Homework 2 Solutions

1. Establish the validity of the following arguments using the rules of inference.

a.

p
$p \rightarrow q$
$\neg q \vee r$

$\therefore r$

1. p
2. $p \rightarrow q$
3. q
4. $\neg q \vee r$
5. r

b.

$(\neg p \vee q) \rightarrow r$
$r \rightarrow (s \vee t)$
$\neg s \wedge \neg u$
$\neg u \rightarrow \neg t$

$\therefore p$

1. $(\neg p \vee q) \rightarrow r$
2. $r \rightarrow (s \vee t)$
3. $(\neg p \vee q) \rightarrow (s \vee t)$
4. $\neg s \wedge \neg u$
5. $\neg u \rightarrow \neg t$
6. $\neg u$
7. $\neg t$
8. $\neg(s \vee t) \rightarrow \neg(\neg p \vee q)$
9. $\neg s \wedge \neg t$
10. $\neg(s \vee t)$
11. $\neg(\neg p \vee q)$
12. $p \wedge \neg q$
13. p

c. $p \rightarrow (q \rightarrow r)$
$p \vee s$
$t \rightarrow q$
$\neg s$

$\therefore \neg r \rightarrow \neg t$

1. $p \vee s$
2. $\neg s$
3. p
4. $p \rightarrow (q \rightarrow r)$
5. $q \rightarrow r$
6. $t \rightarrow q$
7. $t \rightarrow r$
8. $\neg r \rightarrow \neg t$

d. p
$p \rightarrow r$
$p \rightarrow (q \vee \neg r)$
$\neg q \vee \neg s$

$\therefore \neg s$

1. p
2. $p \rightarrow r$
3. r
4. $p \rightarrow (q \vee \neg r)$
5. $q \vee \neg r$
6. q
7. $\neg q \vee \neg s$
8. $\neg s$

2. Write each of the following arguments in symbolic form. Then establish the validity of the argument or give a counter example to show that it is invalid.

a. If Rochelle gets the supervisor's position and works hard, then she'll get a raise. If she gets the raise, then she'll buy a new car. She has not purchased a new car. Therefore either Rochell did not get the supervisor's position or she did not work hard.

p: Rochelle gets the supervisor's position.
q: Rochelle works hard.
r: She gets a raise.
s: She buys a new car.

$(p \wedge q) \rightarrow r$
$r \rightarrow s$
$\neg s$

$\therefore \neg p \vee \neg q$

1. $(p \wedge q) \rightarrow r$
2. $r \rightarrow s$
3. $p \wedge q \rightarrow s$
4. $\neg s$
5. $\neg s \rightarrow \neg (p \wedge q)$
6. $\neg (p \wedge q)$
7. $\neg p \vee \neg q$

b. If Dominic goes to the racetrack, then Helen will be mad. If Ralph plays cards all night, then Carmela will be mad. If either Carmela or Helen gets mad then their lawyer Veronica will be notified. Veronica has not heard from either of these two clients. Consequently, Dominic did not make it to the racetrack and Ralph didn't play cards all night.

p: Dominic goes to the racetrack.
q: Helen will be mad.
r: Ralph plays cards all night.
s: Carmela will be mad.
t: Veronica will be notified.

$p \rightarrow q$
$r \rightarrow s$
$(q \vee s) \rightarrow t$
$\neg t$

$\therefore \neg p \wedge \neg r$

1. $(q \vee s) \rightarrow t$
2. $\neg t$
3. $\neg(q \vee s)$

4. $\neg q \wedge \neg s$
5. $p \rightarrow q$
6. $\neg q \rightarrow \neg p$
7. $\neg q$
8. $\neg p$
9. $r \rightarrow s$
10. $\neg s \rightarrow \neg r$
11. $\neg s$
12. $\neg r$
13. $\neg p \wedge \neg r$

c. If there is a chance of rain or her red headband is missing, then Lois will not mow her lawn. Whenever the temperature is over 20°C there is no chance for rain. Today the temperature is 22°C and Lois is wearing her red headband. Therefore Lois will mow her lawn.

p: There is a chance of rain.
q: Lois' red headband is missing.
r: Lois will mow her lawn.
s: The temperature is over 20°C.

$(p \vee q) \rightarrow \neg r$
$s \rightarrow \neg p$
$s \wedge \neg q$

$\therefore r$

The argument is invalid.
From the premises we get: $\neg(p \vee q)$
But $(p \vee q) \rightarrow \neg r$ does not implie $\neg(p \vee q) \rightarrow r$