

Genetic Algorithms

- components of a GA
 - representation for potential solutions
 - method for creating initial population
 - evaluation function to rate potential solutions
 - genetic operators to alter composition of offspring
- various parameters to control a run



Simple GA - SGA

- a.k.a. Canonical GA
- Operators of a SGA
 - selection
 - cross-over
 - mutation



generate initial population
repeat
 evaluate individuals
 perform reproduction
 select pairs
 recombine pairs
 apply mutation
until end of generations

Representation & Encoding

- population size constant
- individual has one chromosome (haploid)
- chromosome length constant
- individual has a fitness value
- binary genes (0/1)
- generational

Initial Population

random initial population

 \downarrow each gene value for each individual determined randomly to be either 0 or 1with equal probability

Fitness Evaluation

- fitness function
 - objective function(s)
 - constraints
- shows fitness of individual

 degree to which solution candidate meets objective
- apply fitness function to individual

Example Problem: One-Max

Objective: maximize the number of 1s in a string of length 5, composed only of 1s and 0s

⇒ population size = 4 chromosome length = 5 fitness function = no. of genes that are 1

Example Population

individual 1:

chromosome = 11001 fitness = 3

. . . .

<u>individual 3:</u> chromosome = 11111 fitness = 5 individual 4: chromosome = 01110 fitness = 3

chromosome = 00001

individual 2:

fitness = 1

Reproduction

consists of

- selection
 - mating pool (size same as population)
 - possibly more than one copy of some
- individúals
- cross-over
- mutation



Examp	ole Selection
Current Population: i1: 11001, 3 i2: 00001, 1 i3: 11111, 5 i4: 01110, 3	Probability of each individual being selected: prob(i1) = 3/12 = 0.25 prob(i2) = 1/12 = 0.08 prob(i3) = 5/12 = 0.42 prob(i4) = 3/12 = 0.25
Expected copies of each individual in poc i1: (3/12*4) 1 i2: (1/12*4) 0 i3: (5/12*4) 2 i4: (3/12*4) 1	Assume: wheel is turned 4 times 1 copy of i1 2 copies of i3 1 copy of i4 is copied into mating pool



Recombination

- new individuals formed from pairs of parents
- one point cross-over
- probability of cross-over: p_c
 a.k.a. cross-over rate
 - typically in range [0.5, 1.0]



Mutation

- bitwise mutation
- probability of mutation: pm – a.k.a. mutation rate
 - equal probability for each gene
 - chromosome of length L; expected
 - no. of changes: L*p_m
 - typically chosen to be small
 depends on nature of problem



Example Mutation

Assume:

as a result of random draws, 1st gene of i1

are found to undergo mutation

and 4th gene of i3 $\begin{array}{l} \text{i1:} \ \underline{1}1011 \rightarrow 01011 \\ \\ \text{i3:} \ 111\underline{1}0 \rightarrow 11100 \end{array}$

Population Dynamics

- generational GA
 - non-overlapping populations
 - offspring replace parents

Example New Population

individual 1: chromosome =01011 fitness = 3 individual 2: chromosome =11101 fitness = 4

individual 3: chromosome =11100 fitness = 3 **individual 4:** chromosome =01111 fitness = 4

Stopping Criteria

- main loop repeated until stopping criteria met
 - for a predetermined no. of generations $\boldsymbol{\sqrt{}}$
 - until a goal is reached
 - until population converges

Convergence

- progression towards uniformity
- gene convergence: when 95% of the individuals have the same value for that gene
- population convergence: when all genes have converged

 average fitness approaches best

Example Stopping Criteria

Case 1:

- number of generations= 250
 - -loop repeated 250 times
 - best individual at each generation found
 - overall best individual becomes solution

Example Stopping Criteria

Case 2:

objective: maximize the number of 1s **goal:** to find the individual with 1 for all gene locations

- -loop repeated forever
- best individual at each generation found
- -terminates when goal individual found

Example Stopping Criteria

<u>Case 3:</u>

95% of 4 is 4

gene convergence: 4 individuals must have same value for a gene location

population convergence: 5 gene locations must be converged

Example converged populations: Example 1: Example 2: Example 1

Example 1:	Example 2:	Example 3:
i1: 11010	i1: 00000	i1: 11111
i2: 11010	i2: 00000	i2: 11111
i3: 11010	i3: 00000	i3: 11111
i4: 11010	i4: 00000	i4: 11111

Example Problems

Function Optimization

Objective:

Find the set of integers $\mathbf{x}_{\mathbf{i}}$ which maximize the function $\mathtt{f}.$

$f(x_i) = \sum_i x_i^2$ i=1,2,3 -512 < $x_i \le 512$



Function Optimization

Individual:

- function has 3 parameters:
 - X_{1}, X_{2}, X_{3} (-512 < $x_{i} \le 512$)
- 10 bits for each x_i
- chromosome has 30 bits



Function Optimization

what if x_i were real numbers? interval: $-5.12 < xi \le 5.12$

- possible to use binary
 - precision of 2 digits after decimal
 use 1024 different integers (divide
 - number by 100)
- use other representations (e.g. real)



0/1 Knapsack Problem

Example item set:

(1) w= 2, v=10 (2) w= 6, v= 3 (3) w=10, v= 8 (4) w= 7, v=16 (5) w= 4, v=25	Example feas items: {1,2,5} items: {3} items: {4,5}	
Example knapsack capacity: W = 12	items: {2}	value = 41 $\Rightarrow weight = 6$ value = 3

0/1 Knapsack Problem

Representation:

5 items \Rightarrow chromosome length 5

Example chromosomes:

0/1 Knapsack Problem Can fitness be total weight of subset? what if overweight? how to handle overweight subsets? delete? penalize? by how much? make correction?

Exercise Problem

In the Boolean satisfiability problem (SAT), the task is to make a compound statement of Boolean variables evaluate to TRUE. For example consider the following problem of 16 variables given in conjunctive normal form:

 $F = (x_5 \lor \overline{x}_{12} \lor x_{16}) \land (\overline{x}_4 \lor \overline{x}_6) \land (x_2 \lor x_{13} \lor \overline{x}_7 \lor \overline{x}_9 \lor \overline{x}_{14}) \land$ $(x_1 \lor \overline{x}_8 \lor x_{11} \lor x_{15}) \land (x_3 \lor \overline{x}_{10})$

Here the task is to find the truth assignment for each variable x_i for all i=1,2,...,16 such that F=TRUE. Design a GA to solve this problem.

Genetic Algorithms: Representation of Individuals

Binary Representations

- simplest and most common
- chromosome: string of bits
 genes: 0 / 1
- **example:** binary representation of an integer
 - 3: 00011
 - 15: 01111
 - 16: 10000



problem: Hamming distance
 between consecutive integers may
 be > 1

example: 5 bit binary representation

14: 01110 15: 01111 16: 10000 Probability of changing 15 into 16 by independent bit flips (mutation) is not same as changing it into 14! (*hamming cliffs*)

 $\sqrt{1}$ Gray coding solves problem.

Gray Coding

• Hamming distance 1 Example: 3-bit Gray Code

integer	0	1	2	3	4	5	6	7
standard	000	001	010	011	100	101	110	111
gray	000	001	011	010	110	111	101	100
algorithms exist for								
– gray \Rightarrow binary coding								
– binary \Rightarrow gray coding								

Integer Representations

- binary representations may not always be best choice
 - another representation may be more natural for a specific problem
- e.g. for optimization of a function with integer variables

Integer Representations

- values may be
 - unrestricted (all integers)
 - restricted to a finite set
 - •e.g. {0,1,2,3}
 - e.g. {North,East,South,West}

Integer Representations

- any natural relations between possible values?
 - obvious for ordinal attributes (e.g. integers)
 - maybe no natural ordering for cardinal attributes (e.g. set of compass points)

Real-Valued / Floating Point Representations

- when genes take values from a continuous distribution
- vector of real values – floating point numbers
- genotype for solution becomes the vector $<\mathbf{x}_1$, \mathbf{x}_2 , ... , $\mathbf{x}_k \!\!>$ with $x_i \!\!\in \! \Re$

Permutation Representations

- deciding on sequence of events

 most natural representation is
 permutation of a set of integers
- in ordinary GA numbers may occur more than once on chromosome

 invalid permutations!
- new variation operators needed

Permutation Representations

- two classes of problems
 - based on order of events
 - e.g. scheduling of jobs - Job-Shop Scheduling Problem
 - based on adjacencies
 - e.g. Travelling Salesperson Problem (TSP)

 finding a complete tour of minimal length between n cities, visiting each city only once

Permutation Representations

two ways to encode a permutation

- *i*th element represents event that happens in that location in a sequence
- value of *i*th element denotes position in sequence in which *i*th event occurs

Permutation Representations

Example (TSP):

4 cities A,B,C,D and permutation [3,1,2,4] denotes the tours:

first encoding type: $[C \rightarrow A \rightarrow B \rightarrow D]$

second encoding type: $[B{\rightarrow}C{\rightarrow}~A{\rightarrow}~D]$

Genetic Algorithms: Mutation

Mutation

- a variation operator
- create one offspring from one parent
- acts on genotype
- occurs at a mutation rate: pm – behaviour of a GA depends on pm

Bitwise Mutation

- flips bits
 - –0 \rightarrow 1 and 1 \rightarrow 0
- \bullet setting of p_m depends on nature of problem
 - usually (expected occurence)
 between 1 gene per generation and 1
 gene per offspring

Bitwise Mutation (Binary Representations)

 $1 0 1 0 0 0 1 0 \longrightarrow 1 0 0 1 0 0 0 0$

Integer Representations: Random Resetting

- bit flipping extended
- acts on genotype
- mutation rate: pm
- a permissible random value chosen
- most suitable for cardinal attributes

Integer Representations: Creep Mutation

- designed for ordinal attributes
- acts on genotype
- mutation rate: p_m

Integer Representations: Creep Mutation

- add small (positive / negative) integer to gene value
 - random value
 - sampled from a distribution
 - symmetric around 0
 - with higher probability of small changes

Integer Representations: Creep Mutation

- step size is important
 - controlled by parameters
 - setting of parameters important
- different mutation operators may be used together
 - -e.g. "big creep" with "little creep"
 - e.g. "little creep" with "random resetting" (different rates)

Floating-Point Representations: Mutation

- allele values come from a continuous distribution
- previously discussed mutation forms not applicable
- special mutation operators required

Floating-Point Representations: Mutation Operators

 change allele values randomly within its domain

 $< x_1, x_2, ..., x_n > \rightarrow < x'_1, x'_2, ..., x'_n >$

- upper and lower boundaries • U_i and L_i respectively

where $x_i, x'_i \in [L_i, U_i]$

Floating-Point Representations: Uniform Mutation

- values of x_i drawn uniformly randomly from the [L_i,U_i]
 - analogous to
 - bit flipping for binary representations
 - random resetting for integer representations
- usually used with positionwise mutation probability

Floating-Point Representations: Non-Uniform Mutation with a Fixed Distribution

- most common form
- analogous to creep mutation for integer representations
- add an amount to gene value
- amount randomly drawn from a distribution

Floating-Point Representations: Non-Uniform Mutation

- Gaussian distribution (normal distribution)
 - with mean 0
 - user-specified standard deviation
 - may have to adjust to interval [L_i,U_i]

Floating-Point Representations: Non-Uniform Mutation

- Gaussian distribution
 - 2/3 of samples lie within one standard deviation of mean
 • most changes small but probability of very large changes > 0
- Cauchy distribution with same standard deviation
 - probability of higher values more than in gaussian distribution

Floating-Point Representations: Non-Uniform Mutation

- usually
 - applied to each gene with probability 1
 - $-p_m$ used to determine standard
 - deviation of distribution
 - determines probability distribution of size of steps taken

Permutation Representations: Mutation Operators

- not possible to consider genes independently
- move alleles around in genome
- mutation probability shows probability of a string undergoing mutation









1 2 3 4 5 6 7 8 9 1 5 4 3 2 6 7 8 9

Genetic Algorithms: Recombination

Recombination

- process for creating new individual
 two or more parents
- term used interchangably with crossover
- mostly refers to 2 parents
- \bullet crossover rate p_{c}
 - -typically in range [0.5,1.0]
 - acts on parent pair



- two parents selected randomly
- a r.v. drawn from [0,1)
- \bullet if value < p_c two offspring created through recombination
- else two offspring created asexually
 - copy of parents







Binary Representations: Crossover

- positional bias
 - e.g. in 1-point crossover bias against keeping bits at head and tail of string together
- distributional bias
 - in uniform crossover bias is towards transmitting 50% of genes from each parent

Integer Representations: Crossover

- same as in binary representations
- blending is not useful
- averaging even and odd integers produce a non-integer !

Floating-Point Representations: Recombination

- discrete recombination
 - similar to crossover operators for bitstrings
 - alleles have floating-point representations
 - offspring z, parents x and y value of allele i in offspring:
 z_i=x_i or z_i= y_i with equal probability

Floating-Point Representations: Recombination

- intermediate or arithmetic recombination
 - for each gene position
 - new allele value between those of parents $z_{i}\text{=}\alpha x_{i}$ +(1- $\alpha)y_{i}$ where α in [0,1]
 - new allelele values
 - averaging reduces range of values in population

Floating-Point Representations: Arithmetic Recombination

- \bullet sometimes random α
- usually constant $\alpha = 0.5$
 - uniform arithmetic recombination
- 3 types
 - -simple a. r.
 - -single a. r.
 - whole a. r.

Floating-Point Representations: Simple Arithmetic Recombination

• pick random recombination point k

child1:

 $<\!\!{\rm x}_1\,,\,\ldots\,,{\rm x}_k\,,\ \alpha {\rm y}_{k+1}\!+\!(1\!-\!\alpha)\,{\rm x}_{k+1}\,,\ldots\,,\ \alpha {\rm y}_n\!+\!(1\!-\!\alpha)\,{\rm x}_n\!>$

child2:

 $<\!\!\mathrm{y}_1,\ldots,\mathrm{y}_k,\ \alpha\!\mathrm{x}_{k+1}\!+\!(1\!-\alpha)\,\mathrm{y}_{k+1},\ldots,\ \alpha\!\mathrm{x}_n\!+\!(1\!-\alpha)\,\mathrm{y}_n\!>$



Floating-Point Representations: Single Arithmetic Recombination

• pick a random allele k





Floating-Point Representations: Whole Arithmetic Recombination

- most commonly used
- takes weighted sum of alleles from parents

 $Child1 = \alpha . \overline{x} + (1 - \alpha) . \overline{y}$

Child $2 = \alpha . \overline{y} + (1 - \alpha) . \overline{x}$





Multiparent Recombination

- more than two parents
- may be advantageous for some groups of problems
- not widely used in EC
- approaches grouped as:
 based on allele frequencies
 generalizing uniform crossover

Multiparent Recombination

- based on segmentation and recombination (e.g. diagonal crossover)
 - generalizing n-point crossover
- based on numerical operations on real valued alleles (e.g. the center of mass crossover)
 - generalizing arithmetic recombination operators

Genetic Algorithms: Fitness Functions

Fitness

- Fitness shows how good a solution candidate is
- Not always possible to use real (raw) fitness
 - Fitness determined by objective function(s) and constraint(s)
 - Sometimes approximate fitness functions needed

Fitness

- population convergence \Rightarrow fitness range decreases
 - premature convergence
 - good individuals take over population
 - slow finishing
 - when average nears best,
 - not enough diversity
 - can't drive population to optima
 - i.e. best and medium get equal chances

Fitness

- Fitness remapping schemes needed
 - Fitness scaling
 - Fitness windowing

Linear Scaling

If

f: raw fitness and f':scaled fitness
then linear relationship,
f'=af+b

Linear Scaling

a and b chosen such that

 $f'_{avg} = f_{avg} \text{ and } f'_{max} = C_{mult} * f_{avg}$ where

C_{mult}: expected no. of copies of best individual in population (*Note:* Typically for populations of size 50 to 100, C_{mult}=1.2 to 2 is used.)











Genetic Algorithms: Population Models

Population Models

- generational model
- steady state model

Generational Model

- population of individuals : size N
- mating pool (parents) : size N
- offspring formed from parents
- offspring replace parents
- offspring are next generation : size N

Steady State Model

- not whole population replaced
- N: population size (M≤N)
 - M individuals replaced by M offspring
- generational gap
 - percentage of replaced
 - -equal to M/N
- competition based on fitness

Genetic Algorithms: Parent Selection

Fitness Proportional Selection

- FPS
- e.g. roulette wheel selection
- selection probability depends on *absolute* fitness of individual compared to *absolute* fitness of rest of the population

Selection

- Selection scheme: process that selects an individual to go into the mating pool
- Selection pressure: degree to which the better individuals are favoured
 - if higher selection pressure, better individuals favoured more

Selection Pressure

- determines convergence rate
 - if too high, possible premature convergence
 - if too low, may take too long to find good solutions

Selection Schemes

- two types:
 - proportionate
 - ordinal based

Fitness Proportionate Selection

- e.g. roulette-wheel selection (RWS)
- problems with FPS
 - premature convergence
 - almost no selection pressure when fitness values close together
 - may behave differently on transposed versions of same fitness function
 - e.g. consider f(x) and y(x)=f(x)+10;

Fitness Proportionate Selection

- solutions
 scaling
 - windowing









Ranking Selection

- ordinal based
- population sorted by fitness
- selection probabilities based on rank
- constant selection pressure
- how to allocate probabilities to ranks

 can be any linear or non-linear function
 e.g. linear ranking selection (LRS)

Linear Ranking

- parameter s: 1.0 < s ≤ 2.0
 in generational GA s: no. of expected offspring allotted to best
 - Assume best has rank *m* and worst *1* selection probability of individual with rank i:

 $p_{sel}(rank_i) = \frac{(2-s)}{m} + \frac{2i(s-1)}{m(m-1)}$

	Fitness	Rank	FP	LR (s=2)	LR (s=1.5)
А	1	1	0.1	0	0.167
В	5	3	0.5	0.67	0.5
С	4	2	0.4	0.33	0.33
Sum	10		1.0	1.0	1.0

Exponential Ranking

with linear mapping

 $p_{sel}(rank_i) = \frac{1 - e^{-i}}{1 - e^{-i}}$

- range of selection pressure limited
 max s=2 (median fitness has 1 chance)
- if wish to select above average more
 exponential ranking

c: normalization factor

Tournament Selection

- ordinal based
- RWS and SUS uses info on whole population
 - info may not be available
 - population too large
 - population distributed on a parallel system
 - maybe no universal fitness definition (e.g. game playing, evol. art, evol. design)

Tournament Selection

- TS
- relies on an ordering relation to rank any n individuals
- most widely used approach
- tournament size k
 - if k large, more of the fitter individuals
 controls selection pressure
 - k=2 : lowest selection pressure

pegin	
set current member=1:	
while (current member	<m)do< th=""></m)do<>
pick k inividuals r	andomly;
select best from k	individuals;
denote this individ	lual i;
set mating_pool[cur	rent_member]=i;
set current_member=	current_member+1;
od	
end	

Genetic Algorithms: Survivor Selection



- a.k.a. replacement
- determines who survives into next generation
 - reduces (m+l) to m
 - \bullet m population size (also no. of parents)
 - \bullet I no. of offspring at end of generation
- several replacement strategies

Age-Based Replacement

- fitness not taken into account
- each inidividual exists for same number of generations

 in SGA only for 1 generation
- e.g. create 1 offspring and insert into population at each generation

 FIFO
 - replace random (has more performance variance than FIFO; not recommended)

Fitness-Based Replacement

- uses fitness to select m individuals from (m+l) (m parents, l offspring)
 - fitness based parent selection techniques
 - replace worst
 - fast increase in population mean
 - possible premature convergence
 - use very large populations or no-duplicates – elitism
 - keeps current best in population
 - replaces an individual (worst, most similar, etc)