## Nature-Inspired Computing

Genetic Algorithms

Dr. Şima Uyar
September 2006

## Genetic Algorithms

- components of a GA
- representation for potential solutions
- method for creating initial population
- evaluation function to rate potential solutions
- genetic operators to alter composition of offspring
- various parameters to control a run


## Genetic Algorithms

- parameters of a GA
-no. of generations
- or other stopping criteria
- population size
- chromosome length
- probability of applying some operators


## Simple GA

Simple GA - SGA

- a.k.a. Canonical GA
- Operators of a SGA
- selection
- cross-over
- mutation

SGA
generate initial population

## repeat

evaluate individuals
perform reproduction select pairs recombine pairs apply mutation until end_of_generations

## Representation \& Encoding

- population size constant
- individual has one chromosome (haploid)
- chromosome length constant
- individual has a fitness value
- binary genes (0/1)
- generational


## Initial Population

random initial population

$$
\Downarrow
$$

each gene value for each individual determined randomly to be either 0 or 1
with equal probability

## Example Problem: One-Max

Objective: maximize the number of 1 s in a string of length 5, composed only of 1s and 0s
$\Rightarrow$ population size $=4$
chromosome length $=5$
fitness function $=$ no. of genes that are 1

## Example Population

individual 1:
chromosome $=11001$
fitness $=3$
individual 3:
chromosome $=11111$
fitness $=5$
individual 2:
chromosome $=00001$
fitness = 1
individual 4:
chromosome $=01110$
fitness $=3$

## Reproduction

- consists of
- selection
- mating pool (size same as population)
- possibly more than one copy of some individuals
- cross-over
- mutation


## Selection

- uses roulette wheel selection
- fitness proportionate
- expected no. of representatives of each individual is proportional to its fitness
prob $_{i}=\frac{\text { fitness }_{i}}{\sum_{j} \text { fitness }_{j}}, j=1 \ldots$ pop.size

| Example Pairing |  |  |
| :---: | :---: | :---: |
| Current mating pool: |  |  |
| mate 1: 11001 (i1) <br> mate 2: 11111 (i3) <br> mate 3: 11111 (i3) <br> mate 4: 01110 (i4) |  |  |
|  |  | Assume: <br> As a result of random drawing (mate 1, mate 3) |
| Pairs: $\quad$ Pair 2. |  | (mate 2, mate 4) |
| Pair 1: | $\frac{\text { Pair 2: }}{1111}$ | are paired off for reproduction. |
| $\begin{aligned} & 11001 \\ & 11111 \end{aligned}$ |  |  |
|  | 01110 |  |

## Example Pairing

mate 1: 11001 (i1)
mate 2: 11111 (i3)
mate 3: 11111 (i3)
mate 4: 01110 (i4)

## Pairs: <br> Pair 1: <br> 11001 <br> 2 <br> 11111

## Example Selection

Current Population: Probability of each individual
i1: 11001, 3 being selected:
i2: 00001, 1
i3: 11111, 5
i4: 01110, 3
Expected copies of each individual in pool:
i1: $(3 / 12 * 4) 1$
i2: $(1 / 12 * 4) 0$
i3: $(5 / 12 * 4) 2$
i4: $(3 / 12 * 4) 1$
pob(i1) $=3 / 12=0.25$
$\operatorname{prob}(i 2)=1 / 12=0.08$
$\operatorname{prob}(i 3)=5 / 12=0.42$ $\operatorname{prob}(i 4)=3 / 12=0.25$

Assume:
wheel is turned 4 times 1 copy of i1
2 copies of i3
1 copy of i4
is copied into mating pool

## Recombination

- new individuals formed from pairs of parents
- one point cross-over
- probability of cross-over: $\mathrm{pc}_{c}$
- a.k.a. cross-over rate
-typically in range [0.5, 1.0]

| One-Point Cross-Over |  |
| :---: | :---: |
| 00000 11000000 <br> (11101/0000011 | 0000000000011 <br> (1110111100000 |

## Example Cross-Over

## Assume $\mathrm{p}_{\mathrm{c}}=1.0$

## for pair 1:

cross-over site: 3
$110 \mid 01 \rightarrow 11011$
$111 \mid 11 \rightarrow 11101$
for pair 2:
cross-over site: 1
$1 \mid 1111 \rightarrow 11110$
$0 \mid 1110 \rightarrow 01111$
the new individuals:
i1: 11011
i3: 11110
i2: 11101
i4: 01111

## Mutation

- bitwise mutation
- probability of mutation: $\mathrm{p}_{\mathrm{m}}$
- a.k.a. mutation rate
- equal probability for each gene
- chromosome of length L; expected no. of changes: $\mathrm{L}^{*} \mathrm{p}_{\mathrm{m}}$
- typically chosen to be small
- depends on nature of problem


## Bitwise Mutation

$10100100110 \longrightarrow 101010101010$


## Population Dynamics

- generational GA
- non-overlapping populations
- offspring replace parents


## Example New Population

## individual 1:

chromosome $=01011$
fitness $=3$
individual 3:
chromosome $=11100$
fitness = 3
individual 2:
chromosome $=11101$
fitness $=4$
individual 4:
chromosome $=01111$
fitness $=4$

## Stopping Criteria

- main loop repeated until stopping criteria met
- for a predetermined no. of generations $\sqrt{ }$
- until a goal is reached
- until population converges


## Convergence

- progression towards uniformity
- gene convergence: when $95 \%$ of the individuals have the same value for that gene
- population convergence: when all genes have converged
- average fitness approaches best


## Example Stopping Criteria

## Case 1:

number of generations $=250$

- loop repeated 250 times
- best individual at each generation found
- overall best individual becomes solution


## Example Stopping Criteria

## Case 2:

objective: maximize the number of 1 s
goal: to find the individual with 1 for all gene locations

- loop repeated forever
- best individual at each generation found
- terminates when goal individual found


## Example Stopping Criteria

Case 3:
$95 \%$ of 4 is 4
gene convergence: 4 individuals must have same value for a gene location
population convergence: 5 gene locations must be converged

Example converged populations:
Example 1: Example 2: Example 3:
i1: 11010 i1: 00000 i1: 11111
2: 11010 i2: 00000 i2: 11111
i3: 11010 i3: 00000 i3: 11111
i4: 11010 i4: 00000 i4: 11111

## Example Problems

## Function Optimization

## Objective:

Find the set of integers $\mathbf{x}_{\mathbf{i}}$ which maximize the function $\mathbf{f}$.

$$
\begin{gathered}
f\left(x_{i}\right)=\sum_{i} x_{i}{ }^{2} \quad i=1,2,3 \\
-512<x_{i} \leq 512
\end{gathered}
$$

## Function Optimization

## Representation:

- 1024 integers in given interval
- 10 bits needed
$0 \quad: 0000000000(-511)$
1 : $0000000001(-510)$
2 : 0000000010 (-509)
1023 : 1111111111 (512)


## Function Optimization

## Example chromosome:

110010101011000000000000000110
$\mathbf{x}_{1} \quad \mathbf{x}_{2} \quad \mathbf{x}_{3}$
$x_{1}=(810-511)=299$
$x_{2}=(768-511)=257$
$x_{3}=(6-511)=-505$
fitness $=(299)^{2}+(257)^{2}+(-505)^{2}$

$$
=410475
$$

| Function Optimization |  |
| ---: | :--- |
| Example chromosome: |  |
| 110010101011000000000000000110 |  |
| $\mathbf{x}_{1} \quad \mathbf{x}_{2} \quad \mathbf{x}_{3}$ |  |
| $\mathrm{x}_{1}$ | $=(810-511)=299$ |
| $\mathrm{x}_{2}=(768-511)=257$ |  |
| $\mathrm{x}_{3}=(6-511) \quad=-505$ |  |
| fitness | $=(299)^{2}+(257)^{2}+(-505)^{2}$ |
|  | $=410475$ |

## Function Optimization

## Individual:

- function has 3 parameters:
$x_{1}, x_{2}, x_{3}\left(-512<x_{i} \leq 512\right)$
- 10 bits for each $\mathbf{x}_{i}$
- chromosome has 30 bits


## Function Optimization

what if $\mathbf{x}_{\mathbf{i}}$ were real numbers?
interval: $-5.12<x i \leq 5.12$

- possible to use binary
- precision of 2 digits after decimal
- use 1024 different integers (divide number by 100)
- use other representations (e.g. real)


## Function Optimization

what if representation has redundancy?
e.g. interval: $-5.4<x_{i}<5.4$

## 0/1 Knapsack Problem

## Objective:

$\max \sum_{i} v_{i} * x_{i} \quad i=1,2, \ldots$ item_count

$$
\text { subject to } \sum_{i} w_{i} * x_{i} \leq W
$$

$x_{i}=0 / 1$ (shows whether item i is in sack or not)

## 0/1 Knapsack Problem

## Example item set:

(1) $w=2, v=10$
(2) $w=6, v=3$
(3) $w=10, v=8$
(4) $w=7, v=16$
(5) $w=4, v=25$

Example knapsack capacity: $\mathbf{W}=12$

Example feasible solutions:
items: $\{1,2,5\} \Rightarrow$ weight $=12$
value= 38
items: $\{3\} \quad \Rightarrow$ weight $=10$
value $=8$
$\Rightarrow$ weight $=11$
items: $\{4,5\} \quad \begin{array}{ll} & \text { veight }=11 \\ & \text { value }=41 \\ \text { items: }\{2\} & \Rightarrow \text { weight }=6\end{array}$
items: $\{2\} \quad \begin{gathered}\Rightarrow \text { weight }=6 \\ \text { value }=3\end{gathered}$

## 0/1 Knapsack Problem

## Representation:

5 items $\Rightarrow$ chromosome length 5

Example chromosomes:
$11001 \Rightarrow$ items $\{1,2,5\}$ included in sack
$00100 \Rightarrow$ items $\{3\}$ included in sack
$00011 \Rightarrow$ items $\{4,5\}$ included in sack
$01000 \Rightarrow$ items $\{2\}$ included in sack

## 0/1 Knapsack Problem

- Can fitness be total weight of subset?
- what if overweight?
- how to handle overweight subsets?
- delete?
- penalize?
- by how much?
- make correction?


## Exercise Problem

In the Boolean satisfiability problem (SAT), the task is to make a compound statement of Boolean variables evaluate to TRUE. For example consider the following problem of 16 variables given in conjunctive normal form:
$F=\left(x_{5} \vee \bar{x}_{12} \vee x_{16}\right) \wedge\left(\bar{x}_{4} \vee \bar{x}_{6}\right) \wedge\left(x_{2} \vee x_{13} \vee \bar{x}_{7} \vee \bar{x}_{9} \vee \bar{x}_{14}\right) \wedge$
$\left(x_{1} \vee \bar{x}_{8} \vee x_{11} \vee x_{15}\right) \wedge\left(x_{3} \vee \bar{x}_{10}\right)$
Here the task is to find the truth assignment for each variable $x_{i}$ for all $i=1,2, \ldots, 16$ such that $F=T R U E$. Design a GA to solve this problem.

| Genetic Algorithms: |
| :---: |
| Representation of Individuals |
|  |

## Binary Representations

- simplest and most common
- chromosome: string of bits - genes: 0 / 1
example: binary representation of an integer
3: 00011
15: 01111
16: 10000


## Binary Representations

problem: Hamming distance between consecutive integers may be $>1$
example: 5 bit binary representation
14: 01110 15: 01111 16: 10000
Probability of changing 15 into 16 by independent bit flips (mutation) is not same as changing it into 14 ! (hamming cliffs)
$\sqrt{ }$ Gray coding solves problem.

## Gray Coding

- Hamming distance 1

Example: 3-bit Gray Code
$\begin{array}{lllllllll}\text { integer } & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7\end{array}$ $\begin{array}{lllllllll}\text { standard } & 000 & 001 & 010 & 011 & 100 & 101 & 110 & 111 \\ \text { gray } & 000 & 001 & 011 & 010 & 110 & 111 & 101 & 100\end{array}$

- algorithms exist for
- gray $\Rightarrow$ binary coding
- binary $\Rightarrow$ gray coding


## Integer Representations

- binary representations may not always be best choice
- another representation may be more natural for a specific problem
- e.g. for optimization of a function with integer variables


## Integer Representations

- values may be
- unrestricted (all integers)
- restricted to a finite set
-e.g. $\{0,1,2,3\}$
- e.g. \{North,East,South,West\}


## Real-Valued / Floating Point Representations

- when genes take values from a continuous distribution
- vector of real values
- floating point numbers
- genotype for solution becomes the vector $\left\langle\mathrm{x}_{1}, \mathrm{x}_{2}, \ldots, \mathrm{x}_{\mathrm{k}}\right\rangle$ with $\mathrm{x}_{\mathrm{i}} \in \mathfrak{R}$


## Permutation Representations

- deciding on sequence of events
- most natural representation is permutation of a set of integers
- in ordinary GA numbers may occur more than once on chromosome - invalid permutations!
- new variation operators needed


## Permutation Representations

- two ways to encode a permutation
- ith element represents event that
happens in that location in a sequence
- value of $i$ th element denotes position in sequence in which ith event occurs



## Permutation Representations

- two classes of problems
- based on order of events
- e.g. scheduling of jobs
- Job-Shop Scheduling Problem
- based on adjacencies
- e.g. Travelling Salesperson Problem (TSP)
- finding a complete tour of minimal length
between $n$ cities, visiting each city only once


## Permutation Representations

## Example (TSP):

4 cities $A, B, C, D$ and permutation $[3,1,2,4]$ denotes the tours:
first encoding type:
$[C \rightarrow A \rightarrow B \rightarrow D]$
second encoding type:
$[B \rightarrow C \rightarrow A \rightarrow D]$

## Mutation

- a variation operator
- create one offspring from one parent
- acts on genotype
- occurs at a mutation rate: $\mathrm{p}_{\mathrm{m}}$ - behaviour of a GA depends on $\mathrm{p}_{\mathrm{m}}$


## Bitwise Mutation

- flips bits
$-0 \rightarrow 1$ and $1 \rightarrow 0$
- setting of $\mathrm{p}_{\mathrm{m}}$ depends on nature of problem
- usually (expected occurence) between 1 gene per generation and 1 gene per offspring


## Bitwise Mutation (Binary Representations)

$10100100110 \longrightarrow 1001101010010$

## Integer Representations:

Random Resetting

- bit flipping extended
- acts on genotype
- mutation rate: $\mathrm{pm}_{\mathrm{m}}$
- a permissible random value chosen
- most suitable for cardinal attributes


## Integer Representations:

## Creep Mutation

- add small (positive / negative) integer to gene value
- random value
- sampled from a distribution
- symmetric around 0
- with higher probability of small changes


## Integer Representations: Creep Mutation

- designed for ordinal attributes
- acts on genotype
- mutation rate: $\mathrm{pm}_{\mathrm{m}}$

Floating-Point Representations: Mutation

- allele values come from a continuous distribution
- previously discussed mutation forms not applicable
- special mutation operators required

Floating-Point Representations: Mutation Operators

- change allele values randomly within its domain
- upper and lower boundaries
- $\mathrm{U}_{\mathrm{i}}$ and $\mathrm{L}_{\mathrm{i}}$ respectively
$<x_{1}, x_{2}, \ldots, x_{n}>\rightarrow<x_{1}^{\prime}, x_{2}^{\prime}, \ldots, x_{n}^{\prime}>$ where $x_{i}, x_{i}^{\prime} \in\left[L_{i}, U_{i}\right]$


## Floating-Point Representations:

 Non-Uniform Mutation with a Fixed Distribution- most common form
- analogous to creep mutation for integer representations
- add an amount to gene value
- amount randomly drawn from a distribution

Floating-Point Representations: Non-Uniform Mutation

- Gaussian distribution (normal distribution)
- with mean 0
- user-specified standard deviation
- may have to adjust to interval $\left[\mathrm{L}_{\mathrm{i}}, \mathrm{U}_{\mathrm{i}}\right]$

Floating-Point Representations: Non-Uniform Mutation

- Gaussian distribution
$-2 / 3$ of samples lie within one standard deviation of mean
- most changes small but probability of very large changes >0
- Cauchy distribution with same standard deviation
- probability of higher values more than in gaussian distribution


## Floating-Point Representations:

 Non-Uniform Mutation- usually
- applied to each gene with probability 1
- $\mathrm{p}_{\mathrm{m}}$ used to determine standard deviation of distribution
- determines probability distribution of size of steps taken


## Permutation Representations: Mutation Operators

- not possible to consider genes independently
- move alleles around in genome
- mutation probability shows probability of a string undergoing mutation



## Permutation Representations: Insert Mutation <br> $1|2| 3|4| 5|6| 7|8| 9 \longrightarrow \quad 1 / 2 / 5|3| 4|6| 7|8| 9$



## Permutation Representations: Inversion Mutation

$12 / 3 / 4 / 56 / 7|8| 9 \longrightarrow$ 1/5/4/3/2|6|7/8|9


## Recombination

- process for creating new individual
- two or more parents
- term used interchangably with crossover
- mostly refers to 2 parents
- crossover rate $\mathrm{p}_{\mathrm{c}}$
- typically in range [0.5,1.0]
- acts on parent pair


## Recombination

- two parents selected randomly
- a r.v. drawn from $[0,1)$
- if value $<\mathrm{p}_{\mathrm{c}}$ two offspring created through recombination
- else two offspring created asexually - copy of parents


## Binary Representations:

 One-Point Crossover

Binary Representations: Uniform Crossover


## Binary Representations: <br> Crossover

- positional bias
- e.g. in 1-point crossover bias against keeping bits at head and tail of string together
- distributional bias
- in uniform crossover bias is towards transmitting 50\% of genes from each parent


## Integer Representations: Crossover

- same as in binary representations
- blending is not useful
- averaging even and odd integers produce a non-integer!


## Floating-Point Representations: Recombination

- discrete recombination
- similar to crossover operators for bitstrings
- alleles have floating-point representations
- offspring $z$, parents $x$ and $y$ value of allele $i$ in offspring:
$z_{i}=x_{i}$ or $z_{i}=y_{i}$ with equal probability


## Floating-Point Representations: Recombination

- intermediate or arithmetic recombination
- for each gene position
- new allele value between those of parents $z_{i}=\alpha x_{i}+(1-\alpha) y_{i}$ where $\alpha$ in $[0,1]$
- new allelele values
- averaging reduces range of values in population


## Floating-Point Representations: Arithmetic Recombination

- sometimes random $\alpha$
- usually constant $\alpha=0.5$
- uniform arithmetic recombination
- 3 types
- simple a. r.
- single a. r.
- whole a. r.

Floating-Point Representations: Simple Arithmetic Recombination

- pick random recombination point k
child1:
$\left\langle x_{1}, \ldots, x_{k}, \quad \alpha y_{k+1}+(1-\alpha) x_{k+1}, \ldots, \quad \alpha y_{n}+(1-\alpha) x_{n}>\right.$
child2:
$\left\langle y_{1}, \ldots, y_{k}, \quad \alpha x_{k+1}+(1-\alpha) y_{k+1}, \ldots, \quad \alpha x_{n}+(1-\alpha) y_{n}\right\rangle$


Floating-Point Representations: Single Arithmetic Recombination

- pick a random allele $k$
child1:
$\left\langle x_{1}, \ldots, x_{k-1}, \alpha_{y_{k}}+(1-\alpha) x_{k}, x_{k+1}, \ldots, x_{n}\right\rangle$
child2:
$<\mathrm{y}_{1}, \ldots, \mathrm{y}_{\mathrm{k}-1}, \alpha \mathrm{x}_{\mathrm{k}}+(1-\alpha) \mathrm{y}_{\mathrm{k}}, \mathrm{y}_{\mathrm{k}+1}, \ldots, \quad \mathrm{y}_{\mathrm{n}}>$

Floating-Point Representations: Single Arithmetic Recombination


Example: $k=3, \alpha=0.5$

Floating-Point Representations: Whole Arithmetic Recombination


Note: if $\alpha=0.5$ two offspring are identical!

Floating-Point Representations: Whole Arithmetic Recombination

- most commonly used
- takes weighted sum of alleles from parents

$$
\text { Child } 1=\alpha \cdot \bar{x}+(1-\alpha) \cdot \bar{y}
$$

Child $2=\alpha \cdot \bar{y}+(1-\alpha) \cdot \bar{x}$

## Permutation Representations:

 Recombination- requires specially designed operators
- for adjacency representations
- partially mapped crossover (PMX)
- edge crossover
- for order based representations
- order crossover
- cycle crossover


## Multiparent Recombination

- more than two parents
- may be advantageous for some groups of problems
- not widely used in EC
- approaches grouped as:
- based on allele frequencies
- generalizing uniform crossover


## Multiparent Recombination

- based on segmentation and recombination (e.g. diagonal crossover)
- generalizing n-point crossover
- based on numerical operations on real valued alleles (e.g. the center of mass crossover)
- generalizing arithmetic recombination operators


## Genetic Algorithms: Fitness Functions

## Fitness

- population convergence $\Rightarrow$ fitness range decreases
- premature convergence
- good individuals take over population
- slow finishing
- when average nears best,
- not enough diversity
- can't drive population to optima
i.e. best and medium get equal chances


## Fitness

- Fitness shows how good a solution candidate is
- Not always possible to use real (raw) fitness
- Fitness determined by objective function(s) and constraint(s)
- Sometimes approximate fitness functions needed


## Linear Scaling

$\square$

## If

f: raw fitness and $\mathrm{f}^{\prime}$ :scaled fitness then linear relationship,

$$
f^{\prime}=a f+b
$$

## Linear Scaling

$a$ and $b$ chosen such that
$f^{\prime}{ }_{\text {avg }}=f_{\text {avg }}$ and $f_{\text {max }}^{\prime}=C_{\text {mult }} * f_{\text {avg }}$
where
Cmult: expected no. of copies of best individual in population
(Note: Typically for populations of size 50 to 100, $C_{\text {mult }}=1.2$ to 2 is used.)



## Linear Scaling

- In later runs,
- average close to maximum
- some very bad individuals greatly below population average
$\Rightarrow$ possible negative scaled fitnesses
- Solution: map minimum raw fitness to $f^{\prime}{ }_{\text {min }}=0$


## Sigma Scaling

- developed as improvement to linear scaling
- to deal with negative values
- to incorporate problem dependent information into the mapping
- population average and standard deviation


## Sigma Scaling

$$
f^{\prime}=f+\left(\bar{f}-c^{*} \sigma\right)
$$

c: small integer (usually set to 2 )
$\sigma$ : population's standard deviation

$$
\text { if } f^{\prime}<0 \text { then set } f^{\prime}=0
$$

## Window Scaling

- Scaling window
$f^{\prime}=F-f$ where $F$ is a constant and $F>f(x)$ for all $x$
-scaling window W : determines how often $F$ is updated
- $W>0 \Rightarrow F=\max \{f(x)\}$ for the last $W$ generations
- $\mathrm{W}=0 \Rightarrow$ infinite window size, i.e.
$\mathrm{F}=\max \{\mathrm{f}(\mathrm{x})\}$ over all evaluations

|  |
| :---: |
| Genetic Algorithms: |
| Population Models |
|  |

## Population Models

- generational model
- steady state model


## Generational Model

- population of individuals : size N
- mating pool (parents) : size N
- offspring formed from parents
- offspring replace parents
- offspring are next generation : size N


## Steady State Model

- not whole population replaced
- N : population size ( $\mathrm{M} \leq \mathrm{N}$ )
- $M$ individuals replaced by $M$ offspring
- generational gap
- percentage of replaced
- equal to $M / N$
- competition based on fitness



## Fitness Proportional Selection

- FPS
- e.g. roulette wheel selection
- selection probability depends on absolute fitness of individual compared to absolute fitness of rest of the population


## Selection

- Selection scheme: process that selects an individual to go into the mating pool
- Selection pressure: degree to which the better individuals are favoured
- if higher selection pressure, better individuals favoured more


## Selection Pressure

- determines convergence rate
- if too high, possible premature convergence
- if too low, may take too long to find good solutions

| Selection Schemes |
| :---: |
| - two types: |
| - proportionate |
| - ordinal based |
|  |

Fitness Proportionate Selection

- e.g. roulette-wheel selection (RWS)
- problems with FPS
- premature convergence
- almost no selection pressure when fitness values close together
- may behave differently on transposed versions of same fitness function
- e.g. consider $f(x)$ and $y(x)=f(x)+10$;


## Fitness Proportionate Selection

- solutions
- scaling
- windowing



## Stochastic Universal Sampling

- SUS
- one spin of wheel with m equally spaced arms
- cumulative selection probabilities
$\left[a_{1}, a_{2}, \ldots . . ., a_{m}\right]$


[^0]
## Ranking Selection

- ordinal based
- population sorted by fitness
- selection probabilities based on rank
- constant selection pressure
- how to allocate probabilities to ranks - can be any linear or non-linear function
-e.g. linear ranking selection (LRS)

FPS x LRS

| FPS X LRS |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  Fitness Rank FP LR (s=2) LR (s=1.5) <br> A 1 1 0.1 0 0.167 <br> B 5 3 0.5 0.67 0.5 <br> C 4 2 0.4 0.33 0.33 <br> Sum 10  1.0 1.0 1.0 |  |  |  |  |  |

## Tournament Selection

- ordinal based
- RWS and SUS uses info on whole population
- info may not be available
- population too large
- population distributed on a parallel system
- maybe no universal fitness definition (e.g. game playing, evol. art, evol. design)


## Linear Ranking

- parameter s: $1.0<\mathrm{s} \leq 2.0$
- in generational GA s: no. of expected offspring allotted to best
Assume best has rank $m$ and worst 1
- selection probability of individual with rank i:

$$
p_{\text {sel }}\left(r a n k_{-} i\right)=\frac{(2-s)}{m}+\frac{2 i(s-1)}{m(m-1)}
$$

## Exponential Ranking

- with linear mapping
- range of selection pressure limited
- max $s=2$ (median fitness has 1 chance)
- if wish to select above average more
- exponential ranking
$p_{\text {sel }}\left(\operatorname{rank} \_i\right)=\frac{1-e^{-i}}{c} \quad$ c: normalization factor


## Tournament Selection

- TS
- relies on an ordering relation to rank any n individuals
- most widely used approach
- tournament size k
- if $k$ large, more of the fitter individuals
- controls selection pressure
- $k=2$ : lowest selection pressure


## Tournament Selection

begin
set current_member=1;
while (current_member $\leq m$ ) do
pick $k$ inividuals randomly; select best from $k$ individuals; denote this individual i;
set mating_pool[current_member]=i;
set current_member=current_member+1;
od
end
m : population size $\quad \mathrm{k}$ : tournament size

## Survivor Selection

- a.k.a. replacement
- determines who survives into next generation
- reduces ( $\mathrm{m}+\mathrm{l}$ ) to m
- m population size (also no. of parents)
- I no. of offspring at end of generation
- several replacement strategies


## Age-Based Replacement

- fitness not taken into account
- each inidividual exists for same number of generations
- in SGA only for 1 generation
- e.g. create 1 offspring and insert into population at each generation - FIFO
- replace random (has more performance variance than FIFO; not recommended)


## Fitness-Based Replacement

- uses fitness to select $m$ individuals from $(m+l)$ (m parents, I offspring)
- fitness based parent selection techniques
- replace worst
- fast increase in population mean
- possible premature convergence
- use very large populations or no-duplicates
- elitism
- keeps current best in population
- replaces an individual (worst, most similar, etc )


[^0]:    Stochastic Universal Sampling
    begin
    set current_member=i=1;
    pick uniform $r . v$. $r$ from $[0,1 / \mathrm{m}]$;
    while (current_member $\leq m$ ) do
    while ( $x \leq a[i]$ ) do
    set mating_pool[current_member] =parents[i];
    set $r=r+1 / m$;
    set current_member=current_member +1 ;
    od
    set $i=i+1$;
    od
    end
    m : population size $\quad a_{i}=\sum_{1}^{i} P_{\text {sel }}(i) \quad i=1,2, \ldots, m$

