yield conditions during the given load increment. These conditions, given by Equation 12, may be written in matrix form as

\[
[S_{\phi\phi}] [d] + [S_{\phi\phi}] [\phi] + [P_{\phi}] = [0]
\]  

(15)

where

- \([S_{\phi\phi}]\) : a \(m \times 6n\) matrix which represents the effect of nodal displacement components on the incremental yield conditions.

It can be proved by the Betti’s reciprocal theorem that, as long as the plastic deformation vector is normal to the yield surface, this matrix is equal to the transpose of matrix \([S_{dp}]\).
\[
[S_\phi] : \text{a square matrix of order } m. \text{ A typical column of this matrix consists of elements which represent the change in the state of internal forces, i.e.}
\]
\[
A_1 \Delta M_x + A_2 \Delta M_z + A_3 \Delta N \quad \text{or} \quad \Delta M_y
\]
\[
(16)
\]
\] at the plastic sections due to the imposed unit plastic deformation of \( \phi_k = 1 \).

\[
P_{\phi_k} : \text{a vector with } m \text{ elements. The typical element } (P_{\phi_k})_k \text{ of this vector represents the change in the state of internal forces at the plastic section } k, \text{ due to the member external loads.}
\]

The elements of matrices \([S_\phi]\), \([P_\phi]\) are calculated through the equilibrium equations written on the deformed geometry of the member considered (Girgin 1996).

According to the preceding discussion, it can be concluded that the extended system of linear equations which corresponds to a structural system with \( n \) nodes and \( m \) plastic sections becomes
\[
\begin{bmatrix}
[S_{\phi_d}] & [S_{\phi_d}] & \begin{bmatrix} d \\ \theta \end{bmatrix} + [P_{\phi_d}] \\
[S_{\phi_d}] & [S_{\phi_d}] & \begin{bmatrix} d \\ \theta \end{bmatrix} + [P_{\phi_d}]
\end{bmatrix} = \begin{bmatrix} q \\ 0 \end{bmatrix}
\]
\[
(17)
\]

3 ILLUSTRATIVE EXAMPLE

The load increments method developed herein has been utilized to study the non-linear behavior of a twelve-story reinforced concrete building structure up to the first- and second-order limit loads.

The typical floor formwork plan and the system elevation are given in Figures 6 and 7, respectively. In Figure 7, working gravity and earthquake loads are also summarized.

![Figure 6. Typical floor formwork plan.](image)
Roof dead load: 5.25 kN/m²  live load: 1.50 kN/m²
Floor dead load: 6.75 kN/m²  live load: 2.00 kN/m²
Exterior wall load: 5.00 kN/m

Figure 7. System elevation, working gravity and earthquake loads.

First, the building is designed in accordance with the current Turkish reinforced concrete standard and seismic code. As required by these codes, two factored load conditions, such as

1.4D+1.6L  
1.0D+1.0L+1.0E  
(D: dead loads, L: live loads)  
(E: earthquake loads)

are considered in the design procedure. Class C25 concrete (characteristic strength, f\text{ck}=25 \text{ MPa}) and S420 reinforcing steel (yield stress, f\text{yk}=420 \text{ MPa}) are used in the design.

Strength reduction factors of 1.50 and 1.15 are applied for concrete and reinforcing steel, respectively.

The design has been carried out by means of a computer program. The size of beams is chosen as 300×700 mm for the entire building. The column dimensions are determined as given in Table 1. As seen from the table, the column sizes are kept constant in three consecutive stories. The cross-sectional dimensions and the thickness of box-shaped core wall are 3000×3000 mm and 300 mm, respectively.

Table 1. Column dimensions(mm/mm).

<table>
<thead>
<tr>
<th>Story</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A1</td>
</tr>
<tr>
<td>12 -10</td>
<td>400 x 400</td>
</tr>
<tr>
<td>9 -7</td>
<td>400 x 400</td>
</tr>
<tr>
<td>6 -4</td>
<td>500 x 500</td>
</tr>
<tr>
<td>3 -1</td>
<td>500 x 500</td>
</tr>
</tbody>
</table>
After the members have been proportioned to satisfy the code requirements, the structural system is analyzed under constant working gravity loads (1.0D+1.0L) and proportionally increasing earthquake loads. The analysis is carried out by means of a computer program (PARCS-3D) developed for the practical applications of the load increments method. Both the first- and second-order non-linear analyses have been performed.

The results of non-linear analyses have shown that the lateral load parameters (ultimate lateral loads divided by working lateral loads) corresponding to the first- and second-order limit loads are

\[ P_{L1} = 1.578 \quad \text{and} \quad P_{L2} = 1.456 \]

The lateral load parameter versus lateral displacement diagrams obtained through the first- and second-order analyses are given in Figure 8a.

At each step of the load increments, the plastic rotations and plastic twisting angles are calculated and compared with the plastic rotation and twisting angle capacities. The plastic rotation capacities are determined by assuming that the equivalent length of a plastic section is \( l_p = 0.5h \) (h: depth of cross-section) and maximum concrete strain at ultimate curvature is \( \varepsilon_{cu} = 0.006 \).

In the second-order analysis, it is observed that the plastic rotation at \( (A) \) axis end of fourth story, \( (2) \) axis beam is reached to the plastic rotation capacity of \( \theta_p = 0.009516 \) at the lateral load parameter of \( P_{U2} = 1.436 \).

Then, by definition, the second-order collapse load is obtained. The lateral load parameter versus plastic rotation diagram for this plastic section is plotted in Figure 8b.

A summary of numerical results is presented in Table 2.

![Figure 8. Lateral load parameter versus lateral displacement and plastic rotation curves.](image)

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Ultimate Lateral Load Parameter</th>
<th>Number of Plastic Sections</th>
<th>Lateral Load Parameter for The First Plastic Section</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Total</td>
<td>Beam Flexure</td>
</tr>
<tr>
<td>1</td>
<td>First-order</td>
<td></td>
<td>400</td>
</tr>
<tr>
<td></td>
<td>P_{L1} = 1.578</td>
<td></td>
<td>243</td>
</tr>
<tr>
<td>2</td>
<td>Second-order</td>
<td></td>
<td>281</td>
</tr>
<tr>
<td></td>
<td>P_{L2} = 1.456</td>
<td></td>
<td>219</td>
</tr>
</tbody>
</table>

(F): flexure (T): torsion.
Column 2 of this table shows the ultimate lateral load parameters corresponding to the first- and second-order limit loads and collapse loads. In columns 3-7, the total number of plastic sections and their distribution among beams, columns and core wall are given. Similarly, columns 8-10 show the lateral load parameters for the first plastic sections developed in each type of structural element.

The plastic section pattern corresponding to the second-order limit load is given in Figure 9 where solid squares represent the torsional plastic sections. The locations of the first and last plastic sections are also indicated.

Figure 9. Plastic section pattern.

The following conclusions can be drawn from the results of the numerical study.
1. The twelve-story reinforced concrete building structure, which is designed in accordance with the current Turkish codes, has achieved a minimum safety factor of 1.436 for earthquake loads under working gravity loads. Considering the load and resistance factors imposed by the codes, this safety factor is found to be sufficient.
2. Several torsional plastic sections occur before the limit load is reached.
3. The limit load is reduced by 7.7% when second-order effects are considered in the analysis.
4. The plastic rotation capacities are exceeded before the limit load level is attained. The ratio of collapse loads to limit loads varies between 0.954 and 0.986 for the first- and second-order analyses, respectively.
5. The first plastic sections develop in beams in the vicinity of working lateral load level.

4 CONCLUSIONS

A method of load increments for the non-linear static analysis and determination of second-order limit load and collapse safety of reinforced concrete space frame structures is presented. The
method considers both the geometrical non-linearity and the non-linear behavior of reinforced concrete. The classical plastic hinge concept is extended to account for the plastic deformations caused by the combined action of internal forces.

In the course of the study, an effective computer program (PARCS-3D) has been developed for the practical applications of the method. The non-linear analysis of large framed structures can be handled by the use of this program.

The proposed method may be adopted to a comprehensive study of non-linear behavior of building structures designed in accordance with the current codes as well as the seismic safety assessment of existing buildings built in earthquake zones.

REFERENCES


